DYNAMIC INTERACTION BETWEEN SOIL AND A GROUP OF BUILDINGS

by

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SYNOPSIS

An approximate mathematical model is presented to analyze the soil-structure systems, comprising of single or more superstructures. The soil medium is idealized as two dimensional linear-elastic single layer and discretized by means of nodal points on the surface. This model is capable to analyze the ground compliance of multi-foundation systems. For the cases of single and two identical, in-phase rigid foundations, the ground stiffness matrices are obtained. Through these matrices, first mode behaviour of the soil-structure systems, with single and two identical buildings, is analyzed. Numerical results for the case of two buildings are presented, in comparison with the soil-single building system.

INTRODUCTION

In recent years the dynamic soil-structure interaction problem has interested many investigators, as one of the most important research topics in earthquake engineering. It is interesting to note that in almost all studies, the interactive system has been assumed to have a single superstructure. It is evident that the interaction occurs not only between the soil and the buildings, but also between the neighbouring buildings through the soil. The "cross interaction", as called by Kobori and Minai [1], has a great importance especially in connection with the dense construction conditions in big cities.

In this paper an approximate mathematical model is presented which is capable of taking into account the existance of a group of superstructures on the soil. In order to analyze the problem, the soil medium is idealized as two dimensional, linear-elastic, homogenous, isotropic single layer and discretized by means of nodal points on the surface(Fig.1a). The discretization procedure used herein is essentially the same as used by Chopra and Perumalswami [2], for the discretization of half plane. The plain strain problem is solved for displacement type boundary conditions defined as unit displacements of nodal points on the surface and zero displacements on infinitely rigid bed rock(Fig.1b), thus the displacement and the stress fields in the medium are determined. Then by solving the mixed boundary value problem in discrete form, the ground stiffness matrices of single and two identical in-phase rigid foundations, are obtained. By means of these matrices, the effect of the cross interaction on the first natural frequencies of two identical buildings is analyzed.

DYNAMIC CHARACTERISTICS

The discrete model shown in Fig.1 can be treated as a single "finite element", with theoritically infinite but practically finite number of nodal points. The dynamic stiffness influence coefficients of the medium could be determined as nodal forces related to harmonic displacements with unit amplitude. In this study these coefficients are evaluated in an approximate manner by introducing the utilization of the consistent mass matrix concept.

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As known, this concept depends on the approximation that inertia forces in the medium are proportional to the quasi-static acceleration field. Thus approximate stiffness matrix of the medium is defined as,

$$[k]_d = [k]_s - \omega^2 [m] \tag{1}$$

in which $[k]_d$, $[k]_s$ and $[m]$ represent the dynamic and static stiffness matrices and consistent mass matrix respectively, ω denotes the frequency of steady state vibration. The elements of the static stiffness matrix are obtained as nodal forces related to the unit displacements, from the distributed stresses on the surface which determined by the plain strain solution. The elements of the mass matrix, i.e. inertia influence coefficients are evaluated as nodal inertia forces related to unit accelerations, through the static displacement field in the medium. It has been shown that Eq.1 corresponds to the first two terms of the series solution of the steady state dynamic problem with the exception that zero lower limits of real Fourier inversion integrals are replaced by the nondimensional frequency a_o [3]. The first static term $[k(0)]_s$ does not differ from $[k(a_o)]_s$ for low frequencies, whereas the inertia influence coefficients are determined as functions of a_o. In the frequency range of interest, in connection to the earthquakes, taking the first two terms of the series solution is shown to be sufficient.

