A Basic Study on the Behaviour of Long Dimensional Size Buildings during Earthquakes

by Michio IGUCHI *

INTRODUCTION

In analysing the seismic response of structures, the base plate has been treated to be rigid. However, when the structure has a long dimension in plane, the assumption that the base plate is rigid is not realistic. Furthermore, for a structure with long dimension, it is important to take into account the effects of the phase differences between free surface motion at each point. Recently, Oien developed a two-dimensional analysis of a harmonic response of a elastic plate resting on an elastic half space, whose problem was formulated in the terms of dual integral equations. [1] To solve the equations Oien successfully used Galerkin's method.

This paper deals with a theoretical, though approximate, analysis of the vertical behaviour of long dimensional size structures on the ground, when subjected to the obliquely incident earthquake motion. For simplicity a long elastic base plate is considered as a model of structure as shown in Fig. 1. The distinctive features of this paper are as follows; 1) The plane shape of the base plate is rectangular, 2) The elastic deformation of the plate is taken into account, 3) The analysis is three-dimensional. The basic equations are formulated in the form of integral equations, which, in this paper, are solved by Galerkin's method after Oien.

In this analysis, the followings are assumed; 1) The plate deforms only along the long direction of it, and the deformation along the short axis is not taken into account. 2) The incident earthquake wave propagates parallel to the long direction of the plate. 3) The contact shearing stress between the ground and the plate is neglected because only the vertical motion of plate is considered and the shearing stress can be supposed to be smaller than the other stress. 4) The continuity of displacement of the ground and the plate is considered only along the center axis of the plate (x-axis shown in Fig. 1). This assumption would be recognized because a slender plate is considered in the analysis.

INCIDENT WAVE AND GREEN'S FUNCTION OF ELASTIC HALF SPACE

As shown in Fig. 1, coordinates system $(\bar{x},\bar{y},\bar{z})$ is set and nondimensional coordinates (x,y) are defined as follows.

$$x = \bar{x} / c \quad , \quad y = \bar{y} / b \tag{1}$$

where c = half length of the plate, b = half width of the plate.

Vertical component on the ground surface of the obliquely incident harmonic motion can be expressed by

$$w^i(x) = \exp i(\omega t - k_i x) \tag{2}$$

where the displacement amplitude is set to be a unit, ω = circular frequency of the incident earthquake motion, k_i = wave number, in which denoting an apparent surface wave velocity by V_a then $k_i = \omega c / V_a$.

In this analysis, an expression of the vertical displacement on the free ground w_p when subjected to a harmonically excited point load at the

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surface is required. An approximate solution of w_p was presented by Tajimi as follows. [2]

$$w_p = \frac{1 - \nu}{2\pi\mu}\frac{\exp(-i\bar{\kappa}r)}{r} e^{i\omega t} \qquad (3)$$

where, μ = shear rigidity of ground, ν = Poisson's ratio of ground, $\bar{\kappa} = 1.33\omega / V_s$ (V_s = shear wave velocity of ground), r = radial distance from the exciting point. Rewriting Eq. (3) with Cartesian coordinates and replacing w_p by $G(x,y|\xi,\eta)$, it can be expressed by,

$$G(x,y|\xi,\eta) = \frac{1 - \nu}{2\pi\mu c}\,\frac{\exp\{i\kappa\sqrt{(x - \xi)^2 + \alpha^2(y - \eta)^2}\}}{\sqrt{(x - \xi)^2 + \alpha^2(y - \eta)^2}} \qquad (4)$$

where the term $e^{i\omega t}$ is omitted and the other symbols denote as follows.

$$\alpha = b / c \quad , \quad \kappa = \bar{\kappa}c = 1.33\Omega \quad , \quad \Omega = \omega c / V_s \qquad (5)$$

Eq. (4) expresses the vertical displacement at the point (x,y) when excited at (ξ,η), and it indicates the dynamical Green's function of semi-infinite elastic ground. Eq. (4) can be expressed by Sommerfeld's integral and further transformed into a more convenient form as follows. [3]

$$G(x,y|\xi,\eta) = \frac{1 - \nu}{2\pi\mu c}\int_0^\infty K_0(\alpha\sqrt{s^2 - \kappa^2}\,|\eta|)\cos\{s(x - \xi)\}\,ds \;;\; y = 0 \qquad (6)$$

where, $K_0(x)$ = modified Bessel function of the second kind, and y is set to 0 from the assumption 4).

