A PROBABILISTIC APPROACH TO THE STUDY OF LINEAR RESPONSE OF STRUCTURES UNDER MULTIPLE SUPPORT NON-STATIONARY GROUND-SHAKING

by

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SYNOPSIS

Structural behaviour is modelled by a multidegree-of-freedom linear dynamic system. An equation of motion contemplating earthquake loading as a multiple support input and/or propagating ground motion, is presented. Earthquake vibration is idealized as a non-stationary Gaussian stochastic process. Magnitude, source mechanism, distance to fault and local soil conditions are considered. Under those assumptions the structural response is also a non-stationary Gaussian stochastic process, whose most important probabilistic properties are derived.

STRUCTURAL DYNAMICS

The general form of the equation of motion is:

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{p} \qquad 1)$$

in which $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the inertia, damping and stiffness matrices; $\mathbf{q}$ is the vector of generalized coordinates; time derivation is represented by a dot;and $\mathbf{p}$ is the generalized forces vector.

Let $\mathbf{q}^b$ denote the coordinates modelling the points of support of the structure (base) and $\mathbf{q}^f$ the other coordinates; thus, reordering: $\mathbf{q}^T = [(\mathbf{q}^f)^T, (\mathbf{q}^b)^T]$ (where T denotes transposition); $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{p}$ are also reordered and partioned to explicit support displacement: $\mathbf{M}=[[(\mathbf{M}^{ff})^T, (\mathbf{M}^{bf})^T]^T, [(\mathbf{M}^{fb})^T, (\mathbf{M}^{bb})^T]^T]$, $\mathbf{C}=\dots$ and $\mathbf{p}=[(\mathbf{p}^f)^T, (\mathbf{p}^b)^T]^T$; The displacement $\mathbf{q}^f$ are decomposed in a dynamic $\mathbf{q}^d$ and a pseudostatic displacement $\mathbf{q}^s$: $\mathbf{q}^f = \mathbf{q}^d + \mathbf{q}^s$; $\mathbf{q}^s$ is the displacement due to support motion in the absence of inertia forces and is computed from $\mathbf{q}^s = \mathbf{R}^{fb}\mathbf{q}^b$ where $\mathbf{R}^{fb} = -(\mathbf{K}^{ff})^{-1}\mathbf{K}^{fb}$.

For simplicity it will be assumed the system has classical modes of vibration for the free coordinates, with frequencies f_i; and mode shape vectors $\mathbf{z}$; and that only the first n modes are of interest. Let $\mathbf{Z}$ be the rectangular matrix $\mathbf{Z}=[\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n]$. The vector of dynamic modal amplitudes $\mathbf{x}$ will be defined by: $\mathbf{q}^d=\mathbf{Z}\mathbf{x}$. The orthogonality properties and the normalization condition of the modes are expressed by: $\mathbf{Z}^T\mathbf{M}^{ff}\mathbf{Z}=\mathbf{I}$, $\mathbf{Z}^T\mathbf{K}^{ff}\mathbf{Z} = [4\pi^2 f_i^2]_d$ where $\mathbf{I}$ is the identity matrix and $[\ ]_d$ denotes a diagonal matrix. It will also be assumed that damping forces acting on the free coordinates do not introduce modal coupling; so: $\mathbf{Z}^T\mathbf{C}^{ff}\mathbf{Z}=[4\pi\zeta_i f_i]_d$ where ζ_i is the percentual damping of the i-th mode. Let $\boldsymbol{\imath}=\boldsymbol{\Lambda}\mathbf{q}$ be the column vector of design responses (Newmark and Rosenblueth, 1971). Design responses are strains, stresses, bending moments, ... or any quantity of interest that is a linear function of the generalized coordinates. $\boldsymbol{\Lambda}$ is a matrix of influence coefficients. In terms of the base and free coordinates: $\boldsymbol{\imath}=[\boldsymbol{\Lambda}^f, \boldsymbol{\Lambda}^b][(\mathbf{q}^f)^T, (\mathbf{q}^b)^T]^T$ and expliciting the dynamic displacements and modal amplitudes: $\boldsymbol{\imath} = \boldsymbol{\Lambda}^f\mathbf{q}^d + (\boldsymbol{\Lambda}^f\mathbf{R}^{fb}+\boldsymbol{\Lambda}^b)\mathbf{q}^b = \boldsymbol{\Lambda}^f\mathbf{Z}\mathbf{x}+(\boldsymbol{\Lambda}^f\mathbf{R}^{fb}+\boldsymbol{\Lambda}^b)\mathbf{q}^b$. For simetry and generality the loading will also be made a linear function of a vector of generalized loadings: for the present purposes only base motion will be considered; so $\mathbf{p}^f = 0$. Expressing base displacements as a linear function of a vector $\mathbf{u}$ of load motions: $\mathbf{q}^b = \Delta\mathbf{u}$, where Δ is a matrix of influence

