# THE EFFECT OF ASYMMETRIC IMPERFECTIONS ON THE EARTHQUAKE RESPONSE OF HYPERBOLIC COOLING TOWERS

by

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## SYNOPSIS

Linear shell theory predicts that the response of axisymmetric cooling towers to base translation will be only of the beam bending modes, while experimental evidence indicates that higher order shell modes have signifcant response. A linear computer program was used to evaluate several towers with various asymmetric imperfections. The results show that the bending stresses produced by the imperfections can be a substantial fraction of the conventional membrane stresses.

## INTRODUCTION

Although the differential equations for the structural response of thin shells to various loads, static and dynamic, have been well known for some time, the geometry of the hyperbolic cooling towers results in a set of very difficult differential equations. As a result of this complexity, the early analyses ignored bending stresses and found the membrane stresses for various loading conditions. In recent years the high speed digital computer has allowed the numerical solution of the differential equations including bending. Hashish and Abu-Sitta (1) used a finite difference technique to solve the Novozhilov equations for the free vibrations of a hyperbolic cooling tower, with and without a ring-stiffened edge.

The rapid development of the finite-element method, particularly the improved efficiency of eigenvalue solutions, has permitted the solution of many shell problems which were previously intractable. Using axisymmetric nonlinear finite elements Chan and Firmin (2) in 1970 determined the dynamic response with asymmetric loads and buckling, including geometric nonlinearity. Lashkari et al (3) evaluated the effect of uniform external pressure on free vibrations in 1972; the same axisymmetric computer program was used by Weingarten et al (4) to evaluate the effect of gravity loading on the free vibrations.

Linear analysis with axisymmetric cooling towers predicts that the only structural response to earthquake input is in the form of one circumferential wave (beam bending modes). As Abu-Sitta and Davenport (5) stated in their conclusions, "the configuration corresponding to one circumferential wave is the only one which is excitable by horizontal earthquake motions."

Experimental investigations by Weingarten et al (4) and others as noted by Schnobrich (6) have revealed that a significant response in higher circumferential wave numbers does exist. This discrepancy between the linear theoretical results and the experimental results is usually

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[I] Member of the Technical Staff, Jet Propulsion Laboratory, Pasadena, California; formerly Graduate Student, University of Southern California

[II] Professor and Chairman, Department of Civil Engineering, University of Southern California, Los Angeles, California attributed to nonlinear effects, since as noted by Citerley and Ball (7), "the only way Fourier modes $n > 1$ can be excited is through the nonlinear effects and through perturbations in the shape and properties of the shell."

## LINEAR THEORY

The well known differential equation for undamped forced vibration, which in its matrix form is applicable to the finite element formulation, is

$$[M]\{\ddot{x}\} + [K]\{x\} = \{F\} \tag{1}$$

where $\{F\}$ are the nodal forces. After solution of the eigenvalue problem

$$\big[[K] - \lambda[M]\big]\{x\} = \{0\} \tag{2}$$

the nodal displacements can be represented in terms of the collection of the eigenvectors, $[\phi]$, and the generalized coordinates, $\{q\}$,

$$\{x\} = [\phi]\{q\} \tag{3}$$

After differentiating equation 3 twice to obtain $\{\ddot{x}\}$, the expressions for $\{x\}$ and $\{\ddot{x}\}$ are used in equation 1 to obtain

$$[M][\phi]\{q\} + [K][\phi]\{q\} = \{F\} \tag{4}$$

which after premultiplying by $[\phi]^T$ gives

$$[\phi]^T[M][\phi]\{\ddot{q}\} + [\phi]^T[K][\phi]\{q\} = [\phi]^T\{F\} \tag{5}$$

In the computed program used in this study, SAP IV (8), the eigenvectors are normalized with respect to the mass and stiffness matrices such that equation 5 becomes

$$\lceil 1 \rfloor\{\ddot{q}\} + \lceil \omega^2 \rfloor\{q\} = [\phi]^T\{F\} \tag{6}$$

For the case of translational acceleration of the base, $\ddot{x}_b$, the nodal forces are

$$F_i = -m_i\, \ddot{x}_b \tag{7}$$

so equation 6 takes on the form

$$\lceil 1 \rfloor\{\ddot{q}\} + \lceil \omega^2 \rfloor\{q\} = -[\phi]^T\{m_{transl}\}\, \ddot{x}_b = -\{\Gamma\}\ddot{x}_b \tag{8}$$

where $\{\Gamma\}$ is the earthquake participation factor.

