DETERMINATION OF DAMPING OF REAL STRUCTURES

P.Sotirov[I]

SYNOPSIS

Hysteretic damping of a soil-structure system is considered. The model is represented by complex numbers. For damping evaluation of each storey of a building the components are presented as a system of elements with complex stiffness.

INTRODUCTION

In the traditional method of earthquake response analysis of structures with a viscous damping model, damping ratios are given directly to each mode of vibration of the complete system. That leads to unrealistic results, because each part of the structure has different influence on each mode of vibration. Considering hysteretic damping model of structures damping characteristics can be given to each part of the structures in advance, according to the real material properties of the structural components.

EQUATION OF MOTION

A model of a multi-degree of freedom system with swaying and rocking in the soil is shown in fig.1. The equation of motion of the system under earthquake motion ($\ddot{y}_0^*$) can be written as

$$[m]_s\{\ddot{y}^*\}_s + [K^*]_s\{y^*\}_s = -[m]_s\{B\}\ddot{y}_0^* \tag{1}$$

where:

$$[m]_s = \begin{bmatrix} m_f & & \\ & [m] & [m]\{h\} \\ & \{h\}^T[m] & I_\varphi + \{h\}^T[m]\{h\} \end{bmatrix}$$

$[m]_s$ - mass matrix of the soil-structure system, $[m]$-mass matrix of the fixed base structure, m_f - mass of foundation, I_φ - moment of inertia of the foundation, $\{h\}$ - vector of mass heights,

$$[K^*]_s = \begin{bmatrix} K_f^* + \{1\}^T[K^*]\{1\} & -\{1\}^T[K^*] & \\ -[K^*]\{1\} & [K^*] & \\ & & K_\varphi^* \end{bmatrix}$$

$[K^*]_s$ - complex stiffness matrix of the soil-structure system,
$[K^*]$ - complex stiffness matrix of the fixed base structure,

I Research Engineer, Geophysical Institute, Bulgarian Academy of Sciences, Sofia, Bulgaria

K_{f}^{*} - complex stiffness of ground for swaying motion, K_{φ}^{*} - complex stiffness of ground for rocking motion, $\{B\}$ - vector of external excitation,

$$\{\ddot{y}^*\}_s = \begin{Bmatrix} \ddot{y}_f^* \\ \hline \ddot{y}_1^* \\ \ddot{y}_2^* \\ \cdot \\ \cdot \\ \ddot{y}_n^* \\ \hline \ddot{\varphi}^* \end{Bmatrix} ; \qquad \{y^*\}_s = \begin{Bmatrix} y_f^* \\ \hline y_1^* \\ y_2^* \\ \cdot \\ \cdot \\ y_n^* \\ \hline \varphi^* \end{Bmatrix}$$

In order to determine eigenvectors and eigenvalues, the differential equation of free vibration of the system should be analysed

$$[m]_s\{\ddot{y}^*\}_s + [K^*]_s\{y^*\}_s = \{0\} \qquad (2)$$

Assuming a harmonic solution of eq (2),

$$\{y^*\}_s = \{A^*\}\, e^{i\lambda^* t} \qquad (3)$$

and substituting eq(3) into Eq (2) yields

$$[[K^*]_s - \lambda^{*2}[m]_s]\{A^*\} = \{0\} \qquad (4)$$

where: $\{A^*\}$ - Complex eigenvector, λ^* - complex eigenvalue.

The i $\underline{\text{th}}$ natural frequency - ω_i and the corresponding modal damping coefficient α_i can be determined by the real and imaginary parts of the i $\underline{\text{th}}$ complex eigenvalue

$$\lambda_i^{*2} = \omega_i^2 + i\alpha_i\omega_i^2 \qquad (5)$$

The data from the complex eigenvalue estimation can be used in modal response analysis.

Considering eq (1) or (2) one can see that the mass matrix can be readily obtained. However the stiffness matrix is composed of complex elements. To obtain the elements of the stiffness matrix, the stiffness of each storey should be first determined. Complex stiffness of the j-th storey of the structure is

$$K_j^* = K_j' + iK_j'' \qquad (6)$$

where: K_j' - real part of the stiffness, K_j'' - imaginary part of the stiffness, i - imaginary unit.

The real part of the stiffness K_j' can be obtained by the methods of structural mechanics. The imaginary part is

$$K_j'' = \alpha_j K_j' \qquad (7)$$

α_j is a hysteretic damping coefficient of the j-th storey. The question is how to evaluate that coefficient.

