SOIL-PILE INTERACTION

by

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SYNOPSIS

Dynamic stiffness and damping of the soil-pile system results from interaction between the pile and the soil. This interaction was theoretically investigated under the assumptions of linear elasticity or visco-elasticity and small displacements. All the vibration modes including torsion and floating piles were analysed. Impedance functions of the pile cap were established and a comprehensive parametric study was conducted. The theory was compared with experiments.

INTRODUCTION

Piles are used in many cases in which the structure is exposed to dynamic loads such as those produced by earthquakes, wind, sea waves, and operating machines. In such cases, the structural response is affected by interaction between the piles and the surrounding soil. This effect can be incorporated into the theoretical prediction of the structural response if the end reactions of the piles at the level of their heads can be described. These reactions are determined by a set of complex frequency dependent impedance functions that relate harmonic exciting forces and resulting displacements. This relationship also defines discrete stiffness and damping constants of the soil-pile system.

Once these equivalent stiffness and damping coefficients of the soil-pile system are determined, the response of structures can be predicted using the same techniques as in the case of surface footings. This seems possible even with earthquakes because the wave length of seismic waves is much longer than the lateral dimension of the piles and the piles should, therefore not modify the free field ground motion very much. The interaction effects and the relative motion of the pile cap should result primarily from the inertia of the structure.

Interaction between piles and soil was solved by Tajimi (13) who used a linear visco-elastic model of soil, and by Penzien (9) who formulated a discrete nonlinear model. The effect of incidental seismic waves was studied in References 2 and 12. This paper summarizes some theoretical and experimental results achieved at the University of Western Ontario. In all the theoretical developments, vertical piles of circular cross section and linear elasticity or visco-elasticity were assumed. Despite this idealization, the theoretical solution offers fundamental insight into the problem as well as guidance for design.

RESISTANCE OF SOIL TO THE MOTION OF THE PILE

The key to the solution of the interaction between the elastic pile and the soil is the description of the resistance of the soil to the motion of the pile. Two approaches were used to describe that reaction.

The *more rigorous approach* assumes a visco-elastic layer overlying rigid bedrock. This assumption yields the resistance of soil per unit of pile length, p(z), written as a sum of contributions from individual wave modes,

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$$p(z) = \pi \mu \sum_{n=1}^{\infty} \bar{\alpha}_n U_n \sin(h_n z) \qquad (1)$$

where z = depth, μ = shear modulus of soil, U_n = modal amplitude, $h_n = \pi(2n-1)/2\ell$ with n = 1,2... and ℓ = depth of the layer (length of the pile). Parameter $\bar{\alpha}_n$ is a dimensionless measure of the soil resistance to the displacement in one wave mode and hence, can be called the resistance factor. For horizontal displacements, the resistance factor is obtained in the following form (4):

$$\bar{\alpha}_n = \bar{r}_o[(1+iD_s)\bar{h}_n{}^2 - a_o{}^2]\bar{T}_n \qquad (2)$$

where

$$\bar{T}_n = \frac{4K_1(\bar{q}_n\bar{r}_o)K_1(\bar{s}_n\bar{r}_o)+\bar{s}_n\bar{r}_oK_1(\bar{q}_n\bar{r}_o)K_o(\bar{s}_n\bar{r}_o)+\bar{q}_n\bar{r}_oK_o(\bar{q}_n\bar{r}_o)K_1(\bar{s}_n\bar{r}_o)}{\bar{q}_nK_o(\bar{q}_n\bar{r}_o)K_1(\bar{s}_n\bar{r}_o)+\bar{s}_nK_1(\bar{q}_n\bar{r}_o)K_o(\bar{s}_n\bar{r}_o)+\bar{q}_n\bar{s}_n\bar{r}_oK_o(\bar{q}_n\bar{r}_o)K_o(\bar{s}_n\bar{r}_o)} .$$

