ANALYSIS OF THE DAMPED RESPONSE OF A MULTISTOREY BUILDING, CONSIDERED AS A HEAVY TIMOSHENKO BEAM, TO A BIDIMENSIONAL SHAKING MOTION OF THE GROUND

F. Buckens[I]

INTRODUCTION

Vertical motions of the ground are usually neglected in earthquake engineering and it is shown that vertical accelerations are usually small compared to g. On the other hand it is known that natural frequencies and modes are functions of the distributed weight, whose effect is of the same order of magnitude as those of rotating inertia and shear forces (1).

It is interesting to analyze in a plane the effect of two dimensional ground motions, including possibly the presence of vertical accelerations, (as in the case of Rayleigh waves), first without internal damping in the structure, further including a damping of a viscous type.

Such a motion at the base of the structure has horizontal and vertical components $X_o(t)$, $Y(t)$, with a tilt $\varsigma(t)$.

THE EQUATIONS OF MOTION OF AN UNDAMPED STRUCTURE

Whereas pure bending deformations of structures considered as beams imply that plane cross-sections remain plane and normal to the neutral line, the effect of shear strains entails the appearance of an additional angle ψ (Fig.1)

$$\psi = S/\beta = S/\kappa AG \qquad (1)$$

(S, shear force, β, shear modulus, A, cross-section area, κ, shape coefficient).

If y is the deflection of the neutral line and ϕ the slope of the normal to the cross-section, one has

$$Dy = \partial y/\partial x = \phi + \psi \qquad (2)$$

with the constitutive law

$$D\phi = -M/\alpha = -M/EI \qquad (3)$$

I Professor, University of Louvain 1348, Louvain-la-Neuve, Belgium.

(M, bending moment, α = EI bending modulus.

The equations of motion are, with $\dot{y} \equiv \partial y / \partial t$,

$$D(S + N\phi) = m(\ddot{y} + \ddot{z}) \qquad (4)$$

$$-DM + S - N\psi = r(\ddot{\phi} + \ddot{\xi}) \qquad (5)$$

where N is the normal force at a cross-section, m and r are the distributed mass and rotatory inertia, and

$$Z = x\ddot{\xi} - g(\zeta - \zeta_o) + \ddot{y}_o \qquad (6)$$

the axial force N being given at each cross-section by:

$$N = -(\ddot{X} + g)\int_x^L m\,dx \qquad (7)$$

while N(L) = O, and the longitudinal deformation of the structure is neglected.

Eliminating M, S and ψ from equs. (1) to (5) yields:

$$D[\beta Dy - (\beta - N)\phi] = m(\ddot{y} + \ddot{Z}) \qquad (8)$$

$$(\beta - N)Dy - (\beta - N - D\alpha D) = r(\ddot{\phi} + \ddot{\xi}) \qquad (9)$$

In matrix form, the system (8), (9) becomes:

$$K\eta + J(\ddot{\eta} + \ddot{z}) = 0 \qquad (10)$$

where

$$K = \begin{bmatrix} -D\beta D & D(\beta - N) \\ -(\beta - N)D & \beta - N - D\alpha D \end{bmatrix} \quad J = \begin{bmatrix} m & 0 \\ 0 & r \end{bmatrix} \qquad (11)$$

$$= \begin{bmatrix} y \\ \phi \end{bmatrix} \quad z = \begin{bmatrix} Z \\ \zeta \end{bmatrix} \qquad (12)$$

The differential matrix operator K is self-adjoint, that is, according to the Betti-Maxwell reciprocity theorem, the scalar products,

$$<\eta_1, \eta_2> = \int_0^L \eta_2^{\tau} K \eta_1 dx = \int_0^L \eta_1^{\tau} K \eta_2 dx \equiv <\eta_2, \eta_1> \qquad (13)$$

are equal for any two sets of function pairs which satisfy the end conditions, which are assumed such that, as usual:

$$\left[y_2(S_1 + N \Phi_1) - y_1(S_2 + N \Phi_2) - \Phi_2 M_1 + \Phi_1 M_2 \right]_0^L = 0 \qquad (14)$$

(τ meaning transpositions, and L the height of the structure), the verification by part integration being then straight forward.

Standard procedure shows then the orthogonality of the eigenmodes $\eta_n(x)$ defined by setting

$$\ddot{X} = \ddot{z} = 0 \text{ and } \eta = \eta_n e^{jw_n t} \qquad (15)$$

in equ. (10), i.e.,

$$<\eta_n, \eta_m> = 0 \quad \text{if} \quad w_n \neq w_m \qquad (16)$$

and also the reality of these corresponding eigenfrequencies.

When vertical motion accelerations are neglected ($\ddot{X}=0$), the solution of equ. (10) is classical and can be represented by standard Laplace integrals by putting

$$z(s) = \int_{0^-}^{\infty} e^{st} z(t) ds \qquad (17)$$

$$\eta(s) = \int_{0^-}^{\infty} e^{st} \eta(t) ds \qquad (18)$$

with:

$$(K_o + s^2 J) \eta(s) + s^2 J z(s) = 0 \qquad (19)$$

K_o being defined by putting in equ. (11)

$$N_o = - g \int_x^L m dx \qquad (20)$$

instead of N in matrix K.

