DYNAMIC RESPONSE OF RECTANGULAR FOUNDATIONS FOR RAYLEIGH WAVE EXCITATION

by
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SYNOPSIS

The dynamic harmonic response of a rigid massless rectangular foundation welded to an elastic half-space and subjected to the action of a horizontally propagating Rayleigh wave is analyzed. The results obtained indicate that the response of the foundation contains a pronounced rocking component in addition to vertical and horizontal components of motion. The horizontal and vertical components of the response exhibit a marked decrease in amplitude for high frequencies. The results for incident Rayleigh waves are completely different from those obtained on the basis of the usual assumption of vertically incident SH waves.

INTRODUCTION

The study of the dynamic response of foundations to nonvertically incident seismic waves plays an important role in the analysis of the interaction between structures and the soil during earthquake excitation. Although there is a large number of studies on the dynamic response of foundations to external loads, very few investigations have been conducted on the related problem of the diffraction of seismic waves by rigid foundations. Oien[1] has considered the harmonic response of a rigid strip bonded to an elastic half-space and excited by a Rayleigh wave. Kobori, et al[2,3] have analyzed the vibrations of a rigid circular foundation subjected to nonvertically incident plane waves. Studies on the torsional response of a rigid circular foundation and a hemispherical foundation to obliquely incident SH-waves have been presented by Luco[4,5]. Iguchi[6] has considered the response of a flexible rectangular plate to nonvertically incident waves, while Wong[7] and Wong and Luco[8] have studied the motion of a rigid rectangular foundation bonded to an elastic half-space and excited by obliquely incident P, SV and SH waves. The results of these studies indicate that nonvertically incident SH-waves generate a considerable torsional response of the foundation, while obliquely incident P and SV-waves induce a marked rocking component. In general, the amplitudes of the rotational components of the response are the highest for the shallower angles

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of incidence of the seismic waves. Under these conditions it is of interest to examine the response of rigid massless rectangular foundations subjected to horizontally propagating Rayleigh waves.

STATEMENT OF THE PROBLEM

The model considered in this study is illustrated in Fig. 1, and it consists of a rigid massless rectangular foundation of length 2a and width 2b ($|x_1| < a$, $|x_2| < b$, $x_3 = 0$) bonded to an elastic, homogeneous and isotropic half-space ($x_3 > 0$). The seismic excitation is represented by a horizontally propagating harmonic Rayleigh wave impinging on the foundation with angle θ measured as shown in Fig. 1. In absence of the rigid foundation the free-field motion of the half-space surface is given by

$$\begin{aligned} u_1(x_1, x_2, 0) &= R_H \cos\theta \exp\{i\omega[t - (x_1\cos\theta + x_2\sin\theta)/C_R]\} \\ u_2(x_1, x_2, 0) &= R_H \sin\theta \exp\{i\omega[t - (x_1\cos\theta + x_2\sin\theta)/C_R]\} \quad (1) \\ u_3(x_1, x_2, 0) &= i R_V \exp\{i\omega[t - (x_1\cos\theta + x_2\sin\theta)/C_R]\} \end{aligned}$$

in which R_H and R_V are, respectively, the amplitudes of the horizontal and vertical components of motion induced by the incident Rayleigh wave on the surface of the elastic half-space, ω is the frequency of the harmonic excitation, and C_R is the Rayleigh wave velocity. For a Poisson's ratio of 1/3, $R_V = 1.565\, R_H$ and $C_R = 0.9325\beta$, where β is the shear wave velocity in the elastic half-space.

The presence of the rigid foundation modifies the free-field motion by the addition of diffracted waves. The resulting motion of the rigid massless foundation may be described by the six generalized coordinates (U_1^*, U_2^*, U_3^*, φ_1^*, φ_2^*, φ_3^*), as shown in Fig. 1. The coordinates U_1^* and U_2^* correspond to the amplitudes of the horizontal components of motion at the center of the foundation, while U_3^* reflects the amplitude of the vertical component. The angular coordinates φ_1^* and φ_2^* correspond to the amplitudes of the rocking angles about the x_1 and x_2 axes, respectively, while φ_3^* reflects the torsional twist about the x_3-axes. The evaluation of the dynamic response, (U_1^*, U_2^*, U_3^*, φ_1^*, φ_2^*, φ_3^*) $\exp(i\omega t)$, of the foundation entails the solution of a mixed boundary-value problem in elasticity. The approximate method of solution employed by the authors consists in dividing the foundation into a number of equal square sub-regions and in assuming that the contact tractions within each sub-region may be considered to be constant[7,8,9]. This procedure reduces the mixed boundary-value problem to a set of linear algebraic equations that may be readily solved.

