# DYNAMIC BEHAVIOR OF STRUCTURES WITH PILE-SUPPORTED FOUNDATIONS

*by*

**J. Bielak[I] and V. J. Palencia[I]**

## SYNOPSIS

An analysis is made of the steady-state response of a simple pile-supported system as a means of assessing the effects of various parameters entering into the problem of soil-pile-structure interaction. Numerical results indicate that for linear systems compliance of point-bearing piles is negligible for tall buildings but may be significant for medium tall and squatty structures. Interaction reduces the fundamental frequency of the system but the primary effect is to reduce the peak response. This effect becomes more pronounced as soil deformability increases.

## INTRODUCTION

Present knowledge of the effects of soil-structure interaction on earthquake response of structures is derived mostly from studies of buildings on mat foundations[1,2]. Little information is available concerning the behavior of pile supported structures. Systematic studies of dynamic soil-pile interaction have been conducted under various simplifying assumptions. Both discrete nonlinear models[3], and linear elastic[4,5] and viscoelastic layers[6] have been used to represent the soil. Some of these investigations have served as a basis for the analysis of individual building-foundation[7] and bridge-foundation[3] systems.

In this paper a modified version of an approximate theory developed by Novak[5] for pile-soil interaction is used for analyzing the response of a building on piles. The soil is modeled by a set of independent infinitesimally thin horizontal viscoelastic layers with material damping of the frequency independent hysteretic type; the piles by elastic vertical point-bearing elements of circular cross section, and the building structure by a single-degree damped linear oscillator (Fig.1). The steady-state sinusoidal response is calculated for different values of the system parameters to estimate effects of soil-pile-structure interaction during earthquakes. It is assumed that the excitation at the base of the structure is the same as the free-field surface motion, i.e., that a massless pile cap would follow the soil if it were not attached to the building base.

## DYNAMIC STIFFNESS OF THE FOUNDATION

Fundamental to the problem of soil-structure interaction is the evaluation of the dynamic force-displacement relationship for the foundation. Since the procedure used in this section follows closely that given by Novak[5], only an outline will be presented.

Consider first the case in which a pile is excited by horizontal translation and rotation of its head in the vertical plane. Let the origin of the coordinate system be located at the top of the pile, with the vertical axis pointing downwards toward the bottom of the soil layer. When a pile element $dz$ undergoes a complex horizontal displacement $u(z, t)$ at height $z$, it meets a horizontal soil reaction

$$G(S_{u1} + iS_{u2})\, u\,(z,\, t)\, dz \tag{1}$$

[I] Instituto de Ingeniería, Universidad Nacional Autónoma de México, México, D. F.

in which $G=$ shear modulus of soil. Parameters $S_{u1}$ and $S_{u2}$ represent dynamic stiffness and damping of the soil and are functions of Poisson's ratio $\nu$, the loss factor $D$,[5] and the dimensionless frequency $a_o = \omega r_o (\rho_s/G)^{1/2}$ . Shown by continuous lines in Fig. 2 are the stiffness and damping functions $S_{u1}$ and $S_{u2}$ for $\nu = 0.4, D = 0$, and several values of the pile slenderness ratio $H/r_o$, as functions of $a_o$. These curves have been obtained from the functions given in Ref. 5, but modified[3,4,6] in a neighborhood around $a_o = 0$ and the fundamental frequency of the layer, to which the results from the original theory are not applicable[6]. Functions $S_{w1}$ and $S_{w2}$ corresponding to the vertical vibration of the pile have been similarly obtained. Effects of material damping on the horizontal and vertical soil reactions are shown by broken lines, in Fig. 2. (The correspondence principle of viscoelasticity was used for extending the elastic solutions.) Results indicate that material damping produces an increase in the damping coefficients $S_{u2}$ and $S_{w2}$ beyond the fundamental frequency, and also a reduction in the values of $S_{u1}$ and $S_{w1}$. This reduction may be viewed either as a decrease in the soil stiffness or as an effective mass of soil moving along with the pile.

With soil reactions defined by Eq. 1 for horizontal translation, and a similar equation for vertical motion of the pile, the corresponding equations of motion of the pile can be written and solved in a straightforward manner. Stiffness and damping coefficients can be obtained directly from these solutions, and thence extended to a set of piles[5].

