DYNAMICAL CROSS-INTERACTION BETWEEN TWO FOUNDATIONS

by

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SYNOPSIS

This paper deals with the vibrational characteristics of two foundations coupled each other through a soil medium. The vibrational field tests for two identical foundations under the harmonic excitation by a vibration generator were carried out and their responses were measured by the velocity type seismometers. On the other hand, the vibrational characteristics of the two coupled foundations have been analyzed theoretically by making use of the dynamical ground compliance matrix of the foundations on a visco-elastic half-space, and then these theoretical results have been compared with the experimental evidence.

INTRODUCTION

For the aseismic design of structures built in a big city, the effect of the interaction among adjacent structures through soil medium may not be ignored. From this point of view, the studies on the soil-structure cross-interaction have been watched by structural engineers recently. The interaction of two or more foundations has been explored analytically by Warburton et al.[1)] and Kobori et al.[2),3)], and experimentally by MacCalden, and Kobori et al..

The objective of this paper is to evaluate experimentally and theoretically the responses of two coupled foundations on a soil medium, and then to assess the validity of the analytical prediction, by comparing with the experimental results. The experimental model system consists of two identical reinforced concrete square foundations, each of which is 200 cm × 200 cm at bottom and 35 cm in thickness. One of the foundations, which is called an active foundation, is located at a fixed position and excited by a vibration generator, and the other designated as a passive foundation is moved in the vicinity of the active foundation changing the distance between the two foundations. The behaviors of these foundations are measured at various excitation level and distance parameters. In the theoretical analysis, the mathematical model of cross-interaction system consists of two foundations on a visco-elastic half-space and the ground compliance matrix of the square foundations is evaluated analytically as a three dimensional wave propagation problem in a visco-elastic medium. The theoretical response characteristics are calculated by using ground constants which are determined through field tests at the site.

EXPERIMENTAL STUDIES

In order to confirm experimentally the effect of interaction between two structures through a soil medium, we had carried out the vibrational field tests for two identical square foundations at the site in *Morinomiya, Osaka* city, where can be considered as the typical soft soil deposits. As

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the pieces of the concrete block and brick were mixed in the ground at the experimental site, and since the response of foundations would be suffered by the effect of such unusual circumstance, the surface soil was excavated up to 2 meters in depth over 60 meters square area as shown in Fig.1. The boring log, the standard penetration test results and other physical constants of the experimental ground were obtained at the point denoted by * in Fig.1, and a typical example is shown in Fig.2. As indicated in these results, the soil medium is composed of very soft clayey and silty layers up to 20 meters in depth, except for a thin sand layer at about 3 meters in depth. For the soil medium near the surface, which plays an important role in the cross-interaction between the foundations, its density ρ is equal to $1.5\ t/m^3$ and shear wave velocity V_S to 80 m/sec. The configuration of two experimental foundations and the locations of seismometers are shown in Fig.3. During each run of the dynamic tests, the eccentric moment of the vibration generator was kept at a constant level, then the exciting force increases in proportion to the square of the exciting frequency. In this paper, all the experimental results are shown graphycally in the velocity response amplitude per unit force (kine/ton) against the exciting frequency (Hz). The parameter X stands for the distance between two foundations, E.M. denotes the eccentric moment, FA and FP represent the active and passive foundations, respectively, and N,S,E and W denote the locations of seismometers on the foundations.