BASIC AND INTEGRAL EQUATIONS

Denoting the deformation of the plate by W the continuity condition of displacement between the ground surface and the plate can be expressed by,

$$w^i(x) + w^s(x) = W(x) \qquad (7)$$

where w^s indicates a displacement caused by the scattered wave. The contact normal stress between the ground and the plate is caused by the scattered wave, and denoting it by q^s the nondimensional equation of motion of the uniform plate along x can be expressed by

$$D\frac{d^4W}{dx^4} - M\Omega^2W = -\frac{1 - \nu}{\mu}\,b\int_{-1}^{1} q^s(x,y)\,dy \qquad (8)$$

where,

$$D = \frac{E(1 - \nu)}{6\mu}\frac{bh^3}{c^4}\;,\quad M = \frac{2(1 - \nu)}{\rho}\frac{bh}{c^2}\rho_p \qquad (9)$$

in which E = Young's modulus, ρ_p = mass density, h = thickness of the plate. w^s is divided into two component as expressed by,

$$w^s(x) = w^d(x) + w^r(x) \qquad (10)$$

where letting w^d be as given by,

$$w^d(x) = -\,w^i(x) \qquad \text{then} \qquad w^r(x) = W(x) \qquad (11)$$

In the same way as dividing w^s as shown in Eq. (10), q^s is divided into two components;

$$q^s(x,y) = q^d(x,y) + q^r(x,y) \qquad (12)$$

In the above equation, q^d corresponds to the w^d and represents the stress distribution of the diffracted wave of immobile rigid plate. On the other hand, q^r corresponds to w^r and represents the stress of radiated wave by the forced displacement W.

Substituting Eq. (12) into (8), the following equation is obtained.

$$D\frac{d^4W}{dx^4} - M\Omega^2W + \frac{1 - \nu}{\mu}\,b\int_{-1}^{1} q^r(x,y)\,dy = -\frac{1 - \nu}{\mu}\,b\int_{-1}^{1} q^d(x,y)\,dy \qquad (13)$$

To solve the response of the base plate W, it is convenient to expand W and

q^r in the following series.

$$W(x) = \sum_{n=0}^{\infty} A_n W_n(x) \quad , \quad q^r(x,y) = \sum_{n=0}^{\infty} A_n q_n^r(x,y) \tag{14}$$

From Eqs. (11) and (14),

$$w^r(x) = \sum_{n=0}^{\infty} A_n W_n(x) \tag{15}$$

Substitution from Eqs.(14) into (13) leads to the following equation, where upper limit of the series is set to n_w instead of infinite.

$$\sum_{n=0}^{n_w} A_n \left[D \frac{d^4 W_n}{dx^4} - M\Omega^2 W_n + \frac{1-\nu}{\mu} b \int_{-1}^{1} q_n^r(x,y)dy \right] = - \frac{1-\nu}{\mu} b \int_{-1}^{1} q^d(x,y)dy \tag{16}$$

In this analysis, the series of W_n is chosen to be the free vibration mode of the elastic plate as shown,

$$W_n(x) = \begin{cases} \dfrac{\mathrm{Ch}\, \beta_n \cos \beta_n x + \cos \beta_n \,\mathrm{Ch}\, \beta_n x}{\sqrt{\mathrm{Ch}^2 \beta_n + \cos^2 \beta_n}} & ; \; n = \text{even} \\[2ex] \dfrac{\mathrm{Sh}\, \beta_n \sin \beta_n x + \sin \beta_n \,\mathrm{Sh}\, \beta_n x}{\sqrt{\mathrm{Sh}^2 \beta_n - \sin^2 \beta_n}} & ; \; n = \text{odd} \end{cases} \tag{17}$$

where β_n represents the root of the following equation,

$$(-1)^n \,\mathrm{Ch}\, \beta_n \sin \beta_n + \mathrm{Sh}\, \beta_n \cos \beta_n = 0 \tag{18}$$

$\beta_0 = \beta_1 = 0$ are also the roots of Eq.(18), and correspond to the rigid body displacements. Therefore, W_0 and W_1 are chosen as follows.

$$W_0(x) = 1 \quad , \quad W_1(x) = x \tag{19}$$

Thus expressed vibration modes satisfy the following orthogonal relations.

$$\int_{-1}^{1} W_n(x) W_\ell(x) dx = \Delta_{n\ell} = 0 \; (n \neq \ell) \; , \; 2 \; (n=\ell=0) \; , \; 2/3 \; (n=\ell=1) \; , \; 1 \; (n=\ell>1) \tag{20}$$

Based on the definitions of q^d and q^r, the following dual integral equations can be obtained.

$$bc \int_{-1}^{1} \int_{-1}^{1} q^J(\xi,\eta) G(x,y|\xi,\eta) \, d\xi d\eta = w^J(x) \quad ; \quad y = 0 \text{ and } J = d \; , \; r_n \tag{21}$$

From above, the problem in hand results in the analyses of the dual integral equations.