Here, K_o and K_1 are modified Bessel functions of the second kind of order zero and one respectively. The other dimensionless magnitudes are

$$\bar{r}_o = r_o/\ell, \quad D_s = \mu'/\mu, \quad D_v = \lambda'/\lambda, \quad a_o = \omega\ell/V_s, \quad \bar{h}_n = \ell h_n$$

$$\bar{q}_n = \sqrt{\frac{(1+iD_s)\bar{h}_n{}^2 - a_o{}^2}{\eta^2 + i[(\eta^2-2)D_v + 2D_s]}} , \qquad \bar{s}_n = \sqrt{\frac{(1+iD_s)\bar{h}_n{}^2 - a_o{}^2}{1+iD_s}}$$

in which r_o = pile radius, λ, μ and λ', μ' are the real and imaginary parts of the complex Lamé constants, respectively, $\eta = V_\ell/V_s$ = ratio of velocities of P- and S-waves in soil, $i = \sqrt{-1}$ and ω = frequency of excitation. Constants D express material damping in soil considered to be frequency independent. (Material damping ratio is often defined as $\mu'/2\mu$). In Eq. 1, between twenty and a hundred terms are usually needed and the weighing amplitudes U_n have to be established.

In the second approach much simpler expressions for soil resistance were used. They can be obtained under the assumption that the soil can be represented approximately by a set of infinitesimally thin independent horizontal layers that extend to infinity. This assumption, used first by Baranov (1), is equivalent to the assumption of plane strain or taking into accound only horizontally propagating waves. Under this assumption, the soil resistance to horizontal motion having amplitude u(z) is

$$p(z) = \mu(S_{u1} + iS_{u2})u(z) \qquad (3)$$

where S_{u1} and S_{u2} are the real and imaginary parts of the complex function (1,6)

$$S_{u1} + iS_{u2} = 2\pi a_o' \frac{\frac{1}{\sqrt{q}} H_2^{(2)}(a'_o)H_1^{(2)}(x_o) + H_2^{(2)}(x_o)H_1^{(2)}(a'_o)}{H_o^{(2)}(a'_o)H_2^{(2)}(x_o) + H_o^{(2)}(x_o)H_2^{(2)}(a'_o)}$$

where $a'_o = \omega\, r_o/V_s$, ν = Poisson's ratio, $q = (1-2\nu)/2(1-\nu)$, $x_o = a'_o\sqrt{q}$ and $H_n^{(2)}$ = Hankel function of the second kind or order n.

The above expressions indicate the dimensionless parameters describing the soil resistance. Figure 1 shows an example of the resistance factor. The resistance of soil diminishes to zero at the natural frequency of the layer, a_1', if material damping is absent. Material damping reduces the stiffness (real part) at higher frequencies, is the only source of damping at frequencies lower than a_1' but contributes little to the total damping at frequencies higher than $a_1^!$. Under the assumption of plane strain, the soil resistance vanishes for frequencies→0 (with the exception of torsion) but approaches the three-dimensional case as the frequency increases. This explains why the simpler approach based on Eq. 3 often yields satisfactory results.

DYNAMIC CHARACTERISTICS OF THE SOIL-PILE SYSTEM

With the soil resistance defined, the response of a pile to external excitation can be predicted and the impedance functions as well as the equivalent stiffness and damping constants established. The more rigorous approach based on Eq. 1 has the advantage of providing a rather complete picture of soil-pile interaction. The advantages of the simpler approach using Eq. 3 are simplicity and versatility which make it easy to examine a variety of situations including torsion or floating piles and to conduct parametric studies at negligible cost.

The more rigorous solution is described in Refs. 3 to 5. The simpler solution is available in Refs. 6 and 7. The simpler solution seems to agree with the more rigorous one quite well particularly for higher frequencies. A comparison of stiffnesses obtained by the two methods is shown in Fig. 2. The vertical stiffness of one pile $k_w^1 = f_{w1}\ E_pA/r_o$ where E_p, A are Young's modulus and cross sectional area of the pile, respectively. It can be seen that the two approaches agree quite well.