## ANALYSIS OF THE SYSTEM

The system under investigation is shown in Fig. 1. The single-story structure of height $h_1$ is linear, viscously damped and has a base mass supported on piles. The structural base is assumed to be a rigid cylindrical footing, with circular or rectangular cross section. Pile configurations are indicated in Fig. 1b. The idealized structure may be viewed either as a direct model of a one-story building frame, or, more generally, as the first-mode approximation of a multistory structure. In the latter case $m_1$, $k_1$, $c_1$, and $h_1$ must be replaced by the corresponding first-mode generalized quantities.

The base excitation is specified by the free-field motion at the ground surface. Only the effect of ground motion in one direction will be investigated. With these assumptions, the equations of motion for steady-state harmonic response of the building-foundation model shown in Fig. 1 are

$$m_1 \ddot{v}_t + c_1 \dot{v}_1 + k_1 v_1 = 0 \qquad (2a)$$

$$m_1 \ddot{v}_t + m_o(\ddot{v}_o + \ddot{v}_g) + c_v \dot{v}_o + c_{v\phi} \dot{\phi} + k_v v_o + k_{v\phi} \phi = 0 \qquad (2b)$$

$$m_1 h_1 \ddot{v}_t + I_t \ddot{\phi} + c_\phi \dot{\phi} + c_{v\phi} \dot{v}_o + k_\phi \phi + k_{v\phi} v_o = 0 \qquad (2c)$$

In these equations, $v_1$ = horizontal displacement of the top mass relative to the base mass, excluding rotations; $v_g$ = free-field surface displacement; $v_o$ = translation of the base mass relative to the free-field motion; $\phi$ = rotation of base mass, and $v_t$ = total horizontal displacement of top mass with respect to a fixed vertical axis, i.e., $v_t = v_g + v_o + h_1 \phi + v_1$. Functions $k_v$, $k_\phi$, $k_{v\phi}$ and $c_v$, $c_\phi$, $c_{v\phi}$ are the frequency dependent stiffness and damping coefficients of the foundation. These functions are obtained by adding to the terms determined from the previous section the corresponding contributions arising from the contact between the base mass and the soil[8,9].

For steady-state excitation, $\ddot{v}_g(t) = \ddot{v}_g exp(i\omega t)$, Eqs. 2 represent a set of linear algebraic equations that can be solved explicitly once the stiffness and damping coefficients $k_v$, $k_\phi$, $k_{v\phi}$ and $c_v$, $c_\phi$, $c_{v\phi}$ have been determined. Solutions have been obtained for these equations for several combinations of the system parameters; results are shown in Figs.3 to 11.

In plotting the response curves for the system it is convenient to introduce, in addition to the dimensionless parameters defined earlier, the following,

$$\gamma_1 = \frac{k_1 h_1^2}{Ga^3}, \quad \gamma_2 = \frac{h_1}{a}, \quad \gamma_3 = \frac{m_1}{\pi \rho_s a^2 h_1}, \quad \gamma_4 = \frac{m_o}{m_1}, \quad \gamma_5 = \frac{\omega_1 H}{V_p} \qquad (3)$$

in which $a$ = radius of an equivalent circle which has the same area as the actual (circular or rectangular) foundation; $\omega_1$ = circular natural frequency of the superstructure; $V_p$ = longitudinal wave velocity in the piles. $\gamma_1$ = relative stiffness between the superstructure and the soil material; $\gamma_2$ = height ratio of the superstructure; $\gamma_3$ = relative mass density for the structure and the supporting soil; $\gamma_4$ = base mass ratio; and $\gamma_5$ = relative stiffness between superstructure and pile.