RESULTS

For Rayleigh waves propagating in the x_1-direction, i.e., for $\theta = 0^\circ$, the response of the rigid rectangular foundations consists of three components: horizontal motion $U_1^* \exp(i\omega t)$ along the x_1-axis, vertical motion $U_3^* \exp(i\omega t)$, and rocking $\varphi_2^* \exp(i\omega t)$ about the x_2-axis. The calculated values for the real and imaginary parts of the normalized results U_1^*/R_H, U_3^*/R_H and $a\varphi_2^*/R_H$ are shown in Figs. 2.a, b, c versus the dimensionless frequency $a_o = \omega a/\beta$ for two values of the aspect ratio b/a. Figs. 2.b and 2.c show that for low frequencies the horizontal and vertical components of motion of the center of the foundation are equal to the corresponding components of the free-field motion ($R_V = 1.565\, R_H$). For higher frequencies the translational response of the foundation exhibits a very pronounced decrease in amplitude. Thus, for a value of $a_o = 2$ the amplitudes of the horizontal and vertical components of the foundation response are less than 40 percent of the corresponding free-field motion amplitudes. This effect is in agreement with experimental evidence that indicates a marked reduction of the translational response for high frequencies. Perhaps the most important characteristic of the response to Rayleigh waves is the marked rocking component about the x_2-axis shown in Fig. 2.a. The rocking motion induced by the horizontally propagating Rayleigh wave is such that, for $a_o = 2$, the amplitude $a\varphi_2^*$ of the vertical motion at the opposite ends $x_1 = \pm a$ of the foundation is about twice the amplitude of the horizontal component of the free-field motion. Such large rocking component may be of importance for stiff massive structures placed on relatively soft soils in which case the structural response is highly influenced by the rocking of the foundation. The results shown in Figs. 2.a, b, c also indicate that the dimension of the foundation in a direction perpendicular to the direction of propagation of the incident waves has very little effect on the foundation response.

The effects of the horizontal angle of incident θ on the response of a massless rigid square foundation are illustated in Fig. 3. In general, obliquely incident Rayleigh waves excite all six components of the foundation response. In particular, it may be observed that the vertical component of the response U_3^* is not very sensitive to changes in the angle of incidence, and that the torsional component of the response φ_3^* is quite small as compared with the other components.

CONCLUSIONS

The problem of the forced vibrations of a massless rigid rectangular foundation bonded to an elastic half-space and subjected to Rayleigh waves has been analyzed. It has been found that when the angle of incidence of the Rayleigh wave is normal to one of the sides of the foundation a large rocking component of motion is generated in addition to horizontal and vertical translational components. The translational components of the response exhibit a pronounced decrease in amplitude with increasing frequency. The rocking component of the response may induce vertical displacements at opposite ends of the foundation of amplitude twice as high as the amplitude of the horizontal component of the free-field motion. The aspect ratio of the foundation has no marked effect on the response, the important parameter being the length of the foundation in the direction of propagation of the incident wave. Obliquely incident Rayleigh waves excite all six components of motion of the foundation.

ACKNOWLEDGMENTS

This research has been supported by grants from the National Science Foundation, the U.S. Geological Survey and the Earthquake Research Affiliates Program at the California Institute of Technology.