The vertical response is fundamental; it is important not only with vertical excitation but also with horizontal excitation where the cap rotation produces significant vertical displacements of the piles. The stiffness and damping characteristics of vertical motion were found to depend very much on the degree of fixity of the pile tip unless the piles are very long. Figure 3 shows stiffness and damping parameters for floating as well as end bearing piles. The damping parameter f_{w2} defines the damping constant of one pile as $c_w^1 = f_{w2}\ E_pA/V_s$. The relationships shown depend on the ratio of the stiffness of the soil to the stiffness of the pile which can be described as G/E_p or V_s/v_c where $v_c = \sqrt{E_p/\rho_p}$ = the velocity of longitudinal waves in the pile. Essential differences can be noticed between the two types of piles. With increasing length of the pile, the stiffness of an end bearing pile decreases while the stiffness of a floating pile increases. On the other hand, floating piles generate more damping than end bearing piles. Parameters of socketed piles would lie between the limits plotted in Fig. 3 and can be found easily using the simpler approach (7)

Variation due to frequency of the stiffness and damping constants depend on input parameters. It is strongest for weak soil and small material damping (Fig. 4). Material damping reduces the effect of frequency in the vicinity of the natural frequencies of the layer. In many cases, the variations of stiffness and equivalent viscous damping due to frequency are modest and can be neglected in practical applications.

Impedance functions of pile cap. When analyzing structures or footings, impedance functions of a group of piles connected by a cap have to be established from stiffness constants k^1 and damping constants c^1 of individual piles. Also, a correction for the effect of grouping of piles resulting in interaction between them may be desirable.

The stiffness and damping constants of a group of piles are obtained as forces that must be applied at the reference point, preferably the centroid of the cap, to produce a unit displacement or unit velocity at the reference point. Using the notation of Fig. 5 the stiffness and damping constants of the cap are

$$k_{zz} = \sum_r k_{zz}^{1} \quad , \quad k_{xx} = \sum_r k_{xx}^{1}$$

$$k_{\psi\psi} = \sum_r (k_{\psi\psi}^{1} + k_{zz}^{1} x_r^{2} + k_{xx}^{1} z_c^{2} - 2k_{x\psi}^{1} z_c) \tag{5}$$

$$k_{x\psi} = k_{\psi x} = \sum_r (k_{x\psi}^{1} - k_{xx}^{1} z_c)$$

$$c_{zz} = \sum_r c_{zz}^{1} \quad , \quad c_{xx} = \sum_r c_{xx}^{1}$$

$$c_{\psi\psi} = \sum_r (c_{\psi\psi}^{1} + c_{zz}^{1} x_r^{2} + c_{xx}^{1} z_c^{2} - 2c_{x\psi}^{1} z_c) \tag{6}$$

$$c_{x\psi} = c_{\psi x} = \sum_r (c_{x\psi}^{1} - c_{xx}^{1} z_c)$$

The summation is taken over all the piles. The subscript indicates the direction involved.

It is worth noticing that the frequent approach of considering only horizontal stiffness k_{xx} and damping c_{xx} is justified only if the rotatory stiffness $k_{\psi\psi} \to \infty$. This is so if the piles are vertically very stiff and wide spread.

The effect of pile interaction (grouping) is difficult to assess. In terms of elasticity, pile interaction should reduce the total stiffness and damping of the group as displacement of one pile contributes to the displacements of the adjoining piles. This reduction can be incorporated into the above formulas by correcting the individual contributions by interaction factors α_r, so that, e.g.

$$k_{xx} = \sum_r k_{xx}^{1} / \sum_r \alpha_r \quad , \quad c_{xx} = \sum_r c_{xx}^{1} / \sum_r \alpha_r$$

The interaction factor α_r describes the contribution of the r-th pile to the displacement of the reference pile; hence $\alpha_1 = 1$ and the other factors are smaller than unity and decrease with distance between the piles. An approximate estimate of values α_r can be obtained from the static solutions of Poulos (10,11) who treated all the basic modes of displacement. A separate consideration should be given to the possibility of an increase in soil stiffness due to pile driving and grouping.

