DYNAMIC RESPONSE OF ASYMMETRIC
SHEAR WALL-FRAME BUILDING STRUCTURES

by

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SYNOPSIS

This paper describes a method for the computation of the dynamic response for uniform tall building structures, comprising both frames and shear walls, subjected to earthquake ground motions. The example structure given in the paper shows that the dynamic analysis should be done because the dynamic coupling amplifies the torsional response so that a static analysis will not adequately determine stresses and deformations.

Introduction

The purpose of this paper is to describe briefly the mathematical model used in computing the dynamic response of uniform tall building structures subjected to earthquake ground motions. This particular class of building structures includes those whose lateral load resisting system is made up of an asymmetrically arranged grouping of flexural type (shear walls) and shear type (frames) elements. These elements may be located in any asymmetric fashion, but for the purpose of comparison with normal static planar loading building code procedures, the method is developed in detail for the case of one axis of symmetry. This comparison is made with the objective of evaluating the adequacy of such static loading provisions and developing guidelines to define situations for which a detailed dynamic response computation is required.

Description of Mathematical Model

This dynamic analysis of the wall-frame system depends upon the following assumptions:

(i) The floors act as rigid diaphragms in their own planes
(ii) The lateral load resisting elements have the same relative cross-sectional position at all floor levels
(iii) The vertical forces acting on the lateral load resisting elements are negligible
(iv) All the lateral load resisting elements are either flexural or shear type elements
(v) The geometric properties of the lateral load resisting elements are vertically uniform for the entire height of the building
(vi) The structure is fully restrained at the base

Each element is modelled as an elastic continuum, i.e. either as a flexure or shear beam. The differential equations for the statical equilibrium of a shear wall i are given by

$$w_{xi}(z) = EI_{xi} \frac{d^4 x_i}{dz^4} \quad (1.a) \qquad\qquad w_{yi}(z) = EI_{yi} \frac{d^4 y_i}{dz^4} \quad (1.b)$$

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$$w_{\Theta i}(z) = EI_{wi}\frac{d^4\Theta_i}{dz^4} - GJ_i\frac{d^2\Theta_i}{dz^2} \quad (1.c)$$

Similarly the differential equations for the statical equilibrium of a frame j are given by

$$w_{xj}(z) = -GA_{xj}\frac{d^2x_j}{dz^4} \quad (2.a) \qquad w_{yj}(z) = -GA_{yj}\frac{d^2y_j}{dz^2} \quad (2.b)$$

$$w_{\Theta j}(z) = -GJ_j\frac{d^2\Theta_j}{dz^2} \quad (2.c)$$

To formulate the equations of motion of the structure, the dynamic problem is treated as a static problem with the inertial forces added to the elastic forces of the structure. The elements are combined by using equilibrium and compatibility conditions to yield a coupled set of partial differential equations.

The equation of motion governing the case of seismic ground motion applied to the entire structure can be written in the form

$$\begin{bmatrix} \frac{1}{b^2} & 0 \\ 0 & \frac{1}{c^2} \end{bmatrix}\begin{Bmatrix} \ddot{y} \\ r\ddot{\Theta} \end{Bmatrix}_n + \left[\begin{array}{c|c} \frac{\partial^4}{\partial z^4} - \alpha^2\frac{\partial^2}{\partial z^2} & \frac{e_m}{r}\frac{\partial^4}{\partial z^4} - \frac{a_m}{r}\alpha^2\frac{\partial^2}{\partial z^2} \\ \hline \frac{e_m}{r}\frac{\partial^4}{\partial z^4} - \frac{a_m}{r}\alpha^2\frac{\partial^2}{\partial z^2} & \frac{\partial^4}{\partial z^4} - \beta^2\frac{\partial^2}{\partial z^2} \end{array}\right]\begin{Bmatrix} \bar{y} \\ r\bar{\Theta} \end{Bmatrix}_n = -Acc(t)\begin{Bmatrix} \frac{1}{b^2} \\ 0 \end{Bmatrix} \quad (3)$$

Eigenvalues and eigenvectors can be determined for the case of free vibration by using numerical procedures already available in the literature. For earthquake excitation, a numerical integration procedure is used to determine the temporal variation function $T_n(t)$ of the displacements for each mode of vibration.

