ON THE FUNDAMENTAL DIFFERENCES OF THREE BASIC SOIL-STRUCTURE INTERACTION MODELS

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SYNOPSIS

The theoretical modeling of a flexible soil-foundation system, undergoing dynamic interaction with a superstructure, is often represented by a rigid base placed on a soil medium which resembles either an infinite half space, a layered stratum, or a soil medium with finite size. Due to wave interferences caused by the geometrical boundaries, these models possess some marked differences in their dynamic behavior. In this paper, the differences between these models are discussed on the basis of exact solutions for two-dimensional soil-structure interaction problems. The results indicate that the half space model induces "radiation damping" on the structural response because of geometrical spreading of waves. In the other extreme, the finite "box-like" soil medium may trap all or a significant portion of the wave energy within, thus creating unrealistic resonant conditions. For the case of an infinite horizontal layer, some resonant modes occur even though radiation damping is present.

INTRODUCTION

In analyzing complicated problems, such as those of dynamic soil-structure interaction, quite often the capability of the most sophisticated and detailed methods of analysis become insufficient. In such problems, some details must be sacrificed. Generally, the most crucial step in simplifying a soil-structure interaction analysis is the first step, i.e., the selection of an appropriate soil and foundation model. To date, popular models for the soil medium may be classified roughly into three groups: (A) a half space, (B) a layered stratum, (C) a soil medium enclosed partly or totally by a rigid boundary. The latter model is sometimes chosen when numerical methods such as finite element and finite differences are applied; the finiteness of the model dimensions is primarily restricted by the limited core space available in present day computers.

Luco et al. (1974) investigated the differences between Models A and C by first comparing the exact solution for a rigid strip foundation on an elastic half space (Model A) to a two-dimensional finite element solution of a strip over a rectangular soil medium (Model C) fixed at its sides and its bottom. Their conclusions indicated that the characteristics of Model C are entirely different from those of Model A. The impedance (the resistance to loading) of the soil-foundation system for Model C exhibits an oscillatory behavior, caused mainly by the constructive and destructive interferences between the radiated waves and the waves rebounded from the rigid boundaries. These wave interference phenomena were also observed in an analogous three-dimensional analysis with a circular disc foundation (Atalik et al., 1975); the most prominent interferences occur for the symmetrical vibration modes such as the vertical translation mode.

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In the following section, the physical characteristics of the three basic models are examined by means of exact solutions. The differences between models A, B, and C can then be identified by the results of a parametric study without the bias introduced by approximate numerical solutions.

THE MODELS

Consider three, two-dimensional models as illustrated in Fig. 1. For simplicity, only the anti-plane (SH) motion is studied; also, the embedded rigid base is assumed to have a semicircular cross-section of radius a_1. The model in Fig. 1B consists of just one infinite layer of depth h over a rigid basement, while in the model of Fig. 1C, the rigid boundary is assumed to be semicircular with radius a_2. The soil properties used are linear and homogeneous, but both elastic and viscoelastic constitutive laws have been considered.

In the following case studies, we chose to use the impedance function to characterize a particular soil-foundation system subjected to external loading. The complex impedance K, which is the force/displacement relation for the soil-foundation system, can be viewed as (Luco et al., 1975; Wong, 1975)

$$K = k + i\omega c \tag{1}$$

where k and c are real and frequency dependent quantities which represent the stiffness and damping contribution to K, respectively. Therefore, K may be treated as a physical quantity adequate for describing the flexibility of the soil medium.

From an exact solution of the two-dimensional wave equation, the harmonic impedance function for Model (A) of Fig. 1A can be expressed in terms of Bessel functions, J_n and Y_n (Wong, 1975), as

$$K(A) = \mu\pi\kappa a_1 \frac{[J_1(\kappa a_1) - i\,Y_1(\kappa a_1)]}{[J_0(\kappa a_1) - i\,Y_0(\kappa a_1)]} \tag{2}$$

where $\kappa = \omega/\beta$ is the wave number of the soil medium and $i = \sqrt{-1}$. The quantities $\beta = \sqrt{\mu/\rho}$, μ, and ρ are the shear wave velocity, shear modulus, and mass density of the soil, respectively.

