

# A DYNAMIC MODEL FOR THE SOIL-FOUNDATION-STRUCTURE INTERACTION IN THE EARTHQUAKE ANALYSIS OF FRAMED STRUCTURES

by  
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## SYNOPSIS

The analysis that follows refers to the aseismic code calculation of the structures based on seismic conventional spectra and the dynamic characteristics of the systems.

The purpose of this paper is the correction of this calculation with the effect of the interaction between structure, substructure and soil.

The way of determining the dynamic characteristics of the structural system is presented by taking into account the manner of static deformation of the structure, substructure and the soil, that is in fact the consideration of the interaction. The interaction effect concerning the dynamic characteristics of the systems without or with damping is discussed with application to the case of framed structures.

The computer analysis of some structural systems has emphasized the extent to which the interaction effect occurs in some real cases.

## INTRODUCTION

In the seismic conventional computation, procedure which is the most used in designing, and adopted by all the codes in the world, the influence of the foundation soil is wholly appreciated considering the effects over the dynamic characteristics of the structure, which directly influences the magnitude of the seismic force. The total effect may be correct for designing in certain situation, but in many other cases these approximations do not correspond to the reality. The way in which the structure and the foundation participate in the interaction with the soil is not emphasized too. This paper follows the same manner of the conventional computation procedure developing particularly the interaction aspects.

It is assumed that the structural system has a finite number of degrees of freedom and it can be divided for using the techniques of finite elements. Also it is supposed that during the seismic structural system and the soil is maintained and the horizontal forces on the contact surface are wholly withstand by friction.

Mechanical models with linear behaviour were adopted for the soil: linear-deformable-homogeneous -isotropic or transversal anisotropic half-space, the Winkler's model the half-space having the elasticity modulus as variable with the depth.

The considerations which are made refer especially to the frame-structures with isolated foundation, beam-foundations and raft foundations.

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This paper is based on certain results obtained previously [1],[2].

#### DYNAMIC ASPECT OF THE INTERACTION

Considering the interaction between the structural system and the soil, the dynamic equations of the movement can be written as:

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{P^*\} + \{Y^*\} \quad (1)$$

where

$[M]$  - masses matrix

$[C]$  - damping matrix

$[K]$  - stiffness matrix of the structural system formed by structure and foundation

$\{q\}$  - generalized displacement vector

$\{\dot{q}\}$  - speeds vector

$\{\ddot{q}\}$  - accelerations vector

$\{P^*\}$  - nodal forces vector

$\{Y^*\} = \begin{Bmatrix} Y \\ 0 \end{Bmatrix}$ ,  $\{Y\}$  being the equivalent nodal forces from the links.

In the case of the free vibrations  $\{P^*\} = \{0\}$  and the vector of equivalent nodal forces from the links may be written according to the rigidity properties of the soil.

$$\{Y^*\} = -[K_T^*] \{q\} \quad \{Y^*\} = \begin{Bmatrix} Y \\ 0 \end{Bmatrix} = - \begin{bmatrix} K_T & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \quad (2)$$

where  $[K_T]$  is the stiffness matrix of the soil;  $\{q_1\}$  is the displacement vector from the nodal points of the substructure;  $\{q_2\}$  the displacement vector of the structure nodal points. If  $N$  is the number of nodal points of the structural system, then  $N_1$  are the nodal points on the substructure, including the connection points between structure and substructure while  $N_2$  are the nodal points exclusively into the substructure.

The pressures on the soil are equivalently substitute by

$$\{\bar{Y}\} = -\{Y\} \quad (3)$$

they being proportional with the displacements  $\{w\}$  on the soil surface in the direction of the substructure nodal points:

$$\{\bar{Y}\} = [K_T] \{w\} = [K_T] \{q_1\} \quad (4)$$

Having in view these considerations, the equations (1) are written

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + ([K] + [K_T^*]) \{q\} = \{0\} \quad (5)$$

or

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K^*] \{q\} = \{0\} \quad (5)$$

where

$$[K^*] = [K] + [K_T^*] \quad [K_T^*] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} + \begin{bmatrix} K_T & 0 \\ 0 & 0 \end{bmatrix} \quad (6)$$

and  $[K^*]$  is called the stiffness matrix of the system made of

structure, substructure and soil. The effect of the interaction appears by the means of the stiffness matrix  $[K]$ , which, as we mentioned above, is different from the stiffness matrix of the structure used when the interaction is ignored, i.e. this difference lies in the fact that it introduces the substructure rigidity and a soil rigidity. The stiffness matrix  $[K_1]$  of the soil depends on the physical and mechanical properties of the soil, of the foundation system and of the adopted idealized model for the soil.