The two coupled displacements $\bar{y}$ (z,t) and $\bar{\Theta}$ (z,t) can be found for any ground motion excitation and take the form

$$\begin{Bmatrix} \bar{y}\ (z,t) \\ r\bar{\Theta}\ (z,t) \end{Bmatrix}_n = \begin{Bmatrix} \phi^N_{yn}\ (z) \\ \phi^N_{\Theta n}\ (z) \end{Bmatrix} T_n(t) \quad (4)$$

Also, the stress resultants applicable to the system of walls are given by

$$\left[M_w(z,t)\right]_n = \left\{\overline{EI}_y\frac{d^2}{dz^2}\left[\phi^N_{yn}(z)\right] + \frac{e_m}{r}\overline{EI}_y\frac{d^2}{dz^2}\left[\phi^N_{\Theta n}(z)\right]\right\}\cdot T_n(t) \quad (5.a)$$

$$\left[S_w(z,t)\right]_n = \left\{-EI_y\frac{d^3}{dz^3}\left[\phi^N_{yn}(z)\right] - \frac{e_m}{r}\overline{EI}_y\frac{d^3}{dz^3}\left[\phi^N_{\Theta n}(z)\right]\right\}\cdot T_n(t) \quad (5.b)$$

$$\left[B_w(z,t)\right]_n = \left\{\frac{-1}{r}\overline{EI}_w\frac{d^2}{dz^2}\left[\phi^N_{\Theta n}(z)\right]\right\}. T_n(t) \quad (5.c)$$

$$\left[T_w(z,t)\right]_n = \left\{\frac{-1}{r}\,\overline{EI}_w\,\frac{d^3}{dz^3}\left[\phi^N_{\Theta n}(z)\right] + \frac{1}{r}\,\overline{GJ}_w\,\frac{d}{dz}\left[\phi^N_{\Theta n}(z)\right]\right\}\cdot T_n(t) \qquad (5.d)$$

And, the stress resultants applicable to the system of frames are given by

$$\left[S_F(z,t)\right]_n = \left\{\overline{GA}_y\,\frac{d}{dz}\left[\phi^N_{yn}(z)\right] + \frac{a_m}{r}\,\overline{GA}_y\,\frac{d}{dz}\left[\phi^N_{\Theta n}(z)\right]\right\}\cdot T_n(t) \qquad (6.1)$$

$$\left[T_F(z,t)\right]_n = \left\{\frac{1}{r}\,\overline{GJ}_F\,\frac{d}{dz}\left[\phi^N_{\Theta n}(z)\right]\right\}\cdot T_n(t) \qquad (6.2)$$

The total response can be determined by the direct summation for the total number of modes taken into consideration.

Example Structure

The floor plan given in Figure 1 is that for a sixteen storey building first analysed by Mendelson (2). The storey height is 3.00m. Taking the z axis at centre of mass given $e_m = -4.95$m and $a_m = 0.0$. The basic parameters associated with the dynamic properties of the structure are:
$\overline{EI}_y = 14.68 \times 10^6$ t.m^2, $\overline{EI}_w = 534.60 \times 10^6$ t.m^4, $\overline{GA}_y = 10.18 \times 10^3$ t,
$\overline{GJ} = 8633.00 \times 10^3$ t.m^2, $\rho A = 6.26$ t.sec^2/m^2 and $\rho I_m = 208.70$ t.sec^2.

Figure 2 shows the coupled flexural-torsional mode shapes for the first four normal modes.

Normalizing three different earthquake records with a maximum acceleration of 0.1g and assuming a constant damping factor of 0.05, the dynamic response of the example wall-frame structure is computed incorporating the first six modes.

