A THEORETICAL APPROACH FOR COMPUTING RADIATION DAMPING IN END BEARING PILE FOUNDATIONS

by

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SYNOPSIS

The radiation damping and the so-called *effective mass* of soil are obtained using the theory of flexible piles embedded in an elastic stratum. The analysis made in this paper is similar to the one used in the half space theory for a rigid plate vibrating vertically on the surface. The study indicates the existance of large values of horizontal radiation damping for frequencies above the fundamental soil frequency, and a zero value for frequencies below it.

INTRODUCTION

The effects of damping on seismic response, particulary around the resonance frequencies, are well known among engineers who study the dynamic properties of structures subjected to seismic forces. Two important effects are: a) the decrease of the amplification curves representing the response to dynamic loads, and b) the shift of the first resonance frequency. It is also known that the total damping is mainly due to the contribution of two parts: 1) hysteretic or internal damping, due to inelastic behaviour of the soil-structure system, and 2) radiation damping, due to dissipation of energy through the waves generated at the site of excitation.

Internal damping is generally determined by carrying out laboratory or field vibratory tests, and by analyzing the resulting hysteretic loops. Radiation damping, on the other hand, is a function of frequency and its evaluation is made through elastic half-space theory.

For the case of pile foundations, the energy dissipation is made through the piles up to certain depth of the soil stratum. Thus, this mechanism of energy dissipation differs considerably from the case of a rigid mass resting on the surface of the half-space.

An approach has been already presented by Novak[2] to determine the radiation damping in piles; however, Novak's method is based on the assumption that the soil is a non-continuous medium, an assumption which is far from reality. An alternative approach, base on Tajimi's theory[4] which considers the soil as a continuous elastic medium, is presented in this paper.

ASSUMPTIONS OF TAJIMI'S THEORY

Using the wave equation for a homogeneous, isotropic, elastic stratum, subjected to transient movements, Tajimi[4] obtained the stresses and displacements of the soil layer. Based

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on these values, he obtained the lateral vibrations of a cylindrical, flexible body, embedded in the stratum. The assumptions of this theory are:

a) the soil is a linear elastic medium resting on bredock

b) the soil has constant viscous damping in the shear mode with depth

c) the vertical displacement of the soil stratum is neglected

d) the pile has a cylindrical section of radius a, and lenght H

e) the head of the pile is rigidly fixed in the superstructure and only undergoes horizontal translation. The tip of the pile is pinned at the bedrock surface

f) the total shear from the superstructure acts on the head of the pile.

RADIATION DAMPING AND EFFECTIVE MASS OF SOIL

The effective mass of soil and radiation damping can be obtained in a similar way in which they are obtained for the half space theory To do this it is necessary to establish the equilibrium equation of the mass on top of the pile. This equation, for cero internal damping, is given by:

$$M\ddot{u}_{pH} + Ve^{i\omega t} = - M\ddot{u}_g \qquad (1)$$

where M is the supported mass, V is the amplitud of the base shear, $\ddot{u}_p$ H the relative acceleration of the pile head, $\ddot{u}_g$ the ground acceleration, and ω is the forced frequency.

As it is shown in another work[1], Eq 1 can be written in the same form as the one obtained for a single degree of freedom system. The following expression is derived

$$M\ddot{u}_{pH} + K_h\ X_2\dot{u}_{pH} + K_h\ X_1\ u_{pH} = P \qquad (2)$$

where $$K_h = \left[\frac{1}{EI}\left(\frac{2H}{\pi}\right)^4 \frac{2}{H}\ F\,(H,0)\right]^{-1} \qquad (3)$$

(This coefficient represents the static spring constant of the soil-mass-pile system)

EI stiffness of the pile

F(H,0) and F(H,ω) are complex functions defined by Tajimi; they represent the amplitud of displacement due to the shear force at the pile's head

$X_1 = f_1$ real part of the complex function :

$$f_1 + if_2 = \frac{F(H,0)}{F(H,\omega)} \; ; \tag{4}$$

the variation of f_1 with ω, is given by Fig 1

$$X_2 = f_2/\omega$$

where f_2 is the imaginary part of the above defined complex function (Eq 4); the variation of X_2 with ω is given by Fig 2.

P is an equivalent force due to the ground acceleration.

Following the same procedure given by Whitman (1966) for the half space theory, the following simplifications can be made in the coefficients of Eq 2:

1) For very low frequencies, make $X_1 = 1$

2) For intermediate frequencies, but less than ω_g the function X_1 may be approximated as a parabola of the form:

$$X_1 = 1 - \Gamma_2 \left(\frac{\omega}{\omega_g}\right)^2 \tag{5}$$

where Γ_2 is a constant which describes the parabola's shape, and ω_g is the fundamental frequency of the soil stratum.