When the properties of the pile group are established from Eqs. 5 and 6 response to any load can be calculated. The response of a given structure varies with the number and dimensions of the piles. An example of such dependence is shown in Fig. 6 where the dimensionless rotation of a footing due to harmonic horizontal load is plotted for different numbers of piles.

Comparison with Experiments. The theory outlined was compared with field experiments conducted with pile supported rigid footings subject to harmonic excitation. It was found that the theory predicted the general character of the response very well; however, both the resonant frequencies and amplitudes were in most cases overestimated when the shear wave velocity derived from wave propagation in deeper layers of soil was used. The static response was underestimated. The agreement between the theory and experiments was much improved when a lower, effective shear modulus of soil was calculated back from the static displacement and used in the analysis. Such a measure appears desirable to account approximately for the diminishing of the shear modulus toward the surface and the separation of the pile from the soil. The agreement between the theory and experiments was further improved by incorporating the effect of pile interaction and relaxing the tip condition for vertical response (7,8).

CONCLUSIONS

Two theories were developed that yield the impedance functions of the soil-pile system for individual piles as well as groups of piles. A comparison with experiments indicates that the theory yields good results if an effective shear modulus of soil is derived from a static test of a pile rather than wave propagation. Further corrections needed include the effect of pile grouping and the motion of the tip.

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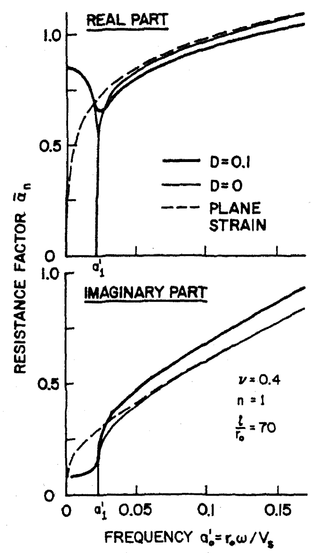

