

LIMIT STATES AND MATHEMATICAL MODELS OF
STRUCTURES DETERMINED FROM SEISMIC RISK
OPTIMIZATION PROBLEM

by

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SYNOPSIS

Some results of the analysis of seismic risk optimization problem are described. It is shown that the safety of structures designed for different seismic zones is unequal at present. A scheme for optimum seismic zoning is suggested which takes into account the probability of recurrence of earthquakes of different intensity.

The problem of choosing mathematical models for the optimum design of structures under earthquake loads is considered.

INTRODUCTION

Though different in details, norms, standards and codes in different countries have much in common in the matter of principle. What is common consists in the fact that the calculations suggested in the norms take into account certain values of the earthquake loads which are much lower than those caused by severe quakes, possible in the region of construction. The criteria for the choice of mathematical models of structures are such that the structural members acted upon by design loads should remain in the elastic stage of deformation.

The question arises: where does the meaning of these calculation lie? What are the real aims achieved as the result of the design based on these calculations? Unfortunately, there are no clear answers to these questions in literature.

Not exceeding the bounds of deterministic notions it is, in principle, impossible to clearly comprehend and to draw up the criteria determining the choice of the mathematical models of constructions and earthquake loads and to sensibly evaluate the trustworthiness of the analysis and the reliability of the design.

No wonder, therefore, that in the last years some researcher's attention is attracted to the problems of seismic risk, to the problems of evaluating structural safety and optimality with respect to the probable frequency of the earthquake recurrences /I-7,9/.

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In the suggested paper some results of the approximated analysis of seismic risk optimization problem are briefly discussed [1,2,3]. They refer, on the one hand, to seismic zoning, and on the other hand, - to the choice of the mathematical models of constructions in conditions of plurality of probable earthquake effects of different intensity during the design life time of the constructions.

SOME RESULTS OF INVESTIGATIONS

Seismic risk is defined in the present paper by the probability of losses connected with seismic danger. The losses are understood in the broad sense. They can be different in terms of their character and consequences. The losses can be economic and non-economic, such as deaths or injuries of people, destruction of spiritual values, social consequences, etc.

Damages of one and the same construction can be partial and complete, they can have only economic or non-economic consequences. These damages can be defined either by economic terms (in the problems of seismic risk optimization), or by physical ones (while drawing up mathematical models of structures).

There exist constructions the damage of which does not threaten with dangers to human life. Such constructions are sometimes called constructions of purely economic responsibility.

We shall define economic risk as the probability of purely economic losses, connected, in this case, with seismic danger.

As the criterion for optimality, the average-probable total costs, ρ , will be considered as connected with seismic danger. The optimum costs are determined by minimizing the optimality criterion

$$\rho = \rho_{\alpha} + \sum_{S_{\min}}^{S_{\max}} K_t T_c \mu_s \rho_{\alpha s} = \min \quad (I)$$

where ρ_{α} - seismic protection costs during construction, which are related to the total costs of the building; K_t - coefficient of the reduction of time-remote costs to the reference year; T_c - life time of the building; μ_s - probability of the occurrence of the earthquake of intensity S during one year; $\rho_{\alpha s}$ - cost of direct and "indirect" damages caused by the earthquake of intensity S

The values of $\rho_{\alpha s}$ are obtained from damage matrices (see Table I, for instance) based on the statistical analysis of the earthquake after-effects [6,8] and on the accepted correlation between the extent of the damage of the building, X and the damages related to the costs of the building (Table 2).

Extremum-minimum of the mathematical expectation is determined from the assumption that $\partial \rho / \partial \rho_{\alpha} = 0$, which gives the equation to estimate the optimum values for the summerized costs ρ , the values for ρ_{α} and for the corresponding optimum damages, $\rho_{\alpha s}$.

It is assumed that the degree of structural damages is a random quantity; it is governed by Gaussian Law. /8/.

The results of the numerical solution of the problem for two regions of Petropavlovsk (Kamchatka) (Curve I) and Andizhan (Middle Asia) (Curve 2) are shown in Fig. I.

The amounts of seismic protection obtained from (I) are sufficient for buildings of pure economic responsibility. For buildings of non-economic responsibility they are necessary but in this case limitations for safety should be taken into consideration.

The probability of failure can be written in the form of

$$P\{X > [X]\} = \sum_{S=7}^{S_{\max}} [1 - F_{(X)}^S] \int_0^{T_c} \mu_s e^{-\mu_s t} dt \quad (2)$$

where X - actual and $[X]$ - ultimate degree of structural damage corresponding the seismic scale classification; $F^S(X)$ - safety of the building under the earthquake of intensity S ; it has normal distribution. The number of the earthquake shakings used to calculate single units is assumed to be governed by Poisson's law.