The vertical velocity amplitude characteristics for the active and passive foundations due to the vertical harmonic excitation are shown in Figs.4(a) and 4(b), respectively. Although in the case where $X=\infty$, the amplitude characteristics at FA-N and FA-S on the active foundation should essentially be identical, the test results do not coincide with each other. This may come from the inhomogenity of the ground conformation at the site. The smaller the distance between two foundations, the larger the effect of the passive foundation on the amplitude characteristics of the active one becomes, and also the larger the difference of the amplitude characteristics at the locations FA-N and FA-S due to the induced rotational motions. Fig.5 describes the horizontal velocity amplitude characteristics due to horizontal excitation. From this figure, it is found that for the case where distance parameter $X=4$ m, the response of the passive foundation becomes small at 15 Hz in frequency, and at that frequency, the ground motion excited by the active foundation becomes small, too. Figs.6(a) and 6(b) show respectively the vertical velocity amplitude characteristics of the active and passive foundations due to the horizontal excitation along the N-S direction which is parallel to the center line through the two foundations. In this case, the rotational excitation is also imposed on the active foundation in proportion to the height of the horizontal exciting force. Hence, the vertical component of the response of the active foundation appears even for the case without passive one. When the passive foundation is very near to the active one, the response of the active foundation effects remarkably on that of the passive foundation. In Fig.7, the effect of the exciting force level on the responses of both foundations is presented. As the eccentric moment of a vibration generator increases from E.M. = 50 kg·cm to 200 kg·cm, the resonant frequency decreases, and its amplitude becomes large. As far as the general tendency is concerned, its pattern is almost unaltered according to the exciting force level. It is apparent, however, that the soil medium shows the nonlinearity under this testing condition. Fig.8 presents the horizontal velocity amplitude characteristics due to horizontal excitation along the E-W direction which is perpendicular to the center line through the two foundations. If two foundations are close each other, the amplitude characteristics of the active foundation near the resonant frequency seem to be dull, but its effect is not clear in details. It is also found that the magnitude of the amplitude characteristics of the passive foundation due to horizontal E-W excitation are generally smaller than that for N-S excitation. Therefore, the effect of cross-interaction depends not only on the exciting level but also the exciting pattern.

ANALYTICAL STUDIES

In order to compare the experimental result with the analytical one, the vibrational characteristics of two coupled foundations are analyzed under the same condition as the field test. In analyzing the vibrational characteristics, we use the dynamical ground compliance matrix each component of which is defined as the complex ratio of the weighted averaged displacement over the surface area associated with a foundation to the amplitude of the integrated exciting force at another foundation area. The mathematical model consists of two foundations on a visco-elastic stratum and half-space as shown in Fig.9 and 10, and the vertical or horizontal harmonic force acts on one of the two foundations. We deal with the cases where the two foundations are on the surface and confine ourselves to the vertical and horizontal components in the X-direction and the rotational component around the Y-axis for both the responses and excitations. If u_i, w_i and ϕ_i are defined as the horizontal, vertical and rotational nondimentional displacements of the i-th foundation, the equations of motion of the i-th foundation are expressed by

$$\begin{aligned} m_i \ddot{u}_i - a_i m_i \ddot{\phi}_i + r_{ui} &= f_{ui} \\ m_i \ddot{w}_i + r_{wi} &= f_{wi} \\ -a_i m_i \ddot{u}_i + (I_i + m_i a_i^2)\ddot{\phi}_i + r_{\phi i} &= -l_i f_{ui} \end{aligned} \qquad (1)$$

in which

$m_i = M_i/\rho B_1^3$: mass ratio ρ : density of soil
$I_i = \mathrm{I}_i/\rho B_1^5$: moment of inertia ratio μ : shear modulus
$a_i = A_i/B_1$: height of center of gravity B_1 : reference length
$u_i = U_i/B_1$, $w_i = W_i/B_1$, ϕ_i: horizontal, vertical and rotational displacements
$r_{ui} = R_{ui}/\mu B_1$, $r_{wi} = R_{wi}/\mu B_1$, $r_{\phi i} = R_{\phi i}/\mu B_1^3$: horizontal, vertical and rotational reactions
$f_{ui} = F_{ui}/\mu B_1$, $f_{wi} = F_{wi}/\mu B_1$: horizontal and vertical exciting forces
$l_i = L_i/B_1$: height of the horizontal exciting force