Eventually the solution $\eta(s)$ provides the actual motion $\eta(t)$ at each cross-section of the structure by the well-known Bromwich inverse integral.

If on the other hand vertical accelerations must be considered, equ. (10) has coefficients which depend on time, and if $\ddot{z}$ and N are periodic with the same period, cases of dynamic instability can arise, which can be analysed by a generalization of Floquet's method.

If the excitation amplitude N_a, ($|N-N_o| \leq |N_a|$), is taken as a parameter dependant on of N_o, the domains of stability and instability (e.g. the motion oscillations tend to decrease or to increase in amplitude with time) are separated by boundaries in the (N_a, N_o) plane, for which the solution η is perildic.

The corresponding curves $N_a = f\ (N_o)$ can be determined by numerical integrations of (10), by the conditions

$$\eta(T) = \eta(0) \qquad (21)$$

$$\dot{\eta}(T) = \dot{\eta}(0) \qquad (22)$$

THE EFFECT OF DAMPING

In a previous paper (2) the effect of damping in the structure deformations was considered in its viscous form, only for the bending deformations. As in the present case the shear deformations are taken into account, it is interesting to introduce the corresponding damping effect which presumablY is relatively more important than the one for bending.

Here viscosity, as an approximation to structural damping, is assumed by replacing in the constitutive equations (1) and (2) the shear and bending coefficients β and α by linear operators in $\partial/\partial t$; thus by replacing β and α by $\beta+p\delta$, and $\alpha+p\gamma$, where $p = \partial/\partial t$ and $\beta, \delta, \alpha, \gamma$ are coefficients which depend only on x. Again we use Fraeijs de Veubeke's characteristic lag method (3); in this method, normal modes of lagging deformations and corresponding external forces (represented here in d'Alembert's inertial acceleration form) are determined for each frequency, and superposition sith their respective characteristic lags λ_n allows the resposse to the actual imposed forces.

Assuming again that $\ddot{X}$ may be neglected, the same lines can be followed as in (2) in order to determine the phase lags λ_k in .

$$\eta = \sum_{k=1}^{\infty} Y_k(x) \sin (wt - \lambda_k) \qquad (23)$$

for a steady periodic response to

$$z = \sum_{k=1}^{\infty} Z_k(x) \sin (wt) \qquad (24)$$

With the previous notations, equ. (10) becomes :

$$(K + pH + p^2 J)\eta + p^2 Jz = 0 \qquad (25)$$

where

$$H = \begin{bmatrix} -D\delta D & D\delta \\ -\delta D & \delta - D\gamma D \end{bmatrix} \qquad (26)$$

Identification yields the two systems of equations

$$(K-w^2J)Y \cos\lambda + wHY \sin\lambda = w^2 JZ \qquad (27)$$

$$(K-w^2J)Y \sin\lambda - wHY \cos\lambda = 0 \qquad (28)$$

Now the boundary conditions for a structure clamped at its base ($y = \phi = 0$ for $x = 0$) and free at its top ($M = S = 0$ for $x = L$) are expressed in matricial form by:

$$\eta = 0 \text{ (for } x = 0), \quad D\eta = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ (for } x = L) \qquad (29)$$

and the combination of system (28) with the systems of equations (29) constitutes an eigenvalue problem for the parameter $\cot\lambda$ which provides the characteristic time lags λ_k ($k = 1, 2, \ldots$).

The corresponding eigenvectors Y_k, defined by (28), determine now the characteristic displacement distributions Z_k.

As in (2), the eigenvectors Y_k form an orthonormal basis whereas the sets of the Y_k and Z_k are biorthogonal. That is :

$$\int_0^L Y_j^T H Y_k \, dx = \int_0^L Y_j (K-w^2J) Y_k \, dx = 0, \qquad (30)$$

and

$$\int Y_j J Z_k = 0 \qquad (31)$$

if $\cot \lambda_k \neq \cot. \lambda_j$.

The rest of the procedure in defining the vector z is the same as in (2) and allows to compute the damped response of the structure to any shaking motion of the ground.

CONCLUSION

The method of characteristic phase lags has been generalized from (2), where the structure was considered as a ordinary beam submitted to shaking inertial forces, to the case of the Timoshenko beam in which not only shear deformations and rotatory inertia, but also axial forces due to the own weight of the structure are taken into account. It is pointed out that the effect of vertical accelerations leads to problems of instability of the parametric type; this point has not been investigated further.

REFERENCES

(1) Cheng, Franklin Y. and Carlton L. Smith, "Static and Dynamic Instability Analysis of Frameworks with Timoshenko members", Symposium on earthquake engineering, Indian Soc. of Earthquake Tech., nov. 14-16, 1970, pp.231-239.

(2) Buckens, F., "Approximating the Characteristic Phase Lags of the Vibration Modes in a Multistorey Building, in Function of the imposed Frequencies", Symposium on earthquake engineering, Ind. Soc. of Earthquake Eng., niv. 14-16, 1970, pp. 240-246.

(3) Fraeija de veubeke, B. "Dephasages caracteristiques et Vibrations forcees d'un Systeme amorti", Acad. Roy. de Belgique, Bull. Cl. des Sc. 5, XXXIV, 1948, pp. 626-641.