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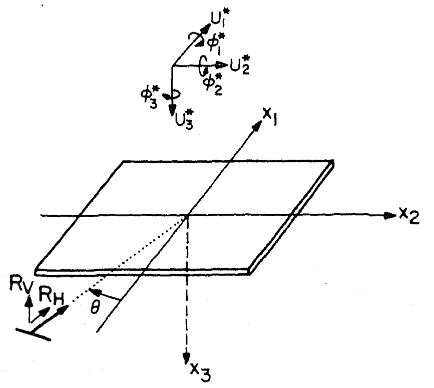
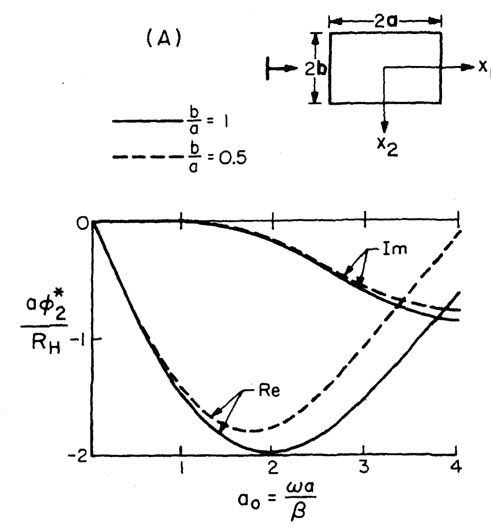


FIG. 1 DESCRIPTION OF THE SYSTEM.



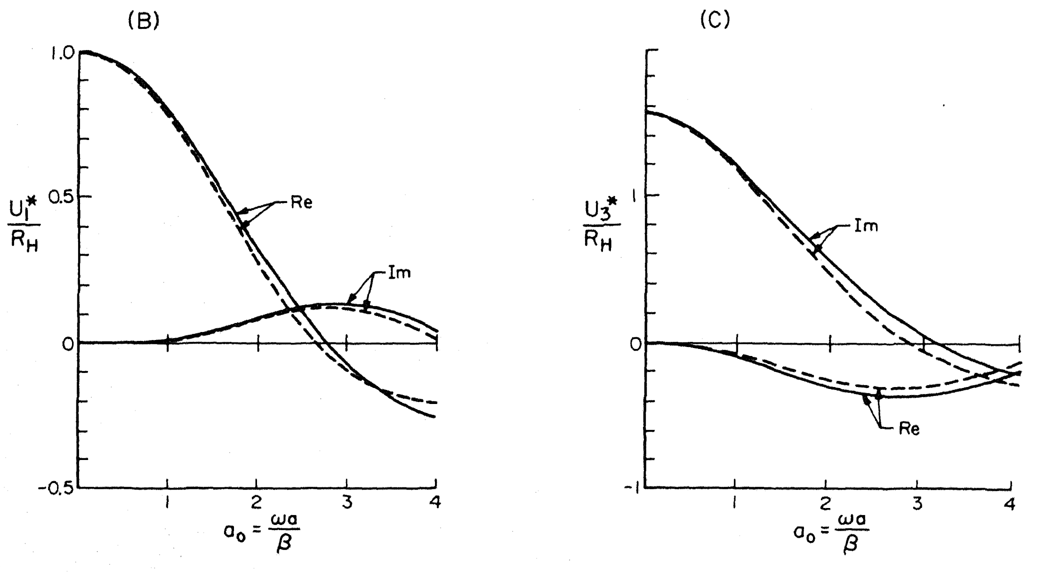


FIG. 2 (A,B,C) RESPONSE OF A RIGID RECTANGULAR FOUNDATION TO A RAYLEIGH WAVE PROPAGATING IN THE x_1 DIRECTION (θ=0, ν=1/3).