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coefficients. Consider, for example, a base rotation θ about a vertical axis Y_3; let q^b_1 be an horizontal base displacement at point P_1 and along a straight line L ; the corresponding coefficient in Δ is the distance between Y_3 and L . Let $\Lambda^x = \Lambda^f \mathbf{Z}$, $\mathbf{M}^z = -\mathbf{Z}^T(\mathbf{M}^{fb} + \mathbf{M}^{ff}\mathbf{R}^{fb})\Delta$, $\mathbf{C}^z = -\mathbf{Z}^T(\mathbf{C}^{fb} + \mathbf{C}^{ff}\mathbf{R}^{fb})\Delta$ and $\Lambda^u = (\Lambda^f \mathbf{R}^{fb} + \Lambda^b)\Delta$. After some algebra and expliciting the time dependence:

$$\iota(t) = \Lambda^x \mathbf{x}(t) + \Lambda^u \mathbf{u}(t) \qquad 2)$$

$$\ddot{\mathbf{x}}(t) + [4\pi\zeta_i f_i]_d \dot{\mathbf{x}}(t) + [4\pi^2 f_i^2]_d \mathbf{x}(t) = \mathbf{M}^z \ddot{\mathbf{u}}(t) + \mathbf{C}^z \dot{\mathbf{u}}(t) \qquad 3)$$

GROUND MOTION IDEALIZATION

Earthquake ground motion is idealized as a vibration irradiating from several lower magnitude earthquakes, whose foci are closely spaced along a fault. The distance between foci is used to modell the intensity of vibration originating in different zones of the fault, and breakage velocity is represented by the time lag between the begining of the activity of consecutive foci. Each focus causes at the site of interest an elementar motion idealized by a stationary Gaussian stochastic process restricted to a time interval. The three components of translation are assumed independent; the influence of magnitude, source properties, focal distance and local site conditions are accounted for in the power spectral density and duration of the elementar motion. The two horizontal components were assumed to have the same power spectral density. Power spectral densities for horizontal and vertical vibrations for a large number of situations are available elsewhere(2). This idealization of earthquake ground motion is discussed in more detail in a companion paper (3).

Motions are usually described in terms of accelerations, velocities or displacements. The corresponding power spectral densities are related to one another by: $S_{\ddot{y}\ddot{y}}(f) = 4\pi^2 f^2 S_{\dot{y}\dot{y}}(f) = (4\pi^2 f^2)^2 S_{yy}(f)$. This is a particularization of the general case of the cross-spectral density between a process and its time derivative: $S_{y\dot{y}}(f) = -2\pi i f S_{yy}(f)$, $S_{\dot{y}y}(f) = +2\pi i f S_{yy}(f)$.