Therefore, the equations of motion for two foundations is given in the following matrix form:

$$-a_0^2 [A]\{\tilde{\delta}\} + \{\tilde{r}\} = \{\tilde{f}\} \qquad (2)$$

in which

$$[A] = \begin{bmatrix} m_1 & 0 & -a_1 m_1 & & & \\ 0 & m_1 & 0 & & 0 & \\ -a_1 m_1 & 0 & I_1 + a_1^2 m_1 & & & \\ & & & m_2 & 0 & -a_2 m_2 \\ & 0 & & 0 & m_2 & 0 \\ & & & -a_2 m_2 & 0 & I_2 + a_2^2 m_2 \end{bmatrix}, \quad \{\tilde{\delta}\} = \begin{Bmatrix} \tilde{u}_1 \\ \tilde{w}_1 \\ \tilde{\phi}_1 \\ \tilde{u}_2 \\ \tilde{w}_2 \\ \tilde{\phi}_2 \end{Bmatrix} \qquad (3)$$

$$\begin{aligned} \{f\}_V^T &= (\ 0\ \ f_{w1}\ \ 0\ \ \ 0\ \ 0\ \ 0\ ) \quad \text{for vertical excitation} \\ \{f\}_H^T &= (\ f_{u1}\ \ 0\ \ -l_1 f_{u1}\ \ 0\ \ 0\ \ 0\ ) \quad \text{for horizontal excitation} \end{aligned} \qquad (4)$$

In the above equations, the symbol ~ represents the Fourier transform with respect to dimensionless frequency $a_0=\omega B_1/V_S$ in which ω is frequency and $V_S=\sqrt{\mu/\rho}$ is the velocity of shear wave. If the ground compliance matrix $[\tilde{C}]$ is known, the relation between the reaction vector $\{\tilde{r}\}$ and displacement vector $\{\tilde{\delta}\}$ is presented as follows:

$$\{\tilde{r}\}=[\tilde{C}]^{-1}\{\tilde{\delta}\} \tag{5}$$

$$[\tilde{C}]=[\tilde{C}^{\zeta k}_{\xi l}] \qquad (\zeta=u,w \text{ and } \phi,\ \xi=H,V \text{ and } R,\ k,l=1,2) \tag{6}$$

in which $\tilde{C}^{\zeta k}_{\xi l}$ is the dynamical ground compliance representing the ζ-component of the weighted averaged displacement on the k-th area to the ξ-component of excitation on the l-th area. Substituting Eq.(5) into Eq.(2) and multiplying the compliance matrix $[\tilde{C}]$ from the left, the displacement vector $\{\tilde{\delta}\}$ is written as follows:

$$\{\tilde{\delta}\}=([I]-a_0{}^2[\tilde{C}][A])^{-1}[\tilde{C}]\{\tilde{f}\} \tag{7}$$

in which [I] is a unit matrix with 6×6 dimension.

The numerical calculation is made for the case of visco-elastic half-space. The analytical results are shown graphically in Figs.11(a), 11(b) and 12. In these figures, the parameters ν, $\eta_p=(V_S/B_1)(\lambda'+2\mu')/(\lambda+2\mu)$, $\eta_S=(V_S/B_1)\mu'/\mu$ and α are Poisson's ratio, dimensionless viscous coefficients for P- and S-waves, in which λ, μ are Lame's constants and λ', μ' are associated viscous constants, and a multiplier to the analytical results considering the nonlinearity of a soil medium. The values of the parameters are shown in each figures, otherwise they are summarized in Table 1. Figs. 11(a) and 11(b) show the vertical velocity amplitude characteristics of the active foundation and the passive foundation due to vertical excitation, respectively, and they correspond to Figs.4(a) and 4(b). It is pointed out that at the vicinity of resonant frequency, the smaller the distance between two foundations, the larger the interaction effect becomes. The results for the horizontal excitation are shown in Fig.12 which correspond to Fig.5. It is noted that for the case where distance parameter $X=4$ m, the decrease of the amplitude characteristics at about 15 Hz, which was found in experimental results, is predicted by the analytical procedure.

CONCLUSIONS

In the preceding sections, the cross-interaction effect of a foundation system on its amplitude characteristics was examined experimentally and analytically, and those results were compared with each other. For the actual soil ground, it is difficult to formulate exactly its mathematical model because of the nonlinearity of the soil medium and of the complexity of layer formation. However, it is noticed that the analytical prediction based on a rather simple model could be coincide with experimental evidence. Also it is pointed out from these studies that the cross-interaction effects of adjacent structures are remarkable over the wide frequency range as the distance becomes small. Therefore, we believe that the consideration on the cross-interaction among adjacent structures would be important in the aseismic design of structures in big cities.