Results presented in Figs. 3 to 11 are divided into two sets. The first group (Figs.3 to 7) corresponds to a circular foundation with pile configuration depicted in Fig. 1b while the second (Figs.8 to 11) refers to the rectangular foundation. All the figures have been calculated for a Poisson's ratio $\nu = 0.4$, soil damping factor $D = 0.1$, $\rho_s/\rho_p = 0.7$, $z_c/a = 0.75$, $\gamma_3 = 0.15$, and $\gamma_4 = 2$. These values are intended to be representative of real systems. With these values fixed, the building foundation system shown in Fig. 1 is specified by the dimensionless parameters $\gamma_1$, $\gamma_2$, $\gamma_5$, $b/\ell$, $H/r_o$, $r_o/a$, and the critical damping ratio of the superstructure, $\eta_1$; the dimensionless excitation frequency $\omega/\omega_1$ is the independent variable, and the amplitude of the displacement of the top mass is measured by the ratio $\omega_1^2 |v_1| / \ddot{v}_g$. Also the base displacements $|v_o| / |v_1|$ and $|h_1 \phi| / |v_1|$ will be analyzed.

The three sets of response curves in Fig. 3 illustrate effects on the structural response of the relative stiffness $\gamma_1$, and of the height ratio $\gamma_2$. Note that $\gamma_1$ is independent of the building height $h_1$ when the structural stiffness $k_1$ is inversely proportional to $h_1^2$. The three values of $\gamma_1$ (3, 6 and 15) were chosen to model hard, medium and soft soils, respectively. Effects of relative pile stiffness $\gamma_5$, and radius, $r_o/a$, on the resonant frequency, $\tilde{\omega}_1/\omega_1$, and peak amplitude of the response are shown in Figs. 4 and 5 as functions of $\gamma_1$. Results corresponding to the foundation without piles ($\gamma_5 = \infty$) are also shown, for comparison.

Even though the building-foundation system under study has several degrees of freedom the frequency response curves shown in Fig. 3 resemble those of a simple oscillator. It is, therefore, convenient to define an effective critical damping ratio $\tilde{\eta}_1$ associated with a simple linear oscillator that has the same resonant frequency as the complete interaction system. The value of $\tilde{\eta}_1$ is chosen in such a way that the amplitude of the response of the associated simple oscillator matches that of the complete system both at resonance and for the limiting static value $\omega/\omega_1 = 0$. With these requirements,

$$\tilde{\eta}_1 = \frac{1}{2} \frac{\ddot{v}_g}{\omega_1^2 |v_1| \max} \qquad (4)$$

Values of $\tilde{\eta}_1$, corresponding to various combinations of $\eta_1$, $\gamma_1$ and $\gamma_2$ are shown in Fig. 6. The influence of the extent of contact between the pile cap and the ground surface on the base displacements is illustrated in Fig. 7 for several values of $\gamma_2$ and relative pile stiffness $\gamma_5$. Both perfect bond and complete separation between pile cap and soil are considered.

Comparison of interaction effects for buildings that exhibit different geometrical and dynamic properties in two orthogonal directions is provided by the response curves shown in Fig. 8. In calculating these curves it has been assumed that the natural period of the superstructure along the length of the foundation is two thirds of that in the short direction. Shown in Fig. 9 are resonant frequencies and peak amplitudes of the response corresponding to various pile distributions. To model the response of structures with different numbers of piles, groups of four piles at each of the nine locations have been considered in addition to the original arrangement of nine individual piles. Effects of changes in structural damping, type of contact, and structural and pile slenderness and stiffness are illustrated in Figs. 10 and 11.

## CONCLUSIONS

1. Point-bearing piles on rock effectively increase the stiffness of the foundation, primarily reducing rocking but also affecting the swaying of the base. The natural frequency of the building approaches that corresponding to a rigid base assumption as a result of these actions.

2. While the peak response generally decreases as a result of interaction actual reduction is strongly dependent upon the system parameters. Reductions are negligible for tall structures ($\gamma_2 \geqslant 4$) but become significant for smaller values of $\gamma_2$. Pronounced effects occur for large $\gamma_1$, corresponding to soft soils or to structures that are relatively rigid compared to the soil. Clearly, increases in pile radius and pile stiffness bring the response closer to that corresponding to a rigid base, as does augmenting the number of piles.

Results of the present study are based on a simple building-foundation model. The observation that interaction tends to reduce the maximum response, however, would seem to remain valid for more complex linear soil-structure models, whenever the ordinates in the design spectrum decrease with increasing period in the region of interest. This effect could be important in design, as it would allow the structure to be designed for smaller excitations than would be required under a rigid-base assumption.