Using a similar procedure (Wong, 1975), the expression for the impedance function of Model (C) can be obtained simply as

$$K(C) = \mu\pi\kappa a_1 \frac{[Y_0(\kappa a_2)J_1(\kappa a_1) - J_0(\kappa a_2)Y_1(\kappa a_1)]}{[Y_0(\kappa a_2)J_0(\kappa a_1) - J_0(\kappa a_2)Y_0(\kappa a_1)]} \;. \tag{3}$$

These expressions for K(A) and K(C) are valid for both elastic and viscoelastic soil properties. For the latter case, the solutions can be revised by replacing μ by $\tilde{\mu} = \mu(1 + i\delta)$, where δ is the viscous damping coefficient. For model C, the viscous dissipation is the only form of energy dissipation.

To compare the numerical values of the impedance functions, K(A) with $\delta = 0$, and K(C) with $\delta = 0, 0.1, 0.25, 0.5$, and 1.0, are plotted in Fig. 4 for $a_2/a_1 = 5$ and in Fig. 5 for $a_2/a_1 = 10$.

Consider first the case where $\delta = 0$, i.e., elastic soil properties. A direct comparison of equations (2) and (3) reveals that K(A) is complex while K(C) is real; this suggests that no "radiation damping" due to geometrical spreading is possible for Model (C) because all the wave energy is trapped within the rigid boundaries. Another important difference between K(A) and K(C) is that both the numerator and denominator of K(C) have a finite number of zeroes as the dimensionless frequency κa_1 increases. At the frequencies where the numerator is zero, K(C) becomes zero, indicating that the waves interfere constructively inside the rigid boundaries and that the soil-foundation system has lost resistance to the applied load at that frequency. However, at the frequencies where the denominator is zero, K(C) is infinitely large; implying that the rigid base is located at the node of a standing wave pattern where the motion is zero. The number of zeroes of K(C) in a given interval of frequency depends mainly on the distance a_2 of the rigid boundary from the foundation, this is shown clearly in Figs. 4 and 5, where the ratio a_2/a_1 is 5 and 10, respectively. For a larger ratio of a_2/a_1, the enclosed medium can accommodate more standing waves than a smaller medium. Therefore, K(C) oscillates with higher frequency (compare Fig. 4 with Fig. 5) when the ratio a_2/a_1 is increased. This result might be surprising for some, because many investigators have speculated that the results should approach the limiting half space solution if the size of the model is increased. Judging from equations (2) and (3), K(A) cannot be obtained from K(C) by limiting a_2 to infinity, this simply indicates the different nature of the impedance for the two models.

Since actual soil has some inherent damping, we shall continue our discussion on Models A and C by studying the viscoelastic soil effects on energy dissipation. Through the complex shear modulus $\tilde{\mu}$, both K(A) and K(C) are complex, but the nature of material damping and radiation damping is quite different as shown by Fig. 4 and Fig. 5. At the lower limit of $\omega a_1/\beta$, the material damping [imaginary part of K(C)] is much lower than the radiation damping because the strain rate of the material is low for low frequencies. This large difference between K(A) and K(C) at low frequency is largely a characteristic of two-dimensional models; but other differences, such as the oscillation of K(C), is common for three-dimensional problems as well (Atalik et al., 1975).

Consider now the intermediate case of an infinite layer over a rigid basement. This particular model, shown in Fig. 1B, is the simplest of all layered models, but it can qualitatively explain some of the basic characteristics of a more sophisticated layer model, expecially when the top layer is much "softer" than those below it.

To obtain the exact solution of K(B) for the model shown in Fig. 1B, we may apply the method of images to represent the reflections from the basement by summing the contribution from image foundations (alternating in sign) so that boundary conditions at both the free surface and the basement are satisfied. Due to the higher harmonics obtained from the boundary reflections, the solution of K(B) resulted in an infinite series of Hankel functions.

With the configuration of Model (B), radiation damping is possible because waves can propagate outward in the horizontal direction, but the reflections from the basement may also form standing wave patterns. Hence, one can expect the characteristics of Model (B) to lie between Models (A) and (C). In Fig. 2, values of K(A) and K(B) are plotted versus the dimensionless frequency, $\omega a_1/\beta$, for $\delta = 0.01$, the layer depth, h, is $5a_1$. Since the layer depth is relatively shallow, and the material damping is low, K(B) is oscillatory and is similar in behavior to K(C). The real part of K(B), i.e., the stiffness, has several zero crossings occurring within the frequency band shown. These zeroes are result of the constructive wave interference at the neighborhood of the foundation; clearly, the strength of the interference will decrease if the layer depth h is increased. In Fig. 3, the layer depth, h, has been increased to $10a_1$, and $\delta = 0.1$ is used. The consequence of this change is that K(B) is remarkably similar to K(A) except for some small oscillations, the imaginary parts of both models also represent damping of the same nature. Hence, for a case where a relatively deep top layer exists, the layered stratum may be modeled adequately by a viscoelastic half space.