For this case, and considering that

$$-\omega^2 u_{pH} = \ddot{u}_{pH}$$

it follows that Eq 2 can be written as:

$$\left(M + \Gamma_2 \frac{Kh}{\omega g^2}\right) \ddot{u}_{pH} + K_h X_2 \dot{u}_{pH} + K_h u_{pH} = P \tag{6}$$

Thus, the additional mass of soil is given by:

$$Me = \Gamma_2 K_h X_2 / \omega_g^2 \tag{7}$$

and the equivalent damping coefficient by:

$$Ce = K_h X_2 = K_h f_2 / \omega \tag{8}$$

3) *For $\omega > \omega_g$, assume zero effective mass of soil.*

Observe in Fig 2 that, for frequencies below the fundamental soil frequency, there is not radiation damping ($X_2 = 0$ in this interval). A physical interpretation of this result is that, at low frequencies, the system does not generate the waves which dissipate energy.

This fact indicates that the response of flexible structures, founded on soil deposits with a fundamental frequency larger than the natural frequency of the soil-structure system, can be considerably deamplified when piles are used in their foundations. Such is the case, for example, of structures with N>10 (N number of stories) founded on very soft deposits of soil, with $C_s = 100$ m/sec, and a thickness less than 25 m.

For $\omega > \omega_g$, the value of X_2 can be approximated as 0.01. For this last value, the equivalent damping ratio results:

$$D_h = \frac{0.01 K_h}{2\ K_h M} = 0.005 \left(\frac{K_h}{M + \frac{K_h}{\omega^2} \Gamma_2} \right)^{1/2} \tag{9}$$

ILLUSTRATIVE EXAMPLE

Considering the data given by Fig 3, and the value of f_1 given in Fig 1 for H = 20 m, from Eq 5 the value of Γ_2 is obtained:

$$\Gamma_2 = \frac{1 - f_1}{(\omega / \omega_g)^2} = 0.05$$

Thus, this is the value of Γ_2 which describes f_1 in the same way as it is plotted, in Fig 1.

From Eq 3, the value of K_h is determined. Substituting values it is obtained:

$$K_h = 7.8 \times 10^4 \text{ kg / cm}$$

The mass on top of the piles is:

$$M = \frac{50 \times 10^3}{980} = 51 \text{ kg sec}^2 / \text{cm},$$

and the fundamental frequency of the soil, computed with

$$\omega_g = \frac{C_s \pi}{2H}$$

results equal to 5.5 rad/sec.

Therefore, from Eq 7, the equivalent mass of soil is:

$$Me = 0.05 \times \frac{7.8 \times 10^4}{30.2} = 120 \text{ kg sec}^2 / \text{cm},$$

and from Eq 9, neglecting the effective mass of soil for simplicity, the damping ratio for the horizontal mode is:

$$D_h = \frac{0.005 \times (7.8 \times 10^4)^{1/2}}{51^{1/2}} = 0.19$$

Thus

$$D_h = 19\,\%$$

CONCLUSIONS

The main conclusions from this paper are:

1) Considerable horizontal radiation damping exists when the forced frequencies are greater than the fundamental frequency of the soil stratum (ω_g), but very little (theoretically zero) when those frequencies are less than ω_g.

2) A corollary of the above conclusion is that the response of flexible structures, founded on soft deposits of soil with a fundamental frequency larger than the natural frequency of the soil-structure system, can be considerably deamplified when piles are used in their foundations.

Experimental data is needed in this subject to determine the accuracy of theoretical values.

ACKNOWLEDGMENT

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REFERENCES

1) Flores, J R, "Dynamic response of point piles subjected to seismic forces", *Ph D Thesis National University of Mexico, written at MIT (1974)*

2) Novak, M, "Dynamic stiffness and damping of piles", *Can. Geotech. J. 11 (1974) 574-98*

3) Richart, F E and Whitman, R R, "Comparison of footing vibration tests with theory", *Journal of Soil Mechanics and Foundation Div. ASCE,* SM6 (Nov 1967)

4) Tajimi, H, "Dynamic analysis of a structure embedded in an elastic stratum", *Procs IV WCEE,* Chile (1969)

5) Whitman, R V, "Analsis of foundations vibrations", Symposium of Vibration in Civil Engineering, *Butterworths,* London (1966)