The maximum stress resultants associated with the equivalent static analysis of the 1975 National Building Code of Canada (4) are computed assuming the structure is located in Zone 3 with factor A of 0.1g, having an importance factor I of 1.0 and a foundation factor F of 1.0 and using the K factor of a wall-frame system (K=0.7).

For this example structure, the core eccentricity being -4.95m with the width of the building of 16.00m, the design eccentricity computed by the 1975 NBC code is 8.225m. Since the eccentricity exceeds a quarter of the width of the building, the 1975 NBC code provision requires doubling the effects of torsion, resulting in an effective design eccentricity of 16.45m.

Comparison between the maximum response parameters due to different earthquake records and the stress resultants based on the equivalent static force approach is shown in Table 1.

The variation in dynamic values is due primarily to the use of earthquake records with different response spectra. It can also be seen that the eccentricity in all the three dynamic cases is very nearly the same and the static design eccentricity, when doubled according to code requirements, is similar to the dynamic one.

A more exact static analysis of uniform asymmetric wall-frame structures is presented by Rutenberg and Heidebrecht (5). This method permits the evaluation of the maximum deformations and the maximum stress resultants applicable to the wall-frame system subjected to lateral loads.

Solving the same example structure statically by the method given in reference (5) for a triangular distributed load, the base stress resultants and the top deformations are calculated and shown in Table 2. This table gives also the corresponding maximum dynamic parameters due to different earthquake records and the magnification (or reduction) factor, normalized to the same base shear, for each parameter.

The smaller dynamic moments justify the use of an overturning moment reduction factor such as that given by the 1975 NBC code. It can also be seen that the dynamic modal coupling amplifies the torsional response, resulting in an approximate magnification factor of 3 for the base torsional moment and an approximate magnification factor of 2 for the top rotation.

Conclusions

The results of this investigation indicate that the following conclusions and recommendations can be made:

1. The method provides a convenient way for computation of the dynamic response of uniform asymmetric wall-frame structures.
2. The dynamic response of eccentric core buildings can give eccentricities much larger than normal values calculated in the code; even doubling the calculated value to get a design eccentricity may not be adequate.
3. Comparing to a static analysis which also incorporates torsional behavior, indicated that the dynamic coupling produces substantial magnifications so that a static analysis will not adequately determine stresses and deformations; dynamic analysis should be done if such coupling may be expected.
4. More investigation needs to be made in order to determine guidelines to show when a dynamic analysis is really necessary and when a static analysis could be adequate.

Acknowledgements

The writers wish to thank the National Research Council of Canada for providing the financial support for this investigation and the Computer Centre of McMaster University for making possible the computations involved in this work.

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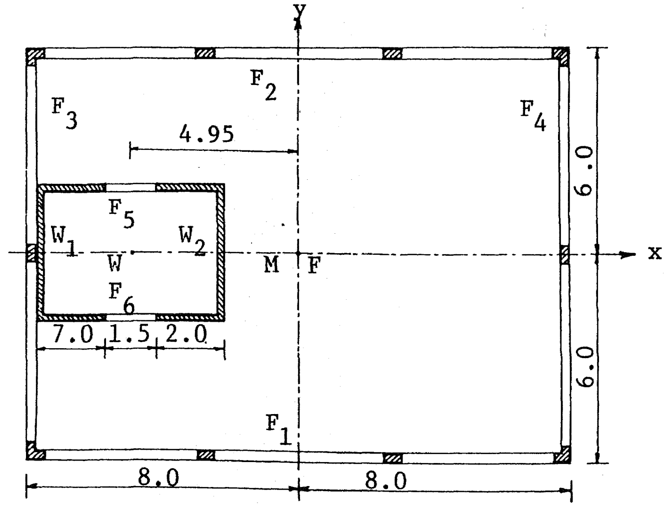