ANALYSIS

Let's solve the dual integral equations by means of Galerkin's method. For this, the contact stress is expanded by series, which is empirically chosen as,

$$q^J(x,y) = \frac{1}{\sqrt{1-y^2}} \sum_{m=0}^{m_T} B_m^J \frac{T_m(x)}{\sqrt{1-x^2}} \quad ; \; J = d \; , \; r_n \tag{22}$$

where, Tm(x) = Tschebysheff polynomials. Multiplying Eq. (21) by $T_\ell(x) / \sqrt{1-x^2}$ ($\ell = 0 \sim m_T$) and further integrating over the interval $-1 < x < 1$, then the following simultaneous equations are finally obtained.

$$\frac{1-\nu}{\mu} b [H_{m\ell}] \{B_m^J\} = \{C_m^J\} \quad ; \quad J = d \; , \; r_n \tag{23}$$

where matrix $[H_{m\ell}]$ is the symmetric one of the size m_T. Making use of the formulae; [4]

$$\left. \begin{aligned} & \int_{-1}^{1} \frac{T_m(x)}{\sqrt{1-x^2}} e^{isx} dx = (-i)^m \pi J_m(s) \\ & \int_{0}^{1} K_0(sx) \frac{1}{\sqrt{1-x^2}} dx = \frac{\pi}{2} I_0\left(\frac{s}{2}\right) K_0\left(\frac{s}{2}\right) \end{aligned} \right\} \tag{24}$$

where $I_0(x)$ = modified Bessel function of the first kind, the element of the matrix can be written by,

$$H_{m\ell} = \pi\int_0^\infty I_0(Z)K_0(Z) \left\{ \begin{array}{l} (-1)^{(m+\ell)/2} J_m(s)J_\ell(s) \\ (-1)^{(m+\ell-2)/2} J_m(s)J_\ell(s) \end{array} \right\} ds \quad \begin{array}{l} ;\ m,\ell = \text{even} \\ ;\ m\cdot\ell = \text{odd} \end{array} \tag{25}$$

where $Z = \sqrt{s^2 - \kappa^2}/2$ and when $m + \ell$ = odd then $H_{m\ell} = 0$. Since integrand of Eq. (25) has a branch point at $s = \kappa$, the integral may be carried out by dividing the interval of the integration into two parts; $0 < s < \kappa$ and $\kappa < s < \infty$. The integration over the interval $0 < s < \kappa$ can be evaluated by using the formulae;

$$I_0(ix) = J_0(x)\ ,\ K_0(ix) = -\frac{\pi}{2}\, iH_0^{(2)}(x) \tag{26}$$

The coefficients of the right hand side of Eq. (23) are expressed by,

$$J = d\ ;\ C_m^d = \int_{-1}^{1} \frac{T_m(x)}{\sqrt{1 - x^2}} w^d(x)\, dx = \begin{cases} -(-1)^{m/2}\ \pi J_m(k_i) & ;\ m = \text{even} \\ i(-1)^{(m+1)/2}\ \pi J_m(k_i) & ;\ m = \text{odd} \end{cases} \tag{27}$$

$J = r_n$;

$$n \geqq 2\ ;\ C_{mn}^r = \int_{-1}^{1} \frac{T_m(x)}{\sqrt{1 - x^2}} W_n(x)\, dx$$

$$= \begin{cases} \pi\, \dfrac{(-1)^{m/2}\ \mathrm{Ch}\ \beta_n\ J_m(\beta_n) + \cos \beta_n\ I_m(\beta_n)}{\sqrt{\mathrm{Ch}^2\ \beta_n + \cos^2\ \beta_n}} & ;\ \begin{array}{l} m = \text{even} \\ n = \text{even} \end{array} \\ \pi\, \dfrac{(-1)^{(m-1)/2}\ \mathrm{Sh}\ \beta_n\ J_m(\beta_n) + \sin \beta_n\ I_m(\beta_n)}{\sqrt{\mathrm{Sh}^2\ \beta_n - \sin^2\ \beta_n}} & ;\ \begin{array}{l} m = \text{odd} \\ n = \text{odd} \end{array} \\ 0 & ;\ m+n = \text{odd} \end{cases} \tag{28}$$

$$n = 0\ ;\ C_{m0}^r = \begin{cases} \pi & ;\ m = 0 \\ 0 & ;\ m \neq 0 \end{cases} \tag{29}$$

$$n = 1\ ;\ C_{m1}^r = \begin{cases} \pi/2 & ;\ m = 1 \\ 0 & ;\ m \neq 1 \end{cases} \tag{30}$$