This paper is concerned primarily with the approximate modal behaviour of the soil-structure systems. It is known that the method of modal analysis has been a common and traditional approach in practical dynamic analyses. Although the method is simple and not time consuming, it has many serious drawbacks concerning the structural idealizations. Particularly, the modal behaviour of the actual structures is only a theoritical assumption due to the complex nature of energy loss mechanisms in structural materials. On the other hand, from the engineering point of wiew, it can be concluded that the method of modal analysis gives a better understanding of structural behaviour under dynamic loading. In the interaction analysis, the assumption of the soil medium as a half space or a single layer, is a simplified idealization for actual soil conditions. But the simplicity of this kind of idealization gives the possibility of showing the general trends in modal interactive behaviour of the systems. At this point, a serious disadvantage of the model arises, since the theoritical normal modes of vibration does never exist,because of the frequency dependent soil behaviour and the energy loss occuring in the medium due to radiation. It is a known fact that the participation of the first natural mode on the total behaviour of the soil-building systems is of prime importance [4,5]. Consequently, the effect of radiational damping in the lowest frequency of vibration is at the minimum level. Under these circumstances, since the emphasis in this paper is to analyze the vibrational behaviour of the first mode of the soil-building systems through an engineering approach, the radiational energy loss mechanism of the soil medium has not been taken into consideration as a result of the quasi-static approximation.

<u>Discrete Solution of the Mixed Boundary Value Problem:</u> In the analysis of rigid foundations resting on the half space, an important problem arises in the application of mixed boundary conditions, especially where the multiple foundation systems exist. In the case of the discrete model used herein, the problem becomes simple and reduces to the application of the rigid body conditions of motion in terms of nodal displacements and a matrix condensation operation for outer nodes of the foundations.

In a two dimensional rigid body, the field displacements can be expressed in terms of three arbitrarily selected displacement components(Fig.2a, 2b). The relationship between the field and rigid body displacements (Fig.3) can be written as,

$$\{d_x\}_f = [T_x]_f \{d_o\}_f \tag{2}$$

where

$$\{d_x\}_f = \begin{Bmatrix} d_x \\ d_y \\ \varphi_z \end{Bmatrix} \quad , \quad [T_x]_f = \begin{bmatrix} 1 & y/2b & -y/2b \\ 0 & (1-x/b)/2 & (1+x/b)/2 \\ 0 & -1/2b & 1/2b \end{bmatrix} \quad , \quad \{d_o\}_f = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} \tag{3}$$

For a common nodal point j on the contact surface of rigid foundation and soil, Eq.2 can be rewritten as,

$$\{d_j\}_f = [T_j]_f \{d_o\}_f \tag{4}$$

where

$$\{d_j\}_f = \begin{Bmatrix} d_{jx} \\ d_{jy} \end{Bmatrix} \qquad [T_j]_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (1-x_j/b)/2 & (1+x_j/b)/2 \end{bmatrix} \tag{5}$$

Eq.4 is generalized for the complete nodal displacements on the contact surface as,

$$\{d\}_f = [T]_f \{d_o\}_f \tag{6}$$

In the case of single foundation, the swaying-rocking motion produces an antisymmetrical soil deformation, thus the matrix $[T_j]_f$ reduces to

$$[T_j]_f = \begin{bmatrix} 1 & 0 \\ 0 & -x_j/b \end{bmatrix} \tag{7}$$

The first step in the discrete solution of the mixed boundary value problem is to reduce the number of degrees of freedom of the contact surface to the rigid body degrees of freedom of the foundation. In the case of single foundation the dynamic stiffness matrix of the soil medium can be written in partitioned form as,

$$[k] = \begin{bmatrix} [k_{ss}] & [k_{sf}] \\ [k_{fs}] & [k_{ff}] \end{bmatrix} \tag{8}$$

where the subscript (f) corresponds to the nodes under the foundation and (s) refers to the outer nodes (Fig.4). Applying the rigid body conditions of the foundation, the degrees of freedom on the contact surface reduces to two, hence the dynamic stiffness matrix becomes as,

$$[k]^* = \begin{bmatrix} [k_{ss}] & [k_{sf}][T]_f \\ [T]_f^T [k_{fs}] & [T]_f^T [k_{ff}][T]_f \end{bmatrix} \tag{9}$$

The number of degrees of freedom is reduced by half, due to the antisymmetrical deformation with respect to the midpoint of the foundation.