Assuming that the motion is due only to shear waves allows an easy derivation of the power spectral densities for base rotations. Let v_s be the shear wave velocity, θ_1, θ_2 and θ_3 the rotations about two horizontal orthogonal axis Y_1 and Y_2 and the vertical axis Y_3, respectively. The cross-spectral densities will be dependent on the polarization of the waves. Let (i, j, k) be an even permutation of (1, 2, 3). It will be assumed that a shear wave propagating along a Y_i - axis will have two probabilisticly independent components, one being an $Y_i\ Y_j$ plane-polarized wave (ppw), the other an $Y_i\ Y_k$ ppw. Motion along Y_i axis will be due to the sum of the $Y_j\ Y_i$ ppw with the $Y_k\ Y_i$ ppw. For obvious physical reasons, the six ppw's will be idealized by six independent stochastic processes. The power spectral density of an $Y_j\ Y_i$ ppw will be assumed equal to the power spectral density of the $Y_k\ Y_i$ ppw; and, thus, equal to $S_{y_i y_i}(f)/2$. The rotations are the components of the vector equivalent to the skew-symetric part of the gradient of the displacements: $\theta_i = (\partial y_j / \partial y_k - \partial y_k / \partial y_j)/2 = (-\dot{y}_j/v_s + \dot{y}_k/v_s)/2$. Hence, the power spectral density for θ_i is:

$$S_{\theta_i\theta_i}(f) = S_{\dot{y}_j\dot{y}_j}(f) / 4 v_s^2 \qquad 4)$$

Attending to the particular properties of the assumed modell the rotations are three independent processes; so $S_{\theta_l\theta_m}(f) = 0$ $(l \neq m)$. The cross-spectral densities between translations and rotations, may also be easily calculated:

$S_{\theta i\ yi}(f) = 0\ ;\ S_{\theta i\ yj}(f) = -\ S_{yj\ yj}(f)\ /\ 2\ v_s$ 5)

The assumption that ground motion is associated to shear waves is debatable, at last, for large focal distances. Rotation about a vertical axis are also associated with Love waves and rotation about horizontal axis with Rayleigh waves. Important rotational components may also be found near the boundary between two geological formations of very different stiffnesses; in this case a strong correlation between translational and rotational components is to be expected.

So far, ground vibration has been described in terms of the motion of a point. However, for extended structures, it may be necessary to idealize differences in ground motion along the base of the structure. Assuming earthquake vibration to be the result of waves travelling in several directions, consider a wave propagating with velocity v along the base of the structure. Let u_i and u_j be two load motion coordinates corresponding to two points separated by distance d, measured along the propagation direction;then $u_i(t) = u_j(t + d/v)$. Describing motion at u_i by a power spectral density S(f), motion at u_j will also be characterized by the same power spectral density. The cross-spectral density between u_i and u_j is easily derived from the correlation function $R_{u_i\ u_j}(\tau) = E\{u_i(t)\ .\ u_j(t+\tau)\} = E\{u_i(t).u_i(t+\tau+d/v)\} = R_{u_i\ u_i}(\tau + d/v)$ (where $E\{\ \}$ denotes expectation). As spectral densities and corresponding correlation-functions are Fourier pairs, then:

$$S_{u_i\ u_j}(f) = \int_{-\infty}^{\infty} R_{u_i\ u_i}(\tau + d/v)\ \exp(-\ 2\ \pi\ if\tau)\ d\tau = \exp(2\ \pi\ if\ d/v)\ S_{u_i\ u_i}(f) \qquad 6)$$

SPECTRAL MOMENT DESCRIPTION OF RESPONSE

Under the previous assumptions the structural response is a non-stationary Gaussian stochastic process. To quantify structural response by the mean and variance of its maximum value, the knowledge of its first three time-dependent spectral moments (Corotis et al., 1972) are required.

As for heavily damped systems stationarity will be an admissible assumption under a wide range of circunstances, only the non-stationarity of lightly-damped systems ($1\% \leqslant \zeta \leqslant 10\%$) will be considered; and because correlations between response and excitation decreases with decreasing damping, the **x** will be assumed independent of the **u** ; assuming, for the moment, stationarity of load and response, from eq. 2) follows:

$$S_{\iota_i}(f) = \sum_{ab} \Lambda^x_{ia} \Lambda^x_{ib}\ S^x_{ab}(f) + \sum_{ab} \Lambda^u_{ia}\ \Lambda^u_{ib}\ S^b_{ab}(f) \qquad 7)$$