FIGURE 1: Floor Plan for the example structure

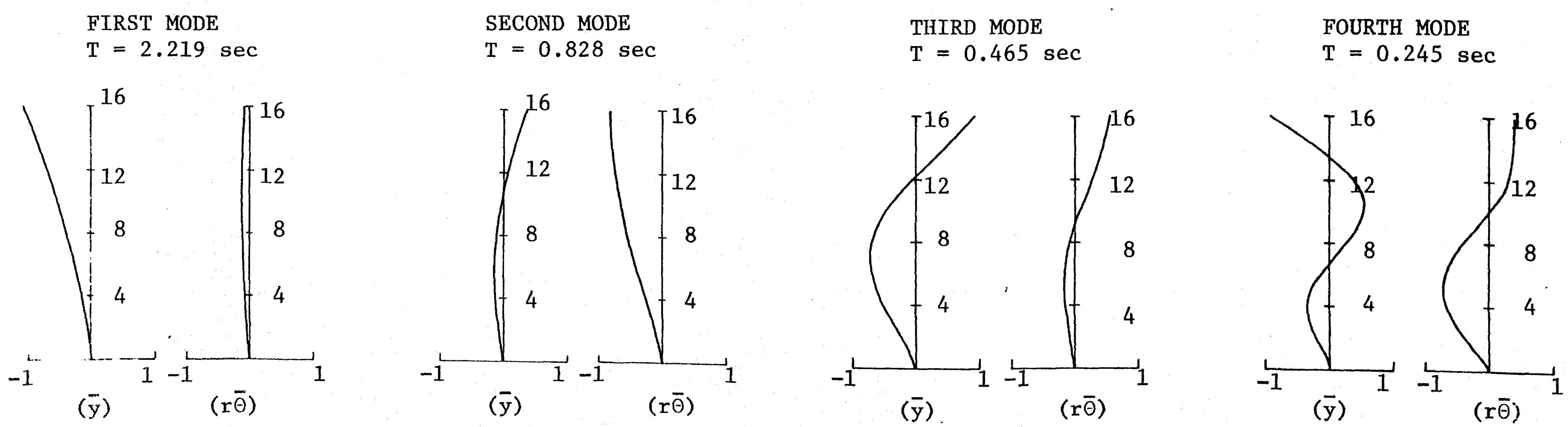

FIGURE 2: The coupled flexural-torsional mode shapes - example structure

Design Parameter	Dynamic Response Analysis			Static Analysis of NBC code 1975
	EL CENTRO *N-S* Max. Value	TAFT *N21E* Max. Value	SAN FRANCISCO *N10E* Max. Value	
Base Shear (t)	168	116	50	80
Base Moment ($t.m$)	3065	1728	888	2228
Design Eccentricity (m)	14.62	15.19	15.73	16.45

Table 1 - Comparison between the Dynamic Response Analysis and the Static Analysis of the 1975 NBC code - Example Structure

Design Parameter	Dynamic Design Analysis						Static Analysis of Reference (5)
	EL CENTRO *N-S*		TAFT *N21E*		SAN FRANCISCO *N10E*		
	Max. Value	*MF (RF)	Max. Value	MF (RF)	Max. Value	MF (RF)	
Base Shear (t)	168	1.0	116	1.0	50	1.0	96
Base Moment ($t.m$)	3065	0.80	1728	0.65	888	0.77	2183
Base Torsional Moment ($t.m$)	2464	2.95	1770	3.07	795	3.18	475
Top Displacement (m)	.0987	0.70	.0610	0.63	.0288	0.68	.0800
Top Rotation (radians)	.00169	2.09	.00109	1.95	.00051	2.10	.00046

Table 2 - Comparison between the Dynamic Response Analysis and the Static Analysis of Reference (5) - Example Structure

*MF : Magnification factor = (Dynamic Parameter/Static Parameter) x (Static Shear/Dynamic Shear)

RF : Reduction factor