VIBRATIONS IN SUSPENSION BRIDGES

A. M. Abdel-Ghaffar[I] and G. W. Housner[II]

SYNOPSIS

A method of dynamic analysis is developed for the vertical, torsional and lateral free vibrations of suspension bridges. The method is based on a linearized theory, a finite element approach and use of the digital computer; it also incorporates certain simplifying features. The objective of the study is to determine a sufficient number of natural frequencies and mode shapes to enable an accurate analysis to be made for practical purposes. The reliability of the analysis is illustrated by computing modes and natural frequencies of a real bridge, and comparing them with the measured natural frequencies for the vertical and torsional modes of vibrations.

I. INTRODUCTION

In recent years few analyses [1, 2, 3] of the free vibrations of suspension bridges have been made by utilizing modern digital computers. Most analysis of this kind was based on idealization of the structure through systems of masses and springs representative of an actual suspension bridge. With the evolution of modern digital computers, of the finite element method of structural analysis, and of techniques of numerical analysis, however, the solution capability for many structural dynamics problems has been significantly enhanced. The purpose of this paper is to outline a method of dynamic analysis for free vibrations of suspension bridges and to exhibit the resulting experimental estimates of the natural frequencies of a suspension bridge. The method of analysis developed employs a digital computer and the finite element technique, and uses a linearized theory which restricts the amplitudes of vibrations to be small. Details of this method can be found in Ref. [4].

II. FINITE ELEMENT APPROACH FOR FREE VIBRATIONS OF SUSPENSION BRIDGES

The finite element approach has been used to: (i) discretize the bridge structure into equivalent systems of finite elements, (ii) select the displacement model most closely approximating the real case, (iii) derive element and assemblage stiffness and inertia properties, and finally, (iv) form the matrix equations of motion and the resulting eigenproblems. The evaluation of the stiffness and inertia properties of the idealized structural element and assemblage is based on the expression of the potential and kinetic energies of the element (or the assemblage) in terms of nodal displacements. This also determines the expressions for the stiffness and mass matrices. Hamilton's Principle is then used to derive the equations of motion with finite degrees-of-freedom. It has proved convenient to separate the investigation of the symmetric modes from that of the anti-symmetric modes of vibrations.

[I] Research Fellow, Civil Engineering, California Institute of Technology, Pasadena, California, 91125, U.S.A.

[II] C F Braun Professor of Engineering, California Institute of Technology, Pasadena, California, 91125, U.S.A.

1. Free Vertical Vibrations

In pure vertical modes of vibration, all points on a given cross section of the bridge move the same amount, and they remain in phase (Fig. 1-a). The suspension bridge is divided into a system of discrete elements which are interconnected only at a finite number of nodal points. Each bridge element consists of cable and girder (or truss) elements connected by two or more rigid suspenders (Fig. 2-b). Since the displacements of each stiffening structure node must equal the displacements of the corresponding cable node, it is appropriate to define only the nodes on the centerline of the stiffening structure (Fig. 2-a). Two nodal degrees-of-freedom (translation and rotation) at each node were considered. The interpolation functions associated with the two nodal degrees-of-freedom are assumed to be cubic Hermitian polynomials (Fig. 2-c). The towers are also divided into small elements (Fig. 2). The top element of the tower must include the equivalent spring which simulates the influence of the restraint of the tower by the main cable. The structural-property and inertia-property matrices are found by evaluating the properties of the individual elements and superposing them appropriately. The matrix equations of motion for the free, vertical-undamped symmetric and antisymmetric vibrations are then derived using Hamilton's Principle.

2. Free Torsional Vibrations

In pure torsional modes, each cross section of the bridge rotates about an axis which is parallel to the longitudinal axis of the bridge and which is in the same vertical plane as the centerline of the bridge. Corresponding points on opposite sides of the centerline of the roadway attain equal displacements, but in opposite directions (Fig. 1-b). The bridge is assumed to be divided into the same system of discrete elements which was used in the analysis of vertical vibrations. In determining the stiffness matrices of the structure, the effect of the cross-section warping associated with torsion was considered. Warping involves the longitudinal movement of points on a cross section (sometimes it is known as bending-torsion), as shown in Fig. 4.

3. Free Lateral Vibrations

In pure lateral motion, each cross section swings in a pendular fashion in its own vertical plane, and, therefore, there is upward movement of the cables and of the suspended structure incidental to their lateral movements (Fig. 1-c). The cable is idealized by a set of string elements, each of which has two nodes, and the suspended structure is idealized by a set of beam elements, each of which also has two nodes. The two sets of elements, connected by rigid hangers, form the bridge elements (Fig. 3) which thus have four nodal-points. For the suspended-structure subelement, there are two nodal degrees-of-freedom at each node: one is the translation of the cross section defined by the node and the other is the rotation of that cross section in the horizontal plane. The cable subelement has only one translational degree-of-freedom at each node. This introduces six degrees-of-freedom for the bridge element. The interpolation functions associated with the two degrees-of-freedom of the nodal-point in the suspended-structure subelement are taken to be cubic Hermitian polynomials. The interpolation displacement polynomial associated with the one degree-of-freedom of the cable nodal-point is taken to be a linear interpolation function. Finally, after the evaluation of the stiffness and inertia properties, Hamilton's Principle is used to derive the matrix equations of motion for the entire bridge structure.