As it is known,

$$F_{(X)}^S = \int_{-\infty}^{[X]} \frac{1}{\sqrt{2\pi} \sigma_x} \exp \left\{ -\frac{[X(\rho_{\alpha}) - m_{X(\rho_{\alpha})}]^2}{2\sigma_x^2} \right\} \quad (3)$$

Integration after substituting $F_{(X)}^S$ in (2) gives the final

$$R = \{X > [X]\} = \sum_{S=7}^{S_{\max}} (1 - \exp(-\mu_s T_c)) \left\{ 0.5 - 0.5 \operatorname{Erf} \left[\frac{[X] - m_{X(\rho_{\alpha})}}{\sigma_x \sqrt{2\pi}} \right] \right\} \quad (4)$$

Where $\operatorname{Erf}(\infty)$ - probability interval; $m_{X(\rho_{\alpha})}$ - mathematical expectation of structural damage degree which is dependent on the seismic protection level ρ_{α} ; σ_x - standard deviation.

Using the expressions (I) and (4) the optimum values ρ_{α} are calculated for the given (permissible) values of the structural damage probability $[P]$ and the damage degree $[X]$. It is also possible to determine minimum values for the structural damage probability at the given values of ρ_{α} and $[X]$.

Society at the present stage of its economic development disposes only of limited researches for human life and health protection. A conventional value for the cost of damage, con-

nected with people's death and injure was derived as follows. Assuming that the cost of non-economic damages is a variable the optimum value of ρ_a was obtained through solving (I) as the function of the damage costs. Then a fixed limit value for the failure probability [P] was accepted as based on the balanced risk principle and the value of ρ_a , corresponding to that limit value, determined. The value of ρ_a was correlated with the obtained value for the cost of the damage, connected with death and injure of people. Using this value the problem of the type of (I) can be solved to get the absolute minimum of ρ_a , in this case without the limitation for safety. The use of this approach in this paper was aimed at equalizing and optimizing the amount of seismic protection measures and at reading the reliability of the buildings of the same type in various seismic regions of the USSR differing in the probable frequency of occurrence of the earthquakes with different intensities.

Graphs in Fig. 2 show the dependence of total damages on the initial costs of ρ_a for the case of purely economic responsibility (Curve 1), non-economic responsibility (Curve 2) and summerized $\rho_{ec} + \rho_{non-ec}$ (Curve 3).

DISCUSSION ON RESULTS

With the existing seismic risk estimation which is based on the earthquake intensity in the given region but does not take into account the earthquake recurrence probability, the values for structural safety differ considerably even in the regions of equal seismic intensity. This is illustrated in Fig. 4, applicably to two regions with intensity 9, which differ in the recurrence of shakings: 1) the city of Petropavlovsk-Kamchatski and 2) the city of Andizhan - for shakings of intensities 7, 8 and 9 at $[X]=3, 2$; $T_c=100$, $G_x=0,64$ (The values for μ_g are given in Table 3).

The calculations have shown that to ensure the optimum balanced seismic risk it is expedient to change the distribution of the amount of seismic protection measures.

The undertaken analysis has revealed the actual meaning of the calculations carried on at present and the real aims achieved as the result of these calculations. It appears that the magnitudes of loads included in the Codes correspond to the earthquakes which take place approximately once during the period of 15-70 years (different for different regions). The limit states and the appropriate design models of buildings are accepted so as not to permit the development of failures in load-bearing structures. This state can be interpreted as intuitively (and not always correctly) identified optimum state from the abovementioned seismic risk optimization problem with regard to only one value of the intensity of the actions out of a plurality of those possible. But during the life time T_c of the building the finite number of seismic effects different in their intensities is predicted. It is reflected in the expression (I) by $\sum \rho_a s$, which takes into account

the actions causing losses. Applicably to the given site the finite number of various predicted effects and a corresponding number of the states of the building are available.

By solving the problem of seismic risk optimization the correspondence between the influences and the optimum states of the building are identified. These states are characterized by the rate of plastic deformations, cracking, and other damages. Hence different mathematical models of one and the same building should correspond to different optimum states.

The idea of two or three-stage analysis sometimes suggested in literature (for example, elastic stage analysis of buildings at weak quakes and elastic-plastic stage analysis at a severe quake) is an reflection of the intuitive attempts aimed at the accounting of the optimum states.

The transition from an economic characteristic of the optimum state of a building to a mathematical description of the model of the building is naturally a complicated separate problem for investigation.