For an axisymmetric structure the eigenvectors have the form of $\sin m\theta$ and the mass terms are identical at any elevation, so the earthquake participation factor for a mode with $m > 1$, $\Gamma_i$, is zero.

## ALUMINUM HYPERBOLOID

Experimental work was conducted by Weingarten et al (4) on a 16 inch high aluminum hyperboloid. In their report, it was said that, Results from the normal mode analysis indicate that the cooling tower should respond only in the beam mode if a beam mode forcing function is applied. To verify this prediction an electrodynamic shaker was applied to a rigid top plate in order to apply a beam type forcing function. A frequency sweep through the natural frequencies of the shell indicated that the shell had a significant response in the beam mode and also in modes other than the beam mode. One explanation of these results is that the slight imperfections in geometry of the shell and the small deviation of the end plate from simulating a rigid place are responsible for exciting resonances other than $n=1$.

The computer program SAP IV (8) was used to model one-half the hyperboloid. First a perfect shell was modeled, and the computed frequencies varied from the frequencies computed using an axisymmetric analysis by 0.2% to 9.3%. The first imperfection tried was a doubling of the shell thickness in a $20^{\circ}$ arc of the throat. When a 5g sine sweep was applied the maximum bending stress was 40% of the maximum membrane stress.

To check the sensitivity of the shell geometry, each of the nodes was perturbed in a radial direction from the nominal location by up to ±0.2" (5.9 times the shell thickness of 0.034"). The 5g sine sweep was repeated and the maximum bending stress was 23.8% of the maximum membrane stress.

## HYPERBOLIC COOLING TOWERS

The first cooling tower to be analyzed was 363 feet high, typical of current American towers (figure 1). The tower has an inside throat diameter of 165 feet. The tower is supported on 60 columns.

The computer program used, SAP IV, was modified to include the geometric stiffness matrices for beam and plate elements. This allows the program to solve for the static stress distributions, and take the altered stiffness of the beam and plate elements into account when performing the eigenvalue analysis.

The first 20 modes of the tower (using perfect geometry) were obtained using three conditions: Not including the column supports and neglecting the dead weight effects; with columns but no dead weight effects; and with columns and the dead weight effects. The first beam bending frequency was 2.437 Hz, 1.693 Hz, and 1.670 Hz for the three cases.

Six different imperfections were tried; all consisted of local thickening at or near the throat over a $24^{\circ}$ arc. The thicknesses used were 7.5" (nominal), 10", and 15". When the 1940 El Centro earthquake was used as input the worst of the imperfections gave a maximum bending stress of 13.1% of the maximum membrane stress. When a 0.5g sine sweep was used the maximum bending stress was 31.5% of the maximum membrane stress.

It is important to note that when the sine sweep was used the maximum membrane stress occurred at an excitation frequency of 1.606 Hz (the first beam bending mode) while the maximum bending moment occurred at an excitation frequency of 1.017 Hz. This illustrates the fact that the bending stresses achieved are <u>not</u> the result of altering the beam bending modes, but rather to the fact that modes other than the beam bending modes are being excited. At an excitation frequency of 1.017 Hz the bending stress is 216% of the membrane stress (since the membrane stress is much lower at this frequency than at the beam bending mode of 1.606 Hz). Thus for an earthquake with strong frequency content at 1.017 Hz the cooling tower could have high bending stresses compared to the membrane stresses.