In the case of the free oscillations, neglecting the effect of the damping, the system of the equations (5) becomes,

$$[M] \{\ddot{q}\} + [K^*] \{q\} = \{0\} \quad (7)$$

From the characteristic equation associated to the system (7), which may be written as follows,

$$\det ([K^*]^{-1} [M] - \frac{1}{\omega_j^2} [I]) = 0 \quad (8)$$

where  $[I]$  is the unitary matrix, it results that the pulsations of the structure  $\omega_j$  with  $n$  degrees of freedom, are influenced by the interaction through the stiffness of the system  $[K^*]$  made of structure, substructure and soil.

If the damping effect is taken into consideration, the system of equations (5) being easy to see that the pulsations of the structure are also influenced by the interaction. It is mentioned that, for decoupling the oscillation modes, one can start from the expressions of the generalized displacements  $\{q\}$  by means of the modal matrix  $[\phi]$  of the normalized natural vibration shapes; the orthogonality condition are written as follows

$$[\phi]^T [M] [\phi] = [I] \quad (9)$$

$$[\phi]^T [K^*] [\phi] = [\omega_j^2] \quad (10)$$

$[\omega_j^2]$  being the diagonal matrix of square values of natural frequencies; from the conditions (10) the influence of the interaction on natural frequency may be directly ascertained:

$$[\phi]^T ([K] + [K_1^*]) [\phi] = [\omega_j^2] \quad (11)$$

$[\phi]^T [K_1^*] [\phi]$  expressing the effect of the soil influence.

In order to have  $[\phi]^T [C] [\phi]$  as an diagonal matrix, then the matrix of the damping coefficients  $[C]$ , must be expressed linearly according to  $[M]$  and  $[K]$ ,

$$[C] = \alpha [M] + \beta [K] \quad (12)$$

$\alpha$  and  $\beta$  being coefficients with a physical meaning [3]. Therefore the effect of the interaction is also influencing the damping

$$[C] = \alpha [M] + \beta [K] + \beta [K_1^*] \quad (13)$$

in the term  $\beta [K]$  the substructure effect, being included while  $\beta [K_1^*]$  represents the soil effect, which increases

the damping capacity of the system.

#### CONSIDERATIONS REGARDING THE DETERMINATION OF THE DYNAMIC CHARACTERISTICS AND OF THE SEISMIC LOADS OF THE FRAMED STRUCTURES

Rectangular frames are considered with the following types of foundations:

- isolated rigid foundations under columns,
- foundations in the form of beam network,
- foundations in the form of general rafts.

For isolated foundations the rigidity matrix of the structural system coincides with the rigidity matrix of the framed structures with observation that the connection sections with the foundations have displacements due to the soil deformation.

In case of the foundations in the form of beam network these divide forming beam elements, which include themselves in the rigidity matrix of the structure. In this sense, the structure and the substructure are considered to form a structural system.

In case of the foundations in the form of general rafts, the plate can be replaced equivalently with a beam system, after which the analysis is made in the same way as in the former case.

The proposed analysis method allows the automatic generation of the matrix  $[K]$  based on the existing programs.

The rigidity matrix  $[K_T]$ , is obtained from the soil flexibility matrix  $[\beta]$ ; is a square and nonsingular matrix and the determination of its elements are based on some relations which give the displacements on the soil surface according to the applied forces. The contact surface between the substructure and the soil divides in subdomains (finite elements), their number being equal to the number of nodal points in the structure. On a subdomain the contact pressures are considered as constant and may be replaced equivalently by their resultants depending on the case in point. These considerations allowed the determination of the elements of the flexibility matrix according to the indications given the previous papers [1] [2] automatic computation programmes with consideration of the soil models mentioned above, being drawn up.

According to the design codes used in present, the seismic load is directly proportional with the design seismic intensity of the zone under consideration, being diminished when fundamental period of vibrations is increased. The repartition of the seismic force along the building height is conditioned, apart from the magnitudes of the masses which oscillates on the considered degree of freedom directions, by the natural mode shapes. Therefore the interaction among structure, foundation and soil will influence the dynamic behaviour of the system, which determinates the magnitude of the seismic loads and its distribution along the structure height. The stresses in structural system and in soil are also influenced, by means of the static interaction effect.

## FINAL REMARKS

The analysis carried out by means of computer, have emphasised the interaction effects in case of some structures having different types of substructures and foundation soils. As an example is given a framed structure, for which the foundation system was considered in two variants: a) isolated blocks and b) foundation beams, while the soil was considered of two types differentiated as regard the deformability modulus  $E_0$ . The soil was considered as a linear deformable homogeneous and isotropic half-space. Some characteristic values (maximum bending moments in the structure and substructure, natural period of oscillation, seismic force) are given in Table 1, where it may be seen that interaction leads to changes which can not be ignored.