## ACKNOWLEDGMENT

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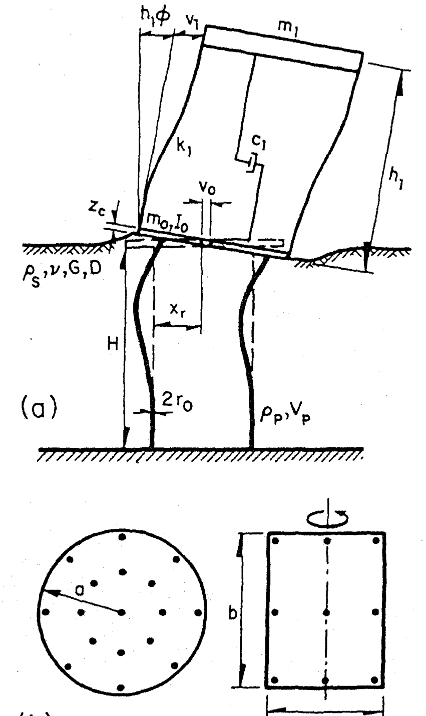



Fig 1

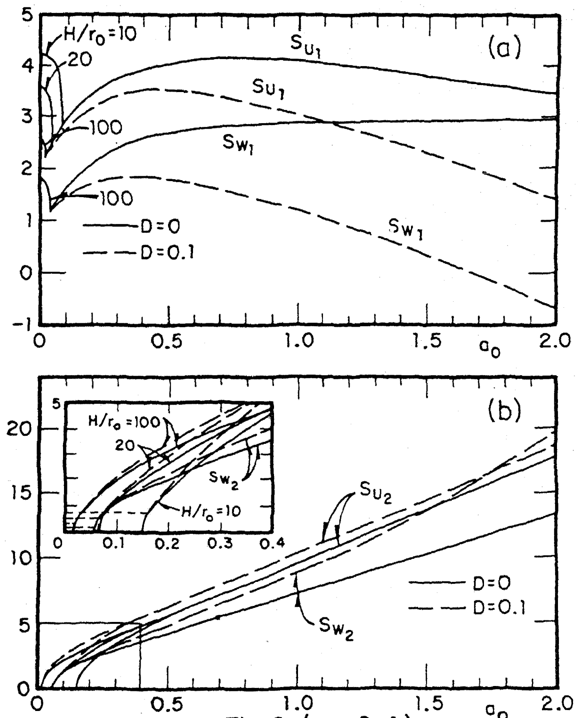



Fig 2 ($\nu = 0.4$)

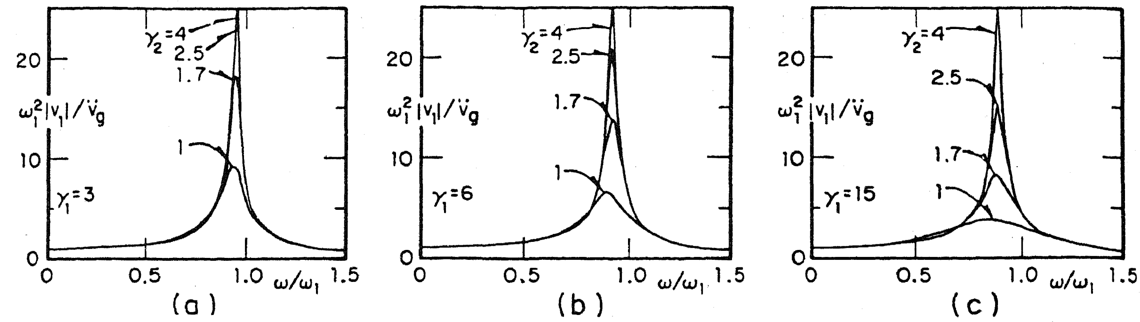
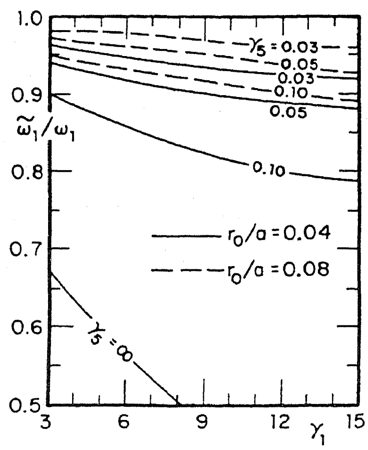