Solving the simultanious equations,the coefficients of the contact stress distributions B_m^d and B_{mn}^r are obtained. Successively, multiplying the both side of Eq. (13) by $W_\ell(x)$ and integrating over the interval $-1 < x < 1$, the following simultaneous equations are given.

$$[F_{n\ell}]\{A_n\} = \{D_n\} \tag{31}$$

where matrix $[F_{n\ell}]$ is a symmetrical one of the size n_W and its element is expressed by,

$$F_{n\ell} = (D\,\beta_n^4 - M\Omega^2)\Delta_{n\ell} + \frac{1 - \nu}{\mu} b\pi \langle C_{jn}^r \rangle \{B_{j\ell}^r\} \quad ;\quad j = 0 \sim m_T \tag{32}$$

in which < > represents a symbol of row vector.

The right hand side of Eqs.(31) can be expressed by,

$$D_n = -\frac{1 - \nu}{\mu} b\pi \langle C_{jn}^r \rangle \{B_j^d\} \quad ;\quad j = 0 \sim m_T \tag{33}$$

NUMERICAL EXAMPLE

The parameters needed in this analysis are five and all nondimensional as follows.

$$\Omega = \frac{\omega c}{V_s}\ ,\ k_i = \frac{\omega c}{V_a} = \frac{V_s}{V_a}\Omega\ ,\ D = \frac{(1-\nu)bh^3}{6\mu c}\ ,\ M = \frac{2(1-\nu)bh}{\rho c^2}\rho_p\ ,\ \alpha = \frac{b}{c} \tag{34}$$

In the numerical analysis, the parameter $\lambda = V_s / V_a$ is used instead of k_i

for convenience. The various constants of the ground and the plate for this numerical analysis are chosen as follows;

$$\rho = 0.15\ \mathrm{t \cdot sec^2/m^4}\ ,\ V_s = 150\ \mathrm{m/sec}\ ,\ \nu = 0.4$$

$$E = 2.1\times10^6\ \mathrm{t/m^2}\ ,\ \rho_p = 0.24\ \mathrm{t \cdot sec^2/m^4}\ ,\ h/c = 0.04\ ,\ \alpha = b/c = 0.25$$

The calculated results are shown in Figs. 2 to 4.

Fig. 2 shows the complex expansion coefficients of W and q^r in Eqs.(14) as a function of Ω and parameter λ. The coefficients stand for frequency transfer functions which indicate the relation between the earthquake motion on the free surface and the free plate modes. From the results the following remark would be made; For the case $\lambda = 0.1$, which coresponds to the case that the incident wave propagates nearly parallel to the ground surface, the displacement response amplitudes for more than the second mode become small. Therefore, for such a case the effect of elastic deformation of the plate becomes less.

Fig. 3 shows the absolute value of displacement response at the both edges and the center of the elastic plate. In this figure, the results for the rigid plate are also shown by dotted line for comparison. This result indicates that the effect of elastic deformation of the plate is significant for this example, and it can not be neglected.

Fig. 4 shows the displacement response along the x-axis of the elastic plate. This result indicates that for large value of Ω the displacement response at the edges may become twice or more as large as that around middle of the plate. Therefore, it can be pointed out that the edges of building with a long dimension in plane should be carefully designed.

CONCLUSION

In this paper, a vertical behaviour of long dimensional elastic plate resting on the ground, when subjected to the obliquely incident earthquake motion, was approximately analysed on the basis of three-dimensional wave propagation theory. The numerical results were given for the harmonic response of the elastic plate. The importance of the elastic deformation of plate with a long dimension in plane was pointed out.

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REFERENCES

[1] M. A. Oien, " Steady Motion of a Plate on an Elastic Half Space", Journal of Applied Mechanics, June 1973, pp 478-484

[2] H. Tajimi, " Basic Theory on Aseismic Design of Structures", Report of the Institute of Industrial Science, Univ. of Tokyo. Vol. 8, no. 4, 1959

[3] Y. Noro, " Special Functions and its Applications", Daily Industrial Press Co., 1962, pp 330-331 (in Japanese)

[4] A. Erdelyi, et al., " Table of Integral Transforms, Vol. 1 and 2", McGraw-Hill, 1954, p122(Vol. 1), p129(Vol. 2)