## BIBLIOGRAPHY

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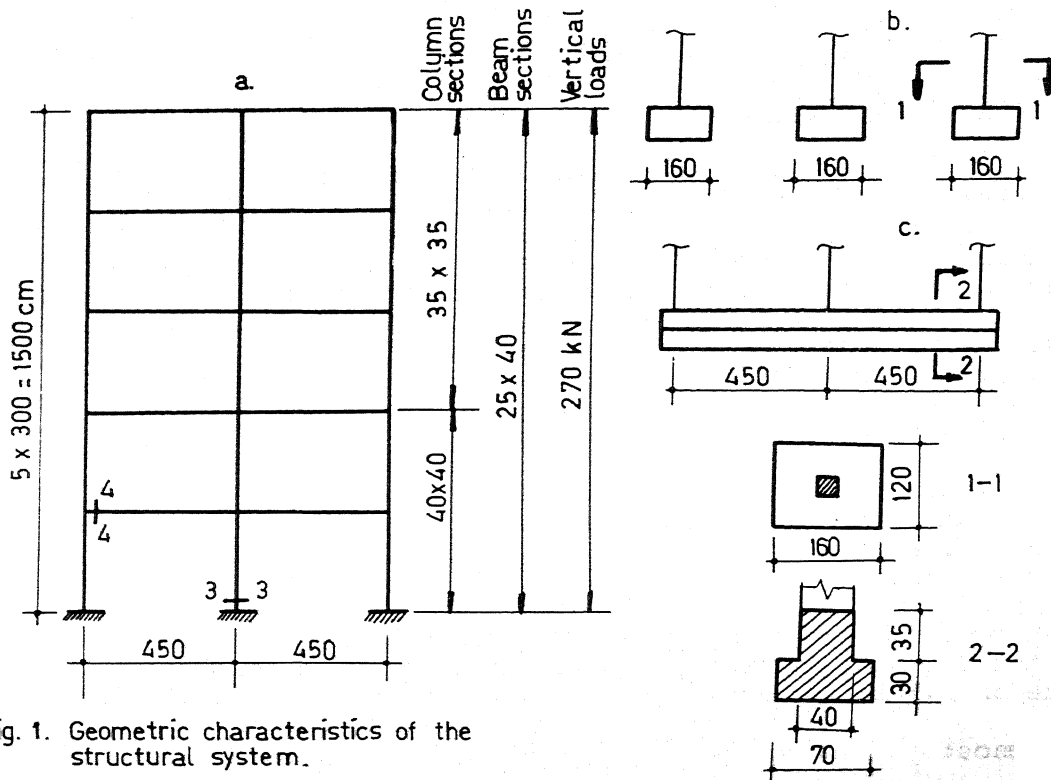


Fig. 1. Geometric characteristics of the structural system.

Table 1.

Type of frame		Frame without interaction		Frame on isolated foundations		Frame on beam foundations	
$\gamma_0 = 0.35$	$E_0, \text{ daN/cm}^2$	100	500	100	500	100	500
Maximal moments in the foundation beam kNm		48,800	40,600	—	—	23,600	25,650
Maximal moments at the base of the central column (3-3) kNm		42,740		14,110	30,670	30,050	36,040
Maximal moments in the marginal beam (4-4), kNm		31,340		29,500	31,900	18,090	22,750
Natural periods sec.	I	0.817		1.269	0.953	1.470	1.166
	II	0.261		0.322	0.289	0.340	0.340
	III	0.146		0.164	0.157	0.195	0.192
Seismic coefficient of the first mode, c%		3.92		2.71	3.55	2.15	2.79
Seismic load of the first mode S, kN		52,930		36,650	47,930	29,040	37,610

## DISCUSSION

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A study of the results given by the authors in table 1 is interesting. In this example, for the structure on beam type of foundation, the effect of soil structure interaction has resulted in a decrease in the seismic force to the extent of 30 to 40 percent of the case without interaction. For the structure on isolated foundations the reduction is not so large. This brings out the **important** fact that when soil - structure interaction is taken into account, the seismic forces developed in a framed structure with continuous foundations is significantly lower than the forces developed in a similar structures on isolated footings. In the authors example, the building on isolated footing has to be designed for approximately 1.25 times the forces in the case of a building on beam foundation. In this context, it would be of interest to note that the Indian Code (1) specifies multiplying factors in the range of 1.0 to 1.5 for buildings on different soil foundation systems. The value of 1.0 is applicable to raft foundations on any type of soil while the highest value of 1.5 is applicable for isolated footings on soft soils.

### Reference:

- (1) IS:1893-1975 - Indian Standard Criteria for Earthquake Resistant Design of Structures.

### Author's Closure

Not received.