where $S_{\iota_i}(f)$ is the power spectral density of response i ; $S^x_{a\ b}(f)$ is the cross-spectral density of dynamic motions x_a and x_b ; and $S_a(f)$ is the cross-spectral density of load motions u_a and u_b. As correlations between design responses will not be considered, there is no need of $S_{\iota_i\ \iota_j}(f)$, and $S_{\iota_i}(f)$ will be used instead of $S_{\iota_i\ \iota_i}(f)$. Let: $\lambda^{\iota}_{ij} = \int_0^{\infty} f^j\ S_{\iota_i}(f)\ df$, be the j-spectral moment of response i ; $\lambda^{\iota x}_{ij} = \sum_{ab} \int_0^{\infty} f^j \Lambda_{ia}\ \Lambda_{ib}\ S_{ab}(f)\ df$, be the j-spectral moment of response i due to dynamic motion; $\lambda^{\iota u}_{ij} = \sum_{ab} \int_0^{\infty} f^j\ \Lambda^u_{ia}\ \Lambda^u_{ib}\ S^u_{ab}(f)\ df$, be the j-spectral moment of the response due to pseudo-static motion; $\lambda^u_{jab} = \int_0^{\infty} f^j\ S^u_{ab}(f)\ df$, the j-spectral moment of load motions u_a and u_b. Then: $\lambda^{\iota u}_{ij} = \sum_{ab} \Lambda^u_{ia} \Lambda^u_{ib} \lambda^u_{jab}$. Let $\lambda^{\varphi\psi f\zeta}_{jab\ k} = \int_0^{\infty} f^j\ S_{ab}(f) |H_k|^{-2}\ df$

(where $\varphi, \psi = \ddot{u}$, $\dot{u}$ and $H_k = (f_k^2 - f^2 - i2\zeta_k f_k f)^{-1}$ is the transfer function of a linear oscillator with frequency f_k and percentual damping ζ_k) be the dynamical spectral moments of the load motions (see above for time derivation in terms of spectral densities). Let $\mathbf{p}(t) = \mathbf{M}^z \ddot{\mathbf{u}}(t) + \mathbf{C}^z \dot{\mathbf{u}}(t)$ be the vector of the modal forces equivalent to load motions $\mathbf{u}(t)$. Let $\lambda^{pf\zeta}_{jabk} = \int_0^{\infty} f^j S^p_{ab}(f) |H_k|^{-2} df$ be the dynamical spectral moments of the modal forces p_a and p_b; then,

$$\lambda^{pf\zeta}_{jabk} = \sum_{cd} (M^z_{ac} M^z_{bd} \lambda^{\ddot{u}\ddot{u}f\zeta}_{jcdk} + C^z_{ac} C^z_{bd} \lambda^{\dot{u}\dot{u}f\zeta}_{jcdk} + M^z_{ac} C^z_{bd} \lambda^{\ddot{u}\dot{u}f\zeta}_{jcdk} + C^z_{ac} M^z_{bd} \lambda^{\dot{u}\ddot{u}f\zeta}_{jcdk}) \tag{8}$$

The relationship between spectral densities of excitation and response is a well known result of random vibration theory. For the present case:

$$S^x_{ab}(f) = H_a(f) H^*_b(f) S^p_{ab}(f) \tag{9}$$

where $*$ denotes conjugate. The real and imaginary part of $H_a(f) H^*_b(f)$ may be expressed as the sum of a function of $|H_a(f)|^2$ with a function of $|H_b(f)|^2$. Following Vanmarcke (1972):

$$\mathrm{Re}[H_a(f) H^*_b(f)] = \frac{1}{32\pi^4} \frac{A_{ab} - B_{ab}(1 - f_a^2/f^2)}{(f_a^2 - f^2)^2 + 4\zeta_a^2 f^2 f_a^2} + \frac{A_{ba} - B_{ba}(1 - f_b^2/f^2)}{(f_b^2 - f^2)^2 + 4\zeta_b^2 f^2 f_b^2} \tag{10}$$

where A_{ab}, A_{ba}, B_{ab} and B_{ba} depend only on f_a, f_b, ζ_a and ζ_b (for its values see (5)). Similarly for the imaginary part:

$$\mathrm{Im}[H_a(f) H^*_b(f)] = \frac{1}{32\pi^4} \frac{E_{ab} f + F_{ab} f^{-2} + G_{ab} f^{-3}}{(f_a^2 - f^2)^2 + 4\zeta_a^2 f^2 f_a^2} + \frac{E_{ba} f + F_{ba} f^{-1} + G_{ba} f^{-3}}{(f_b^2 - f^2)^2 + 4\zeta_b^2 f^2 f_b^2} \tag{11}$$

where $E_{ab}, \ldots, G_{ba}$ depend only on f_a, f_b, ζ_a, and ζ_b, and may be found in (2).