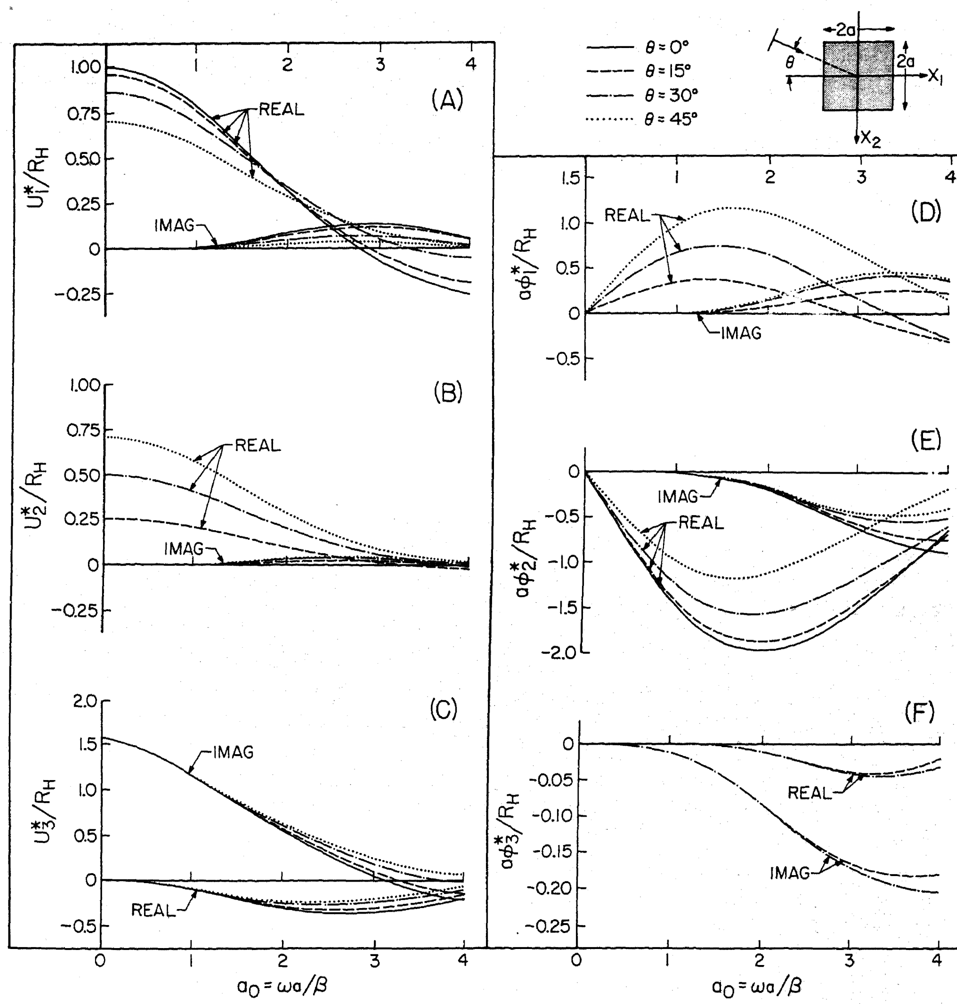


FIG 3 RESPONSE OF A RIGID SQUARE FOUNDATION TO OBLIQUELY INCIDENT RAYLEIGH WAVES $\left(\nu = \frac{1}{3}\right)$

DISCUSSION

K.G. Bhatia (India)

The discussor is interested in knowing from the authors that do they recommend the use of Rayleigh wave over and above the vertically propagating shear waves. Also the authors may highlight the effect of the mass and stiffness parameters of the structure on the conclusions derived in the paper where the foundation has been taken as a rigid and massless foundation. What would be the observation if the foundation is not rigid.

Author's Closure

With regard to the question of Mr. Bhatia, we wish to state that in regards to the first part of the question the authors believe that the type of seismic excitation to be used should result from a detailed study of the source, propagation path and local soil conditions. In general, the seismic excitation will consist of a combination of obliquely incident body waves (P, SV, SH) and of surface waves (Rayleigh, Love). In absence of detailed studies as to the type of seismic excitation, bounds may be obtained by considering that the excitation consists purely of Rayleigh waves (rocking response), or horizontally incident SH waves or Love waves (torsional response).

Referring to the second part of the question the authors must emphasize that the results presented correspond to the input to the complete soil-structure interaction problem. Solutions for the complete soil-structure interaction problem for Rayleigh wave excitation are in agreement with the conclusions stated in the paper. It has been found that due to the increased rocking of the foundation the response at the upper levels of the superstructure may be considerably higher for Rayleigh wave excitation than for vertically incident SH waves.

Finally, in regards to the effects of the flexibility of the foundation it can be stated that the assumption of a rigid foundation is adequate in estimating the overall response of the superstructure, on the other hand, marked localized effects at the lower levels of the superstructure may be associated with the flexibility of the foundation. In considering the flexibility of the foundation it must be kept in mind that the presence of the superstructure creates a stiffening effect.