The second cooling tower evaluated was a 1000 foot tower as analyzed by Weingarten et al (4). The imperfection used on this tower was similar to the imperfections tried on the 363 foot tower. The thickness of a $20^{\circ}$ arc in the throat was doubled, with a 50% increase in thickness used for 10" on each side. A 0.5g sine sweep was made and the maximum bending stress was found to be 16.4% of the maximum membrane stress.

## CONCLUSIONS

It has been shown that asymmetric geometric imperfections will cause significant response of modes other than the beam bending modes when a hyperbolic cooling tower is excited by an earthquake. Current cooling tower design provides only limited reinforcement to resist bending moments. Since some imperfections have been shown to produce bending stresses higher than the membrane stresses at certain frequencies, it would seem that analysis of cooling towers should include these effects. Increased reinforcement could be provided to better resist possible bending stresses.

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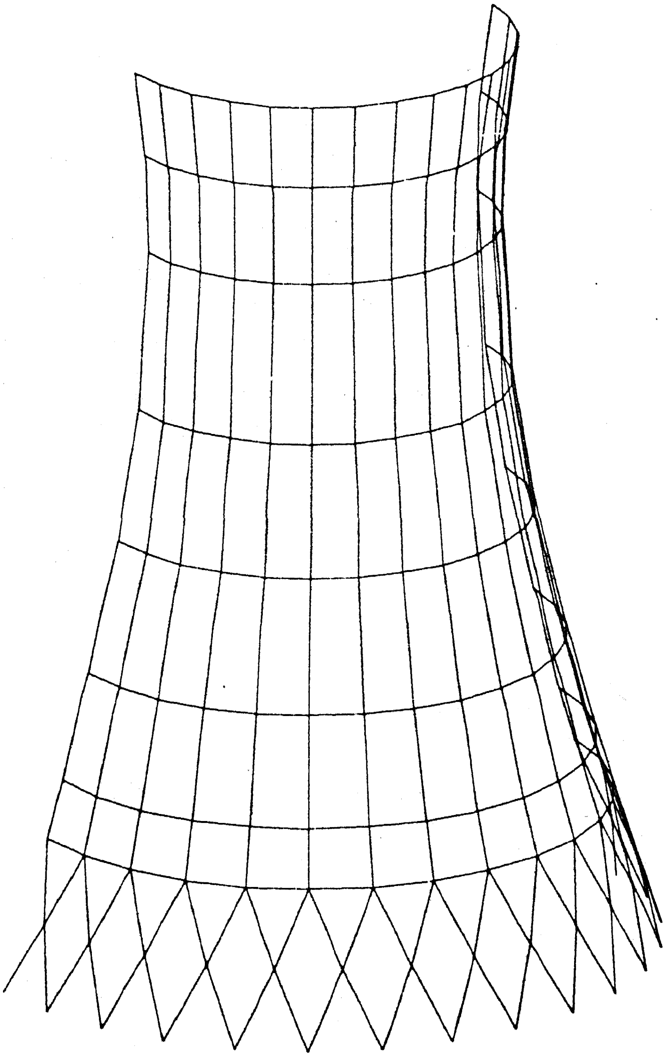

Figure 1

363 Foot Hyperbolic Cooling Tower

## DISCUSSION

### M.L. Kalra (India)

It has been stated in the paper that for 363 ft. high tower, the bending stress is 216% of the membrane stress. Weingarten et al. (4) in their study for a 100 ft. tower stated that maximum bending stress was found to be 16.4% of the maximum membrane stress. The practical designer is mainly concerned with the ratio of maximum bending stress to maximum membrane stress. The authors are requested to please give the ratio for maximum bending and membrane stresses in case of 363 ft. high tower analysed by them.

The imperfections of the shell can be at any point. What were the guiding factors for adopting imperfections at the throat in their study.

The authors are requested to state the boundary conditions adopted in the analysis at the junction of shell and columns.

### Author's Closure

Not received.