CONCLUSION

The theoretical modeling of a soil medium by either (A) a half space, or (B) a layered stratum, or (C) a bounded medium has been discussed. Since each of these models have distinctly different characteristics, the initial selection of a model for a detailed analysis must be made with the following consequences in mind: (1) the selection of a model without wave transmitting boundaries terminates the paths of wave propagation, and the resulting model is characterized mainly by standing wave patterns and hence, resonant type behavior. (2) The material damping introduced into the soil model cannot stimulate the radiation damping consistently, therefore, the proper model must be used if the foundation site is not surrounded by relatively rigid boundaries.

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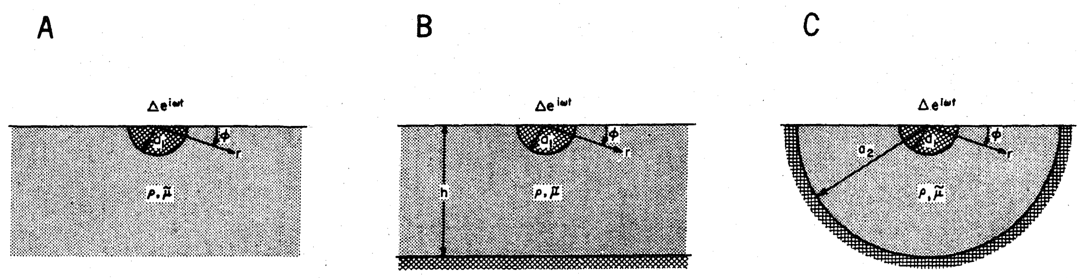

Fig. 1. Simplified two dimensional models: (A) half space, (B) layer over rigid ground, (C) enclosed soil medium.

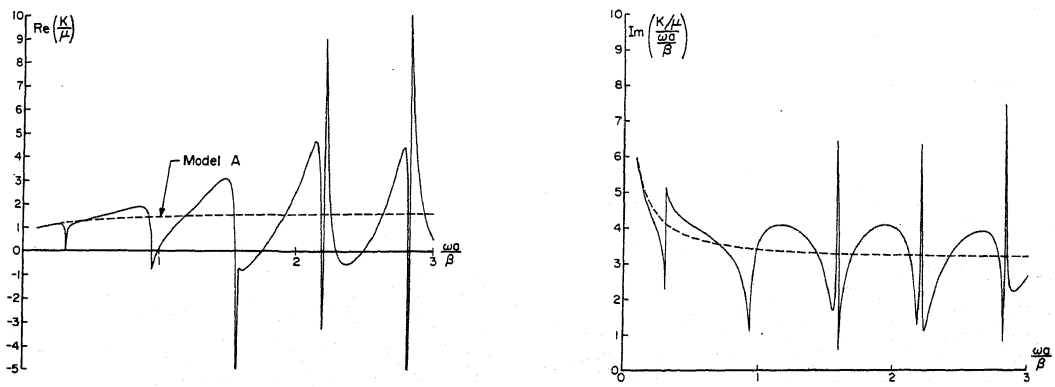

Fig. 2. Impedances for model B; $h/a_1 = 5$, $\delta = 0.01$.

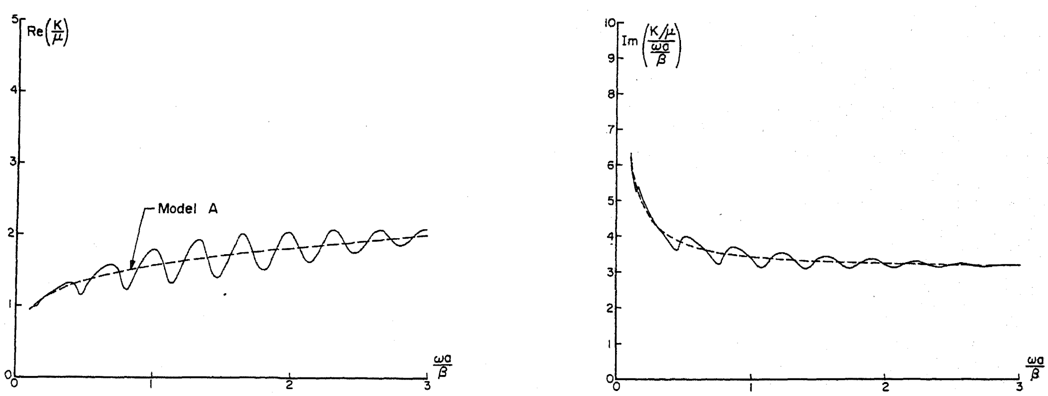

Fig. 3 Impedances for model B; $h/a_1 = 10$, $\delta = 0.1$.