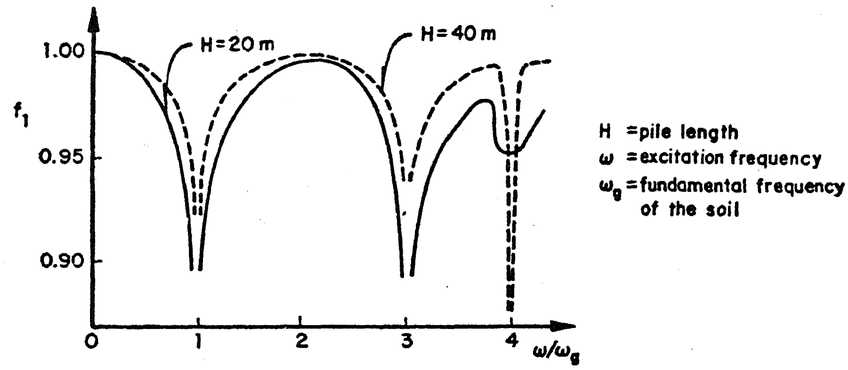

Fig 1. Function f_1 vs. the frequency ratio (ω/ω_g)

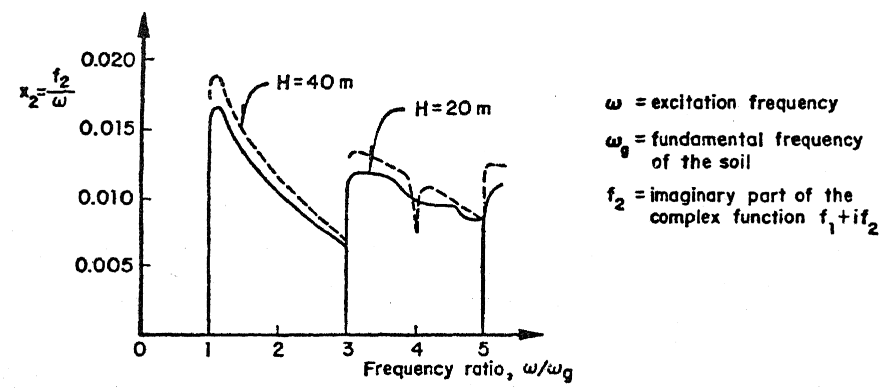

Fig 2. Function X_1 vs. the frequency ratio ω/ω_g

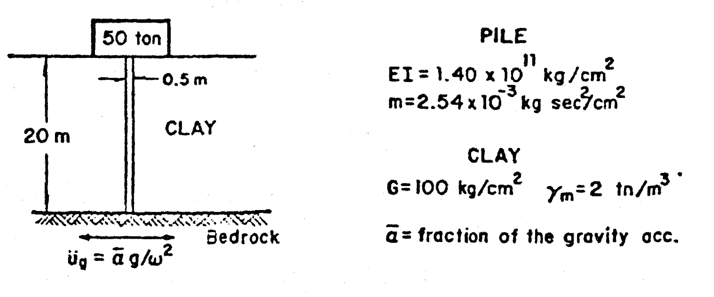

Fig 3. Pile example

DISCUSSION

M. Novak (Canada)

Based on Tajmi's theory the author derives an approximate formula for the equivalent damping ratio of a pile foundation (Eq. 9) and uses the formula to calculate the damping of a 50 ton footing supported by one 0.5 dia pile.

A few comments appear desirable:

1. The author obtains a damping ratio of 19 percent which seems too high.

2. If one just includes the equivalent mass of soil neglected by the author, Eq. 9 yields a damping ratio of about 10 percent instead 19 percent calculated by the author.

3. The footing considered can never respond to horizontal translation as assumed by the author. Coupled motion composed of horizontal translation and rotation in vertical plane will result.

   Two damping ratios are associated with two natural modes of vibration of the footing. Using the writer's theory the damping ratios are about 5 percent for the first mode and 32% for the second.

4. The variations of stiffness and damping with frequency can dramatically differ from that shown by the author in Figs. 1 and 2 if material damping of soil is included. The sharp minima completely disappears due to modest material damping such as tan = 0.1 and significant damping is obtained below the first natural frequency of the soil layer. These facts throw some doubt on the general validity of the author's conclusions.

5. The author misinterpretes the assumptions of the writer's approximate theory (Ref. 2).

The soil reactions assumed in this reference are mathematically accurate for a plane strain case, that is for an infinitely long pile undergoing a uniform translation. Thus, the physical model of soil is continuous.

Author's Closure

Not received.