Fig 3

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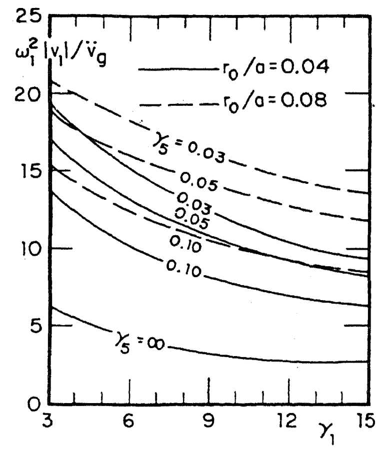


Fig 5

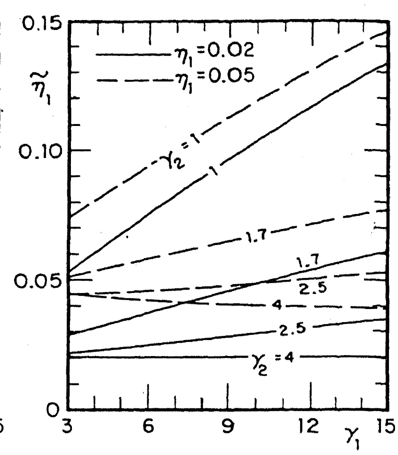


Fig 6

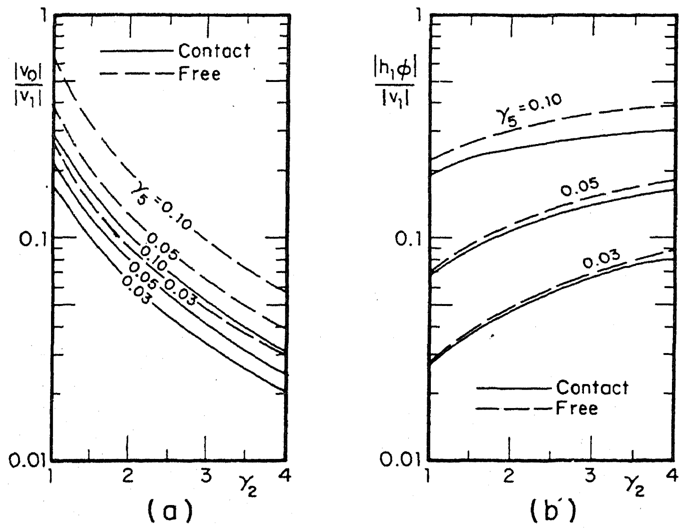


Fig 7

Unless noted otherwise
$r_0/a = 0.04$
$H/r_0 = 60$
$\gamma_2 = 1.7$
$\gamma_5 = 0.05$
$\eta_1 = 0.02$