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Table 1 Values of parameters for numerical example.

		Active F. $i=1$	Passive F. $i=2$
mass ratio	m_i	2.95	2.20
m. of inertia ratio	I_i	0.997	0.754
height of center	a_i	0.222	0.175
height of force	l_i	0.65	

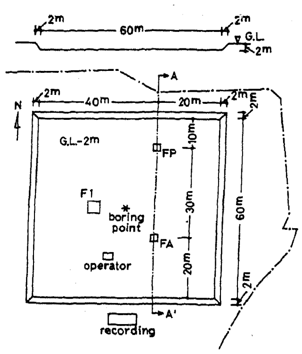


Fig.1 Site for the experiment.

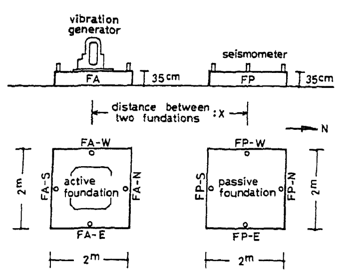


Fig.3 Model of active (FA) and passive(FP) foundations.

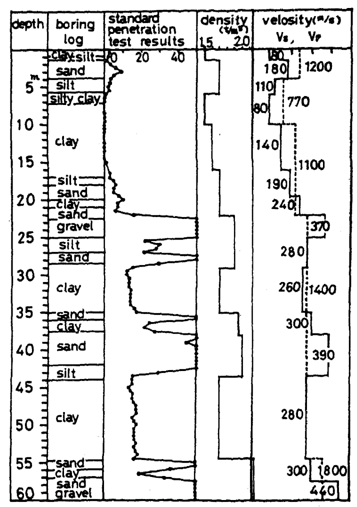
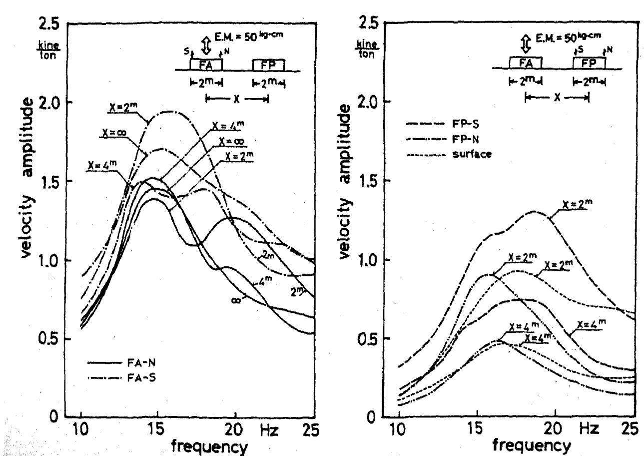


Fig.2 Soil profile.



(a) Active foundation (b) Passive foundation

Fig.4 Vertical amplitude to vertical excitation.

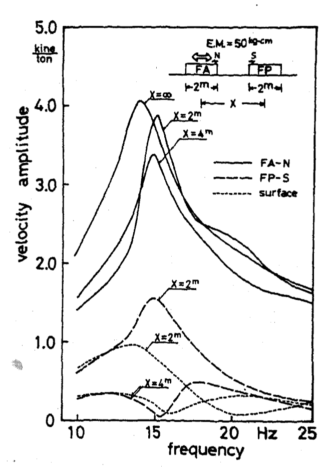
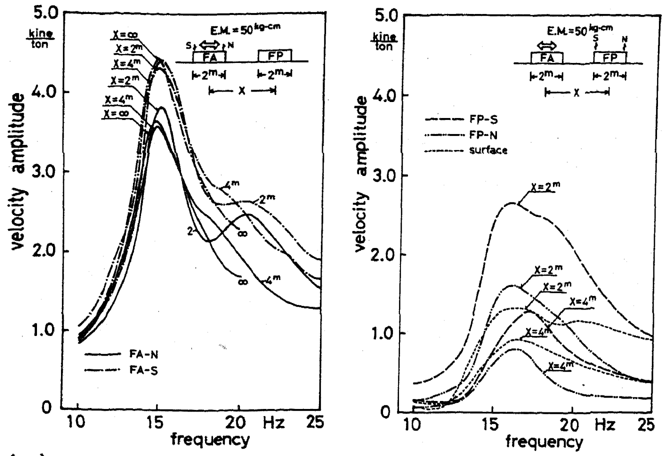


Fig.5 Horizontal amplitude to horizontal NS excitation.