The practical application of the probabilistic estimates of the abovementioned type requires some change in the approach to structural design for seismic regions. One of the main terms for correct designing is thought to consist in the ensuring of the "equal strength" of structural members. This approach impedes the prediction of the character of damages progressing in the structures under earthquakes. It is difficult to comprehend the number of the possible states. This impedes the elaboration of the practical methods for designing complex multistoreyed buildings for severe earthquakes. The giving up of the "equal strength" concept is one of the terms of changing the approaches to structural design for seismic regions. The specification of the failure scheme and sequence of crackings and of plastic deformation zones already at the stage of designing will allow to carry out directed optimum design of rational systems for earthquake protection.

The local damage and plastic hinges in this case should be provided with appropriate constructing, so that the mathematical models of the structure at its various states be most simple, accessible for qualitative analysis.

CONCLUSIONS

1. Basing on the approximated analysis of seismic risk optimization problem it is concluded that at present the distribution of the amount of seismic protection measures among different seismic zones is not optimal. A method for zoning according to the degree of seismic risk danger, is proposed instead of the zoning based on seismic scale intensity.

2. In the analysis of seismic response and in the design of structures several different optimum states and mathematical models of one and the same building should be taken into consideration as corresponding to the quakes of different intensity.

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Table 1. - Damage Matrix for Panel Buildings¹⁾ - (\mathcal{X})

Cost of building protection ρ_a	MSK intensity ^{I)}		
	9	8	7
B	3,5	2,5	1,35
B5-8	2,85	1,85	0,9
B -9	2,05	1,25	0,55

I) Taken from /8/. According to N.V.Shebalin - classification of failures based on seismic scale

Table 2. - Correlation between the degree of failures in buildings (\mathcal{X}) and the losses related to the costs of the building

\mathcal{X}	Damage degree
1	0
1	0,05
2	0,15
3	0,6
4	1,2
5	2,5
	None
	Light
	Moderate
	Heavy
	Total
	Collapse

Table 3. - Risk of intensity of earthquake in any year ($\mu_s \cdot 10^3$)

Region	MSK intensity			
	6	7	8	9
Petropavlovsk	2040	71,3	21,8	3,33
Andizhan	820	41,0	8,04	1,35

Table 4. - Damage probabilities ($\rho \cdot 10^3$)

Region	MSK intensity		
	7	8	9
Petropavlovsk	0,6	3,82	17
Andizhan	0,37	1,41	7,25

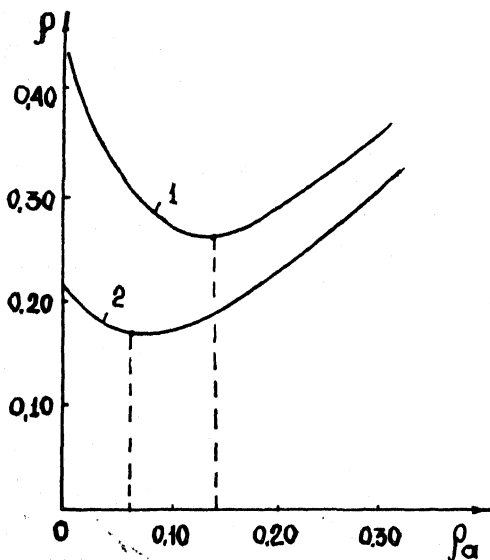


Fig. 1

Relationships between total losses ρ and seismic protection costs ρ_a

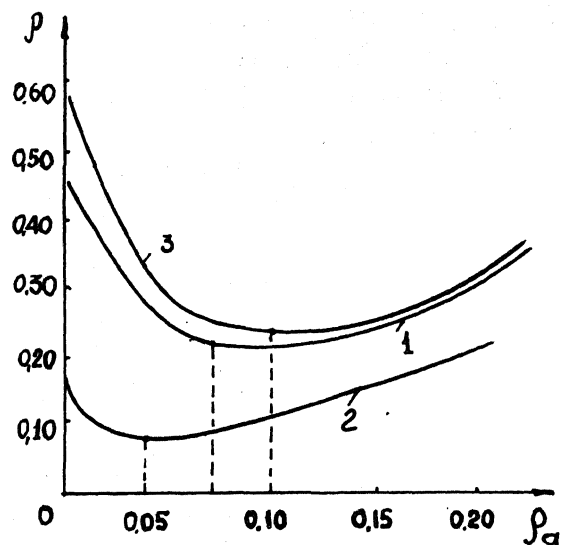


Fig. 2

Relationships between total losses ρ and seismic protection costs ρ_a