HYSTERETIC DAMPING COEFFICIENT OF AN INDIVIDUAL STOREY

Each storey of a building has typical components resisting horizontal loadings, such as shear walls, steel frames, RC frames etc., and nonresisting components - partition walls, made of different building materials. During the vibration the total energy, dissipated by a given storey of a building, is equal to the sum of the energy, dissipated by all components of the storey.

The complex stiffness of the i-th component of the j-th storey can be expressed as

$$K_{ij}^{*} = K_{ij}' + i\bar{\bar{\alpha}}_{ij} K_{ij}' \qquad (8)$$

The components of a storey, resisting lateral loadings, can be regarded as a system of complex stiffness elements, connected in parallel - fig 2. The imaginary part of the stiffness is conventionally represented as springs, drawn by a dotted line. It is supposed that the stiffness of the slabs in their plane is comparatively very large. Hence, the complex stiffness of the j-th storey is

$$K_j^{*} = \sum_{i=1}^{n} K_{ij}^{*} \qquad (9)$$

The real part of eq (9) gives the real stiffness of the j-th storey

$$K_j' = \sum_{i=1}^{n} K_{ij}' \qquad (10)$$

From the imaginary part of eq (9) the hysteretic damping coefficient of the j-th storey can be determined

$$\alpha_j = \frac{\sum_{i=1}^{n} \bar{\bar{\alpha}}_{ij} K_{ij}'}{\sum_{i=1}^{n} K_{ij}'} \qquad (11)$$

The hysteretic damping coefficients $\bar{\bar{\alpha}}_{ij}$ of the components of each storey can be taken by the test data of typical shear walls, steel and RC frames, partition walls etc. The test data should be used with caution. Some test data can be seen in reference /3/.

It is more complicated to find the real damping of the soil and to include it in the complex stiffness of the ground for swaying and rocking motions. Tajimi's theory for evaluation of complex stiffness of the ground gives realistic results /4/.

CONCLUSIONS

A simple method for evaluation of complex stiffness elements of stiffness matrix has been presented. Modal dampings of a building can be estimated by damping of the components. The calculation of some numerical examples gave satisfactory results. Investigations and comparisons of experimentally determined and calculated modal dampings for different type of structures are going on.

REFERENCES

1. Sotirov, P., Hysteretic Damping of a Soil-Structure System for Determination of Modal Dampings, Proc. ECEE, Istanbul,1975.

2. Sotirov, P., The Influence of the Soil on the Damping of the Soil-Structure System, Proc. II - National Congress on Theoretical and Applied Mechanics, Varna, 1973.

3. Sorokin, E. S., Dynamics of Structures, Gostroizdat, Moscow, 1956.

4. - , Evaluation of Complex Stiffness of Ground under Foundation of Hamaoka Reactor Building, Civil and Architectural Engineering Section, Chubu Electric Power Company, September, 1972.

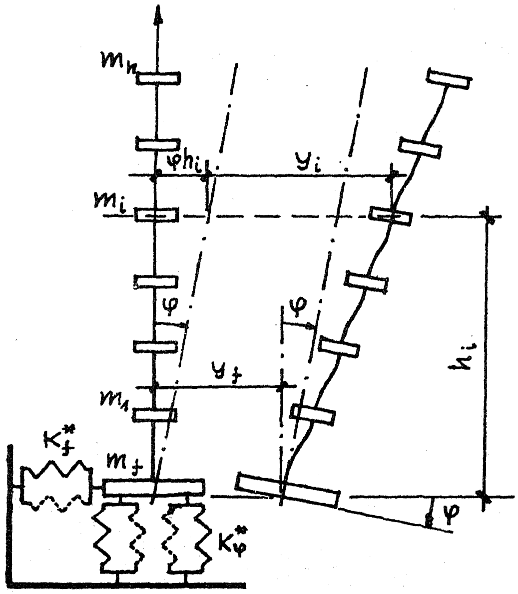

Fig. 1 Model of multi-degree of freedom system

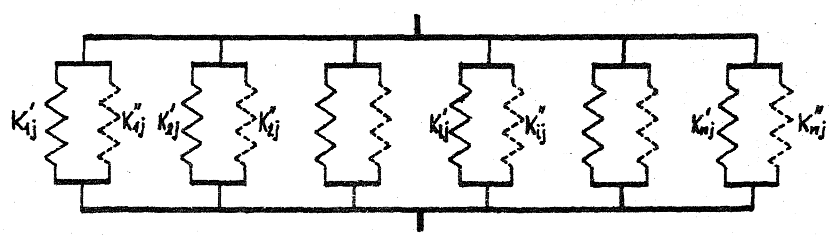

Fig. 2 Model of j-th storey