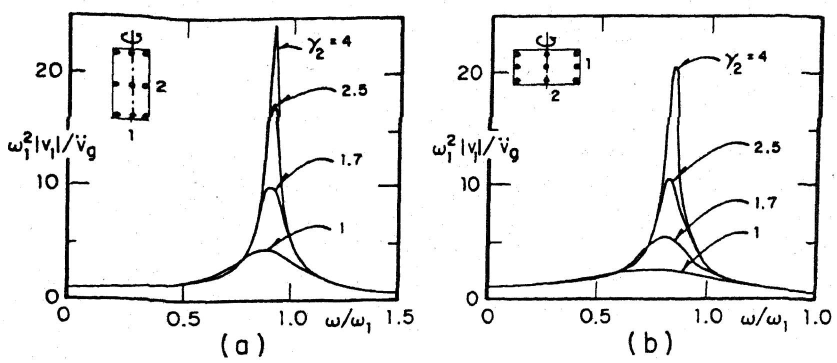

Fig 8

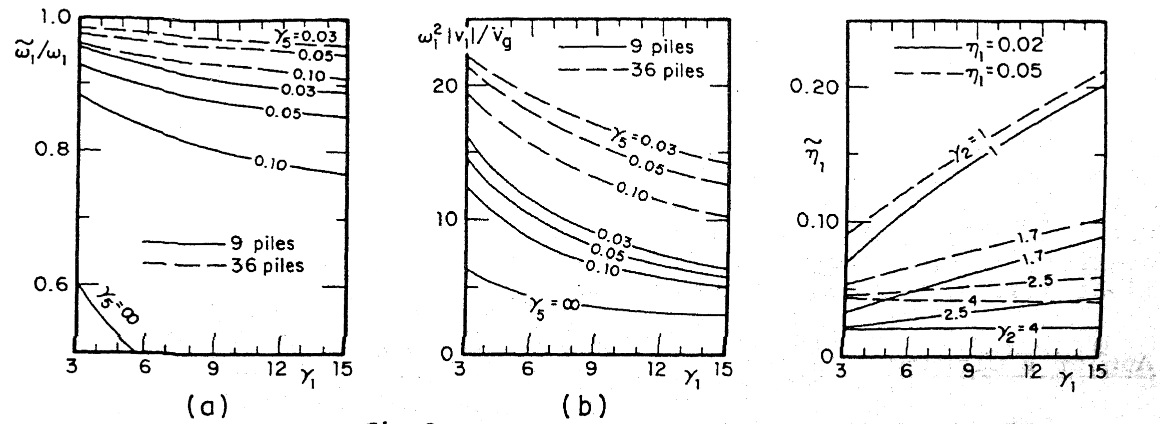

Fig 9

Fig 10

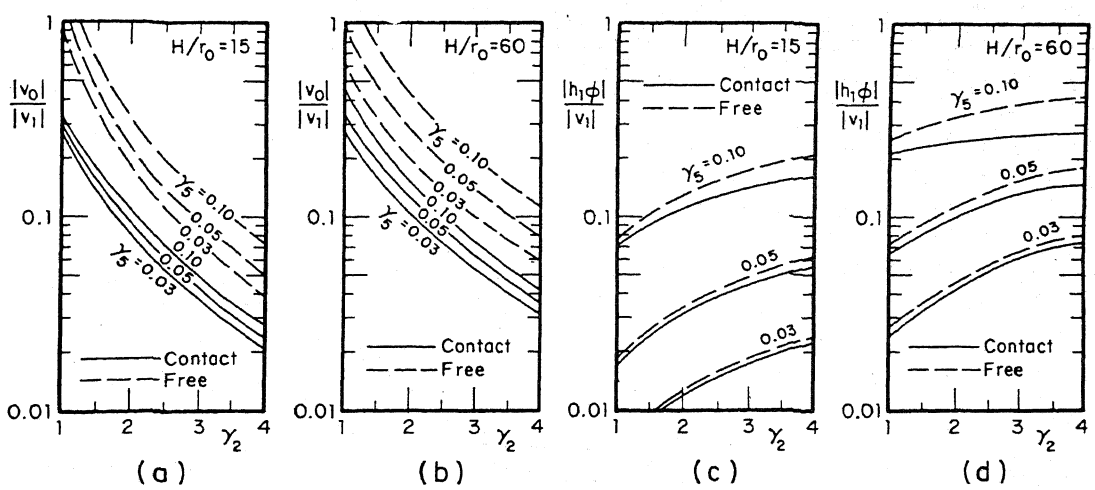

*Fig 11*

Unless noted otherwise: $r_0/a = 0.04$, $H/r_0 = 60$, $b/\ell = 2$, $\gamma_1 = 6$, $\gamma_2 = 1.7$, $\gamma_5 = 0.05$, $\eta_1 = 0.02$, 9 piles

## DISCUSSION

### S.L. Agarwal (India)

In this paper the soil has been represented by shear modulus, poisson's ratio and damping coefficients. In another method, the soilis represented by Winkler model, known commonly as p-y curves. For the offshore piles, subjected to wave loads, some of the codes like API, recommend the use of p-y curves and are silent regarding the method used by authors in this paper. Authors may elaborate whether their method has been checked using the conventional winkler model method. The correlation, if any, claimed by using different models may be elaborated.

### D.V. Reddy (Canada)

In a recent investigation at Memorial University of New Foundland, the discussor found considerable discrepancies about 50% between values computed from linear and non-linear analysis. The discussor would, therefore, question the validity of linear analysis. While it is agreed that non-linear analysis in more complicated, non-realistic approximate solutions have very little value.

### Author's Closure

Not received.