III. NUMERICAL APPLICATIONS

To demonstrate the applicability of the analysis, the natural frequencies and mode shapes of free vibrations of the Vincent-Thomas Suspension Bridge (between San Pedro and Terminal Island, California) have been computed. The eigenproblem, for each type of vibration, was solved on the Caltech digital computer (IBM 370/158). Some of the mode shapes for the symmetric and antisymmetric vibrations are shown in Figs. 5 and 6 for the vertical, torsional and lateral (center span only) vibrations. The information concerning the San Pedro Bridge has been provided by the Bridge Department of the State of California.

IV. COMPARISON BETWEEN THE COMPUTED AND THE MEASURED FREQUENCIES

The natural frequencies of the San Pedro Bridge were accurately determined by measuring traffic-excited vertical vibrations with sensitive seismometers mounted at various locations on the bridge (Fig. 7). The Fourier amplitude spectrum of the recorded vertical movements was computed and plotted; Fig. 8 gives an example of one test result. The measurements revealed a wide band of natural frequencies. The results for the vertical and torsional natural frequencies were correlated with the computed frequencies, as shown in Fig. 8. The results of the field measurements showed reasonable agreement with the computed values which confirms the validity of this method of dynamic analysis developed for suspension bridges.

REFERENCES

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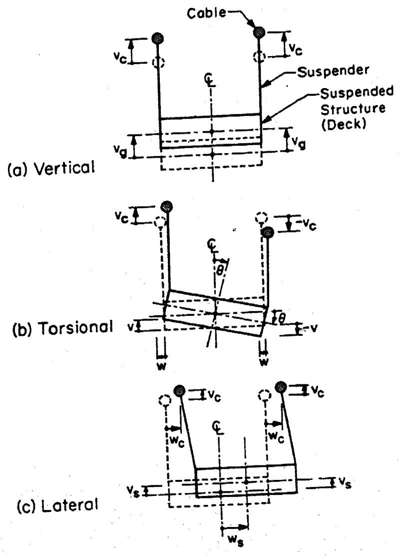
(4) Abdel-Ghaffar, A. M., "Dynamic Analyses of Suspension Bridge Structures and Some Related Topics," Ph.D. Thesis, Civil Engineering, California Institute of Technology, May 24, 1976.



TYPES OF VIBRATIONAL MOTION
IN SUSPENSION BRIDGES

FIGURE 1

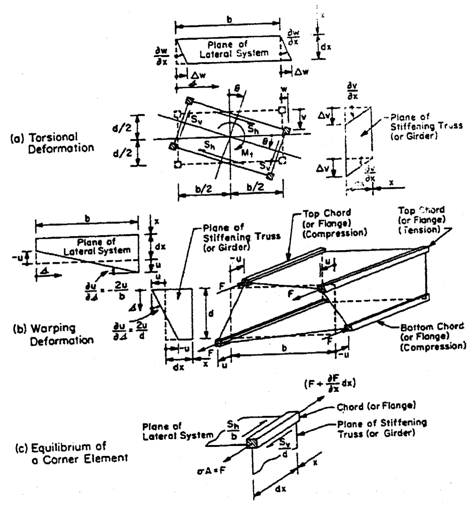


FIGURE 4

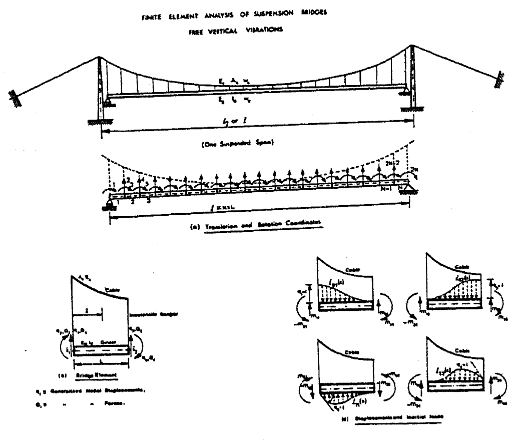


FIGURE 2

FINITE ELEMENT ANALYSIS OF SUSPENSION BRIDGES
FREE LATERAL VIBRATIONS

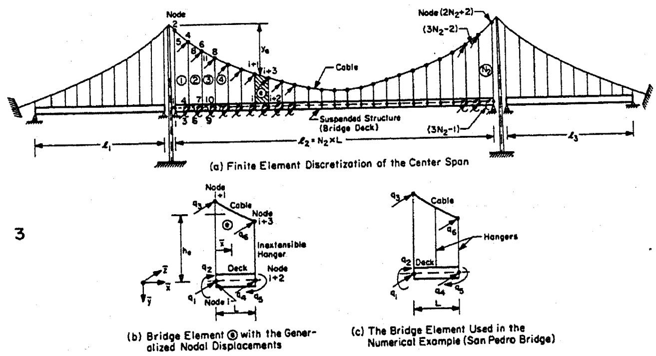


FIGURE 3