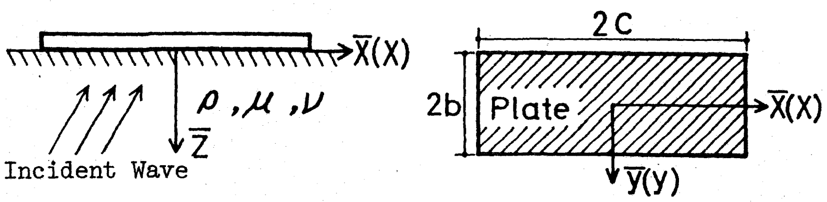

Fig. 1 COORDINATE SYSTEM AND RECTANGULAR PLATE ON THE GROUND

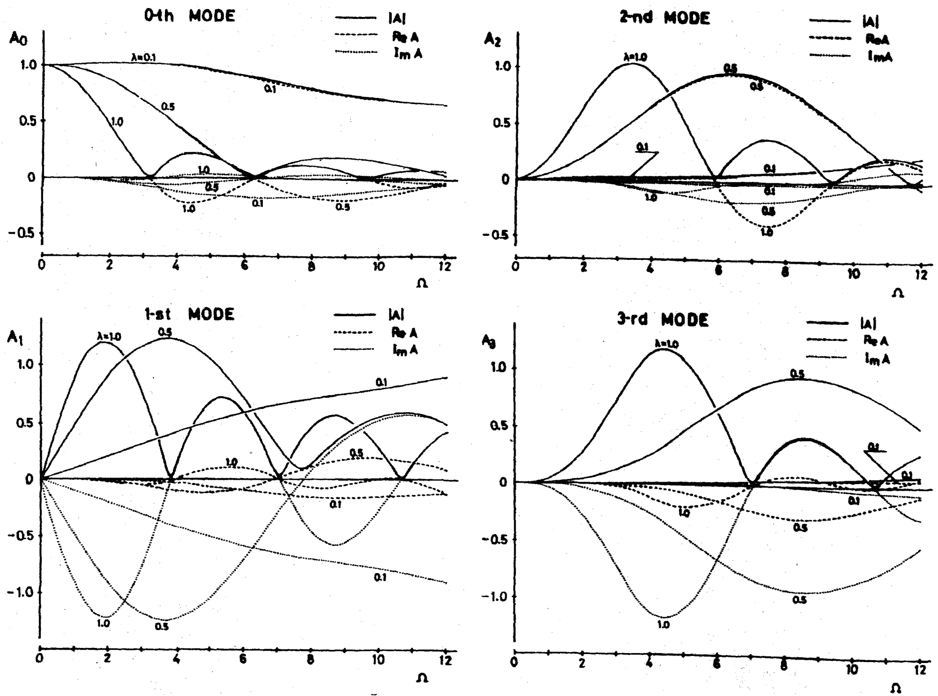

Fig. 2 RESPONSE AMPLITUDE OF EACH MODE. ABSOLUTE VALUE (SOLID LINE), REAL PART (BROKEN LINE), IMAGINARY PART (DOTTED LINE).

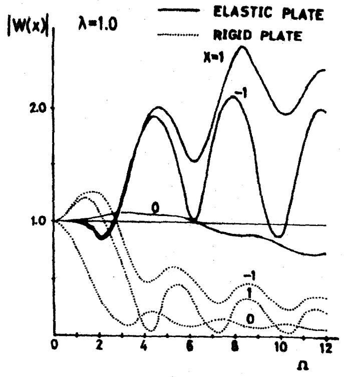

Fig. 3 DISPLACEMENT RESPONSE AT THE EDGES AND THE CENTER OF ELASTIC PLATE (SOLID LINE) AND COMPARABLE RIGID PLATE (DOTTED LINE)

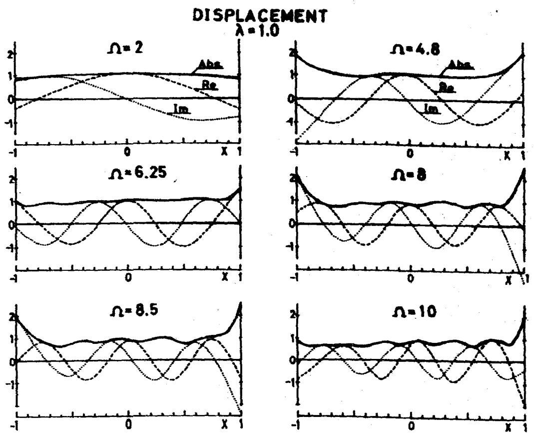

Fig. 4 DISPLACEMENT RESPONSE OF THE ELASTIC PLATE ALONG X.