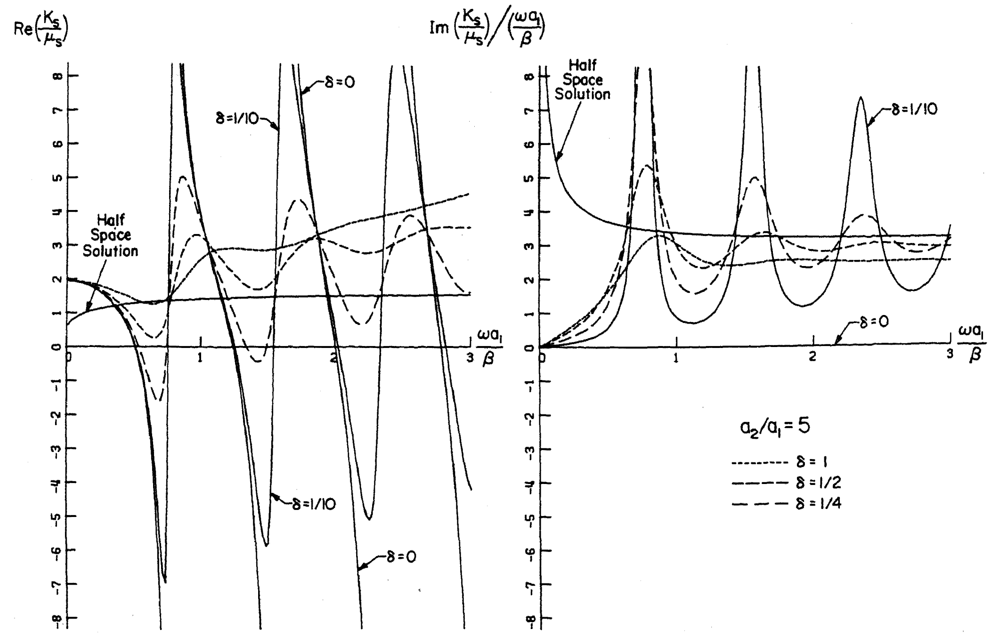

Fig. 4. Impedances for model C, $a_2/a_1 = 5$.

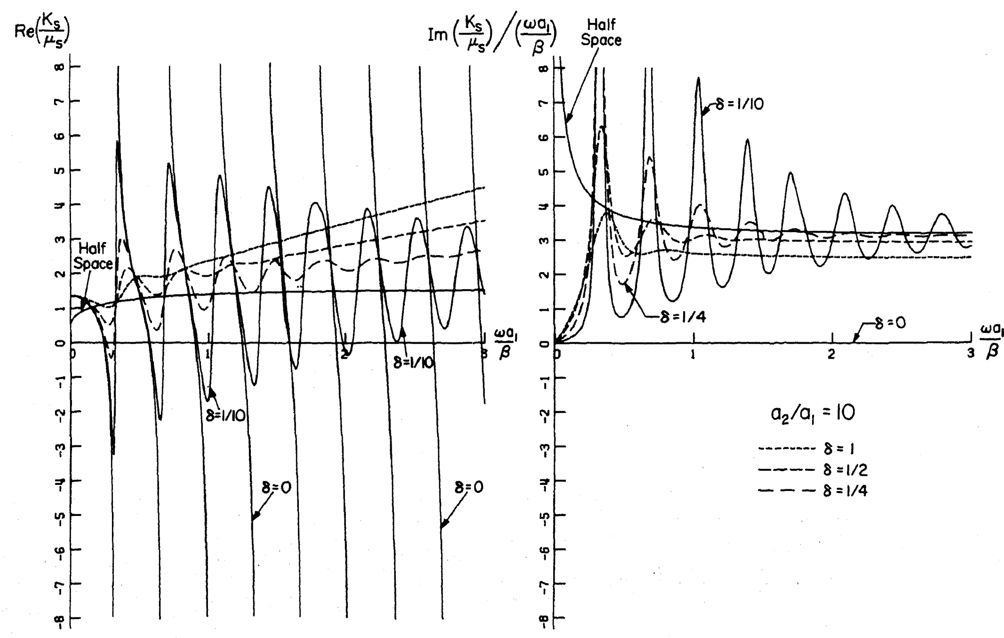

Fig. 5. Impedances for model C, $a_2/a_1 = 10$.