(a) Active foundation (b) Passive foundation
Fig.6 Vertical amplitude to horizontal NS excitation.

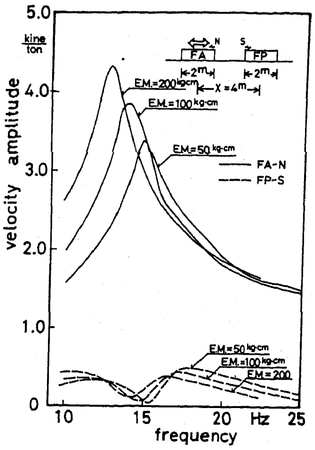


Fig.7 Horizontal amplitude to horizontal NS excitation.

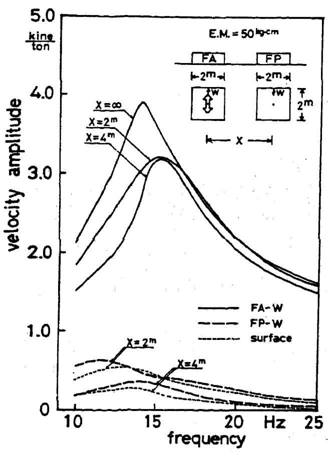


Fig.8 Horizontal amplitude to horizontal EW excitation.

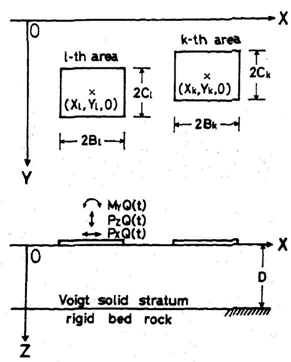


Fig.9 Soil-foundation system considered.

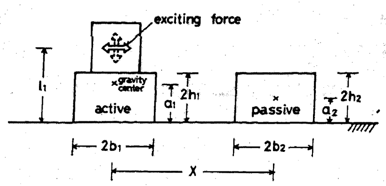
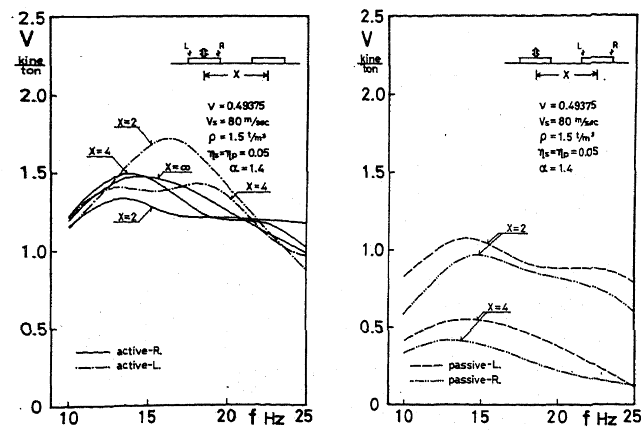


Fig.10 Foundation-soil-foundation system considered.



(a) Active foundation (b) Passive foundation
Fig.11 Vertical amplitude to vertical excitation.

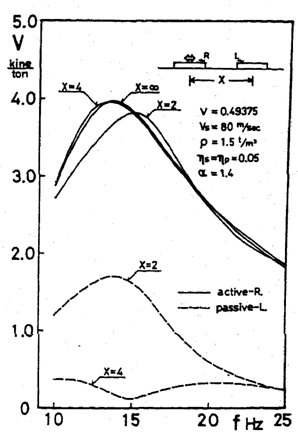


Fig.12 Horizontal amplitude to horizontal excitation.