Obviously for $a = b$ coefficients $E_{ab}, F_{ab}, \ldots, G_{ba}$ are zero. Combining equations 9), 10) and 11) and integrating in f, the spectral moments of response due to dynamic motion are expressed as a linear function of the dynamical spectral moments of the modal forces: $\lambda^{\iota x}_{ij} = \sum_{ab} \Lambda^x_{ia} \Lambda^x_{ib} \mathbf{a}_{ab} \cdot \mathbf{l}_{abj}$ where $\mathbf{a}_{lm}$ is the vector $[\hat{\mathbf{a}}_{lm}, \hat{\mathbf{a}}_{ml}]$ in which $\hat{\mathbf{a}}_{lm} = [A_{lm} - B_{lm}, -B_{lm} f_l^2, E_{lm}, F_{lm}, G_{lm}] / 32\pi^4$; and $\mathbf{l}_{lmj}$ is the vector: $[\mathbf{r}^T_{lmj}, \mathbf{i}^T_{lmj}, \mathbf{r}^T_{mlj}, \mathbf{i}^T_{mlj}]^T$ in which $\mathbf{r}_{lmj} = \mathrm{Re}[\lambda^{pf_l\zeta_l}_{jlm} \; \lambda^{pf_l\zeta_l}_{j-2,lm}]^T$ and $\mathbf{i}_{lmj} = \mathrm{Im}[\lambda^{pf_l\zeta_l}_{j+1,lm}, \lambda^{pf_l\zeta_l}_{j-1,lm}, \lambda^{pf_l\zeta_l}_{j-3,lm}]^T$, and as the dynamical spectral moments of the modal forces are a linear function of the dynamical spectral moments of load motions (eq. 8)), it is concluded that the spectral moments of design response are a linear function of the spectral moments of the moments of one-degree-of-freedom oscillators acted by the load motions. Combining dynamic and pseudostatic response, the spectral moments of the response are (from eq. 7)):

$$\lambda^{\iota}_{ij} = \lambda^{\iota x}_{ij} + \lambda^{\iota u}_{ij} \qquad 12)$$

NON-STATIONARY IDEALIZATION

For the present purposes non-stationarity of the response of a linear oscillator acted by a stationary Gaussian stochastic process between instants t_1 and t_2, will be modelled by the stationary response of the oscillator multiplied by a suitable envellope function. Let $t_1 = -\infty$; after t_2, the amplitude of the oscillator's motion will be decreasing as $\exp(-2\pi f \zeta t)$. Hence, for $t \geq t_2$: $\lambda_j(t) = \lambda_j^s \exp(-4\pi f \zeta (t-t_2))$ where λ_j^s is a stationary spectral moment of the response, and $\lambda_j(t)$ the corresponding time-dependent spectral moment. Suppose a second stationary Gaussian stochastic process begining to act on the oscillator exactly at t_2. From the assumption that the response will not be altered by the transition from the first to the second process, follows the expression for the time variability of spectral moments: $t \leq t_1$: $\lambda_j(t) = 0$; $t_1 \leq t \leq t_2$: $\lambda_j(t) = \lambda_j^s (1 - \exp(-4\pi \zeta f (t-t_1)))$; $t \geq t_2$: $\lambda_j(t) = \lambda_j^s (\exp(-4\pi f \zeta (t-t_2)) - \exp(4\pi f \zeta (t-t_1)))$; These values are assimptotically correct.