FIG. 1 Resistance factor $\bar{\alpha}_n$

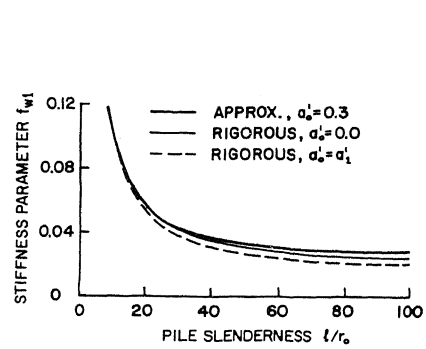

FIG. 2 Vertical stiffness of pile

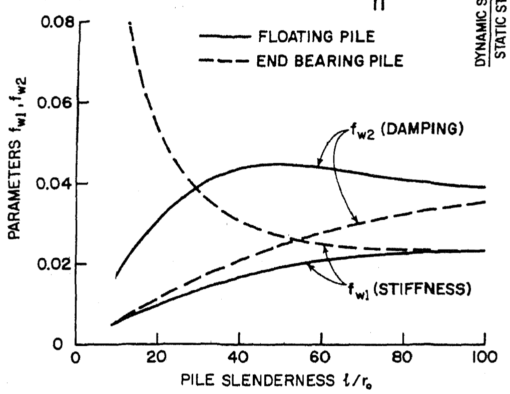

FIG. 3 Floating and end bearing piles

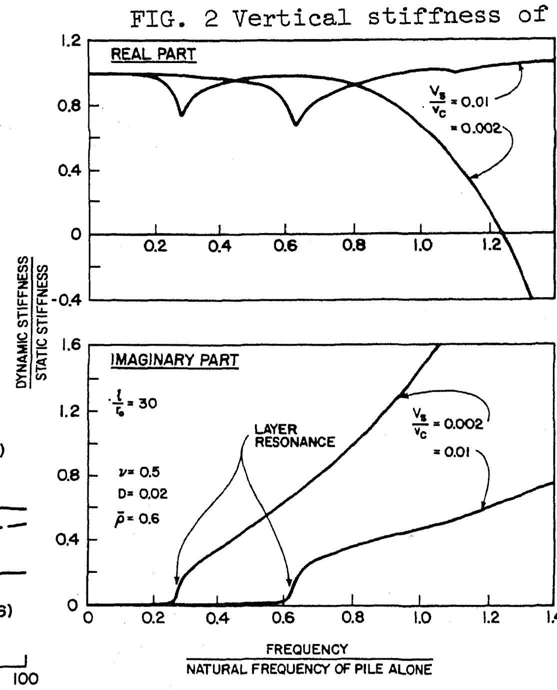

FIG. 4 Horizontal stiffness

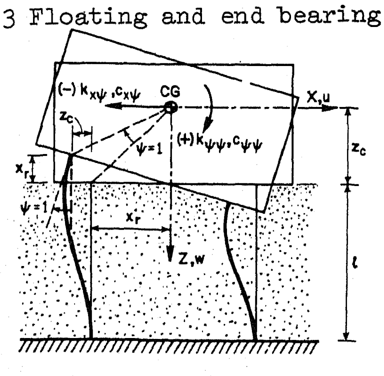

FIG. 5 Rotation of pile cap

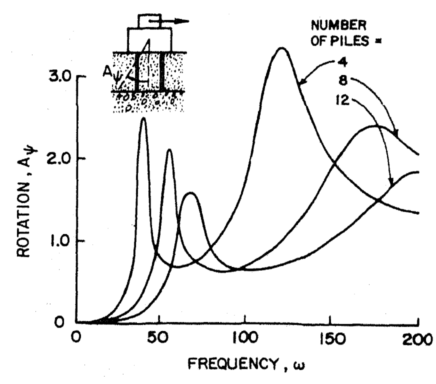

FIG. 6 Effect of number of piles

DISCUSSION

R.K.M. Bhandari (India)

The discussor's question is regarding the behavior of cast-in-situ driven piles in sand where a two pile group is used. The piles could be considered to have adequate embeddment in the bearing stratum. Since the compaction of the sand due to driving of only two piles would not be sufficient, it is expected that the piles may loose lateral support due to liquefaction during earthquake. Would Prof. Novak like to comment whether in the event of loss of lateral support the pile is likely to buckle under the seismic forces and only a partial loss of lateral support takes place and the balance of the support provided by the surrounding soil would suffice to prevent buckling of the piles.

S.L. Agarwal (India)

For a satisfactory solution of any soil-pile interaction problem, two aspects are generally considered, namely (a) mathematical analysis of the pile behaviour yielding to displacements and stresses along its length and (b) determination of the pertinent pile-soil properties.

Regarding the mathematical analysis presented by the author a designer would like to know the limitations of using this method. The maximum amplitude of displacement at the load level or mudline in relation to pile diameter and/or length needs to be defined. Mass of the soil vibrating with the pile does not seem to have been included in the analysis.

Regarding the determination of the pertinent soil properties (equivalent stiffness and damping co-efficients) the method of determining these co-efficients is not given, whether these co-efficients were derived from a static test on an instrumented pile or from samples, is not clearly defined. By characterising these co-efficients from static test and using them for a dynamically loaded pile, may introduce some error. The author may elaborate on this point.

The author may also indicate whether this method is applicable to layered soil media or is restricted to only one soil layer overlying rigid bedrock.

M.V. Ratnam (India)

Prof. Novak has mentioned that, in respect of pile foundation subjected to horizontal vibration, the characteristics of the top most soil/soils have a more prominent effect on the stiffness of the pile than those of soil layers below.

Normally the strength of soil tends to improve with depth. As such it is desirable from a designer's point of view, if Prof. Novak, based on his studies, experience and judgement, can quantify the range of top soil depth (as a fraction of total pile depth) whose properties predominantly effect the stiffness of pile interaction to its response to horizontal vibrations.

Does this aspect vary from a friction pile group to a bearing pile group ? The author may kindly clarify.

<u>Author's Closure</u>

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