In the case of two identical, in-phase foundations with a given spacing (Fig.5), the dynamic stiffness matrix of the soil medium is,

$$[k] = \begin{bmatrix} [k_{ss}] & [k_{sf}] & [k_{sa}] \\ & [k_{ff}] & [k_{fa}] \\ \text{Sym.} & & [k_{aa}] \end{bmatrix} \tag{10}$$

where the subscript (a) corresponds to the nodes between the foundations. The number of degrees of freedom is also reduced by half due to the antisymmetrical in-phase soil deformation with respect to the midpoint between the foundations. Applying the rigid body conditions of the foundations, the dynamic stiffness matrix of the soil medium is obtained as follows:

$$[k]^* = \begin{bmatrix} [k_{ss}] & [k_{sf}][T]_f & [k_{sa}] \\ & [T]_f^T [k_{ff}][T]_f & [T]_f^T [k_{fa}] \\ \text{Sym.} & & [k_{aa}] \end{bmatrix} \tag{11}$$

The second step in the discrete solution of the mixed boundary value problem is the matrix condensation operation for the degrees of freedom excluding the rigid body degrees of freedom. This operation can be done by a standart condensation procedure. As a result of these operations on the matrices of Eq.9 and Eq.11, (2x2) and (3x3) ground stiffness matrices are obtained respectively. It is worth to note that in the case of single foundation the swaying and rocking degrees of freedom and in the case of two foundations all degrees of freedom are coupled.

Equations of Motion: In the soil-building systems with rigid foundations, the total building displacements can be expressed as a sum of two types of displacements,

$$\{d\}_b^t = \{d\}_b^q + \{d\}_b \tag{12}$$

in which $\{d\}_b^q$ represents the quasi-static building displacements due to both free field motion and the deformation of the soil and $\{d\}_b$ is the relative dynamic displacement vector. The quasi-static displacements of the building are related to the foundation degrees of freedom through

$$\{d\}_b^q = [T]_{bf}\{d_o\}_f \tag{13}$$

in which $[T]_{bf}$ represents the quasi-static transformation matrix of order (3xn), where (n) denotes the number of stories of the building. For a typical story (i), the related row of the matrix is,

$$[T_i]_{bf} = [\,1 \quad h_i/2b \quad -h_i/2b\,] \tag{14}$$

where h_i and b are shown in Fig.3 and Fig.6. The equations of motion of the soil-building system can thus be expressed as,

$$\begin{bmatrix}[m_{ff}] & [m_{fb}]\\ [m_{bf}] & [m_{bb}]\end{bmatrix}\begin{Bmatrix}\{\ddot{d}_o\}_f\\ \{\ddot{d}\}_b\end{Bmatrix} + \begin{bmatrix}[k_{ff}] & [0]\\ [0] & [k_{bb}]\end{bmatrix}\begin{Bmatrix}\{d_o\}_f\\ \{d\}_b\end{Bmatrix} = -\begin{bmatrix}[m_{ff}]\\ [m_{bf}]\end{bmatrix}[U]_h\,\ddot{d}_g \tag{15}$$

in which $[k_{bb}]$ and $[k_{ff}]$ are the lateral stiffness matrix of the building and ground stiffness matrix defined in the previous section, respectively. The off-diagonal stiffness matrices vanish, due to rigid foundation. The foundation mass submatrix is obtained as,

$$m_{ff} = \int_v [T_x]_f^T [I]_f [T_x]_f \, dv + [T]_{bf}^T [m_{bb}][T]_{bf} \tag{16}$$

in which the first term represents the mass matrix of the rigid foundation itself and the second term represents the interactive mass matrix of the foundation related to the quasi-static displacements of the building [3]. $[I]_f$ is the square matrix of order (3x3) defined as,

$$I_f = \begin{bmatrix}\rho_f & 0 & 0\\ 0 & \rho_f & 0\\ 0 & 0 & 0\end{bmatrix} \tag{17}$$

where ρ_f represents the mass density of the foundation material. represents the lumped mass matrix of the building and the off-diagonal mass submatrices are obtained as,

$$[m_{fb}] = [T]_{bf}^T [m_{bb}] \tag{18}$$

which represent the inertia interaction between the building and the foundation soil. In Eq.15 $\ddot{d}_g$ corresponds to horizontal earthquake excitation and $[U]_h$ represents the transformation matrix of lateral motion.

NUMERICAL RESULTS

As a numerical example, an actual ten story building frame is considered where the undamped fundamental frequency on infinitely rigid soil is found to be 7.75 sec^{-1}. Two types of soils are taken into consideration with the shear wave velocities of V_s 100 m/sec. and 300 m/sec. Three cases of spacings of two identical buildings were considered, where the ratios of c/b were taken as 1.5, 1.0 and 0.5. For all cases, the ratio of the total building height to the half width of the foundation was kept constant as 5. The nondimensional thickness of the soil layer was taken as $H/b = 10$ and the hysteretic damping ratio of the building was assumed to be $\beta = 0.01$. The soil-structure systems were excited by harmonic free field motions for each case. The amplitude transfer functions of horizontal displacements of the tenth story and the foundation base of the building were plotted at around the fundamental frequencies of the systems. Fig.7 represents the displacement transfer function of the tenth story of the building on infinitely rigid soil. Figs.8,9 and Figs.10,11 show the transfer functions of the tenth story and the foundation base, respectively. The dotted curves in the figures represent the response of the single building whereas the solid curves correspond to the response of two identical buildings with various spacings. The frequency shifts and corresponding amplitude attenuations between the cases of single and two buildings are appearent. It is observed through numerical analyses that the rocking stiffnesses of the two foundations increase and the swaying stiffnesses decrease with the decreasing spacing. Consequently, as shown in Figs.8 and 9, the fundamental frequencies of the buildings tend to increase due to dominant effect of rocking displacements and the tenth story displacements slightly decrease as the spacing is decreased. Whereas, the amplitudes of the horizontal displacements of the foundations increase with the decreasing spacing, due to relative flexibility of the foundation soil in swaying motion(Figs.10 and 11). Though it is not shown here, for very rigid and low buildings with small height/width ratios, the increase of the horizontal displacements with the decrease of the spacing is appearent and the order of the frequency shifts can reverse according to the relative stiffnesses of the soil and the building.

CONCLUSIONS

In this paper an attemp was made to show the general trends of the interactive behaviour of two building-soil systems. Through numerical results presented herein, the following general remarks can be made:

1) The fundamental frequency of the two building-soil system increases relatively compared with the single building-soil system.

2) The building response due to rocking motion decreases and the response due to swaying motion increases with the decreasing spacing of the two identical buildings. Consequently, "cross interaction" affects rather low buildings with considerable rigidity.

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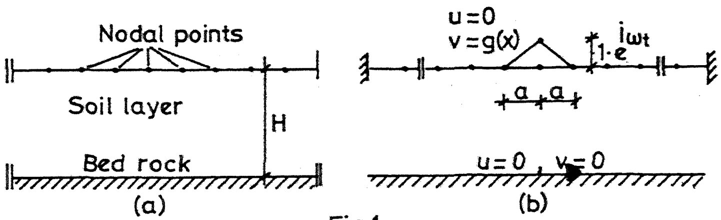

Fig.1

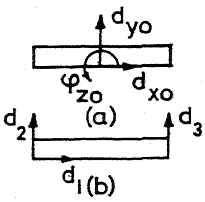

Fig.2

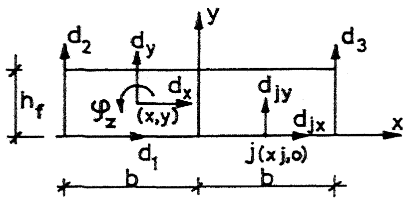

Fig.3

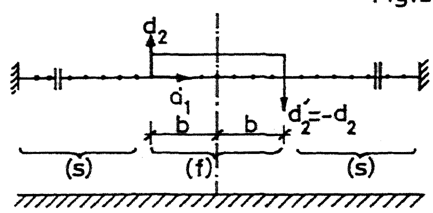

Fig.4

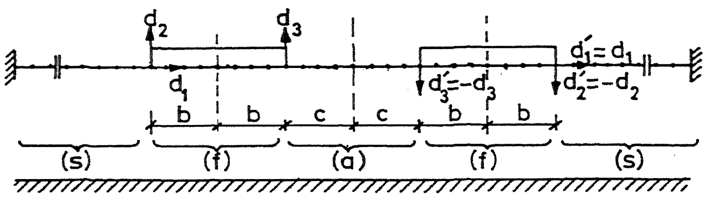

Fig 5

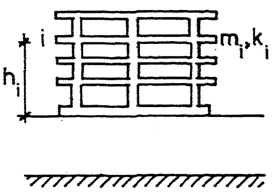

Fig.6

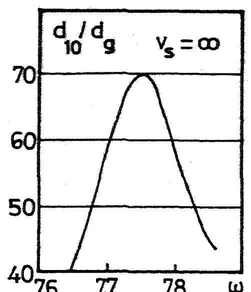

Fig.7

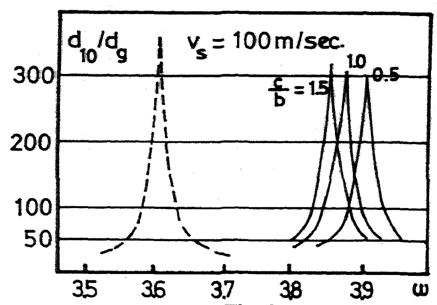

Fig.8

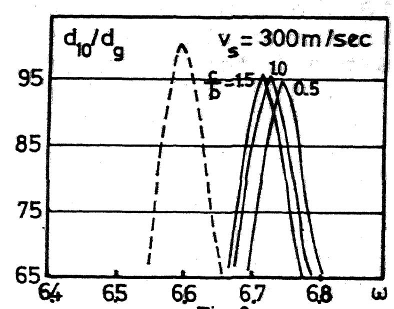

Fig.9

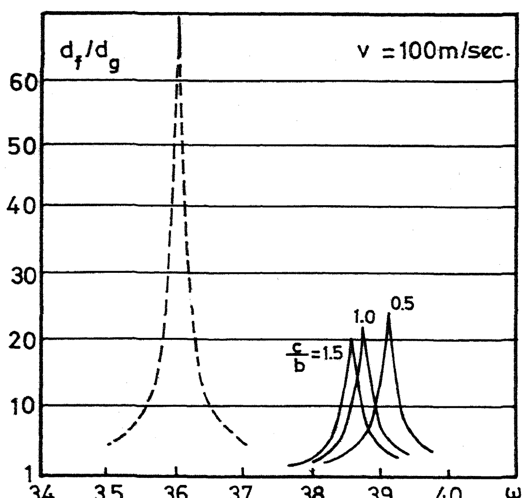

Fig.10

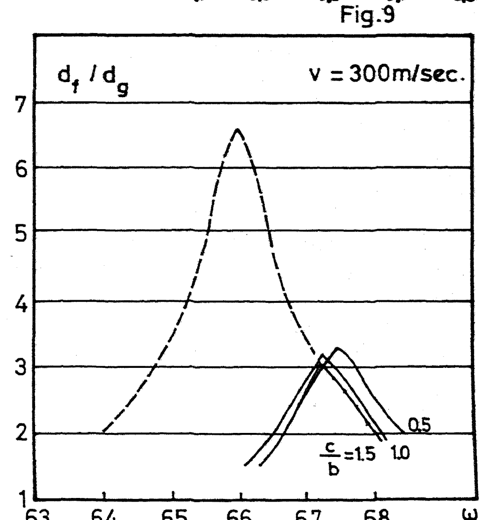

Fig.11