Some insight on the errors envolved in this approach may be gained from comparison with available results. For the problem of a quiescent oscillator suddenly exposed to white noise excitation, the Corotis et al. (1972), small damping approximation gives the same value for the 0-th moment, while their 1-st and 2-nd moment differ by the presence of some time fluctuating terms which decrease exponentially with time; these terms express the changing in the shape of the power spectral density of response, from wide-band at the start, to its assimptotic narrow-band form. The influence of these terms on the reliability of a one degree-of-freedom system, may be also assessed from the same paper; for $1\% \leq \zeta$ the absence of these terms does not affect sensibly the reliability (at last for $f \cdot (t_2 - t_1)$ somewhat greater than 1). It should be remarked that the values of 1-st and 2-nd spectral moment of the response of a multidegree-of-freedom system will be much more dependent on the values of the 0-th spectral moment of high frequency modes than on the values of the 1-st and 2-nd spectral moment of low frequency modes; hence the suitability of the present approach to complex structures.

Concluding: for the present purposes, time dependent spectral moments can be computed directly from stationary spectral moments; eq. 12) is generalized as:

$$\lambda^{\iota}_{ij} = \sum_{ab} \Lambda^{x}_{ia} \Lambda^{x}_{ib} \, a_{ab} \, I_{abj}(t) + \lambda^{\iota u}_{ij}(t)$$

in which $I_{lmj}(t) = [e_1(t) | r^T_{lmj}, i^T_{lmj}], e_m(t) [r^T_{mlj}, i^T_{mlj}]]^T$ where $e_1(t) = 0$, $(1-\exp(4\pi f_e \zeta_e (t-t_1))$ and $(\exp(-4\pi f_1 \zeta_1 (t-t_2)) - \exp(-4\pi f_1 \zeta_1 (t-t_1))$ for $t \leq t_1$, $t_1 \leq t \leq t_2$ and $t_2 \leq t$, respectively, and $\lambda^{\iota u}_{ij}(t) = 0$, $\lambda^{\iota u}_{ij}$, 0 for the same time intervals.

RESPONSE COMPUTATION

In the present analysis response is described in terms of its time dependent spectral moments. Let λ^{l}_{ijk} (t) be the time dependent j-spectral moment of response i due to the elementar vibration produced by focus k. Then, as vibration irradiating from different foci were assumed independent, the total spectral moment will be $\lambda^{l}_{ij}(t) = \sum_k \lambda_{ijk}(t)$. From the knowledge of the λ^{l}_{ij} (t) follows the probability that response ι_i was not higher than a level L (Vanmarcke, 1975).

$$P(|\iota_i| < L) = \exp\left(-\int_0^{\infty} \alpha_i(t)\,dt\right)$$

in which $\alpha_i(t) = 2\nu_L(t)\,(1-\exp(Lq(t)(\pi/2\,\lambda^{l}_{io}(t))^{1/2}/(\exp(-L^2/\lambda^{l}_{io}(t)-1))$ is the time-dependent mean failure rate of response ι_i ; $\nu_L(t) = (2\pi)^{-2}(\lambda^{l}_{i2}(t)/ /\lambda^{l}_{io}(t))^{1/2} \exp(-L^2/\lambda^{l}_{io}(t))$ is the mean rate of up-crossing of level L and $q(t) = (1-\lambda^{l2}_{i1}(t)/\lambda^{l}_{io}(t)\lambda^{l}_{i2}(t))^{1/2}$ is a unitless measure of variability in frequency content (Vanmarcke, 1972).

Assuming that the probability distribution of the maximum of the response is a Gumbel distribution: $P(|\iota_i| < x) = \exp(-\exp(-a(x-u)))$, follows the value of the mean and variance of the maximum response:

$\bar{\iota}_i = u + \gamma/a$ ($\gamma = 0.57722$ is Euler's constant) $\sigma^2_{\iota_i} = \pi^2/6a^2$

Values of u and a may be computed from the probabilities of non--crossing of levels L_1 and L_2:

$a = \ln(\ln P(|\iota_i| < L_1)/\ln P(|\iota_i| < L_2))/(L_2-L_1)$ and $u = L_1 + \ln(-\ln P(|\iota_i| < L_1))/a$.

The need for characterizing response by its mean and variance has been discussed elsewhere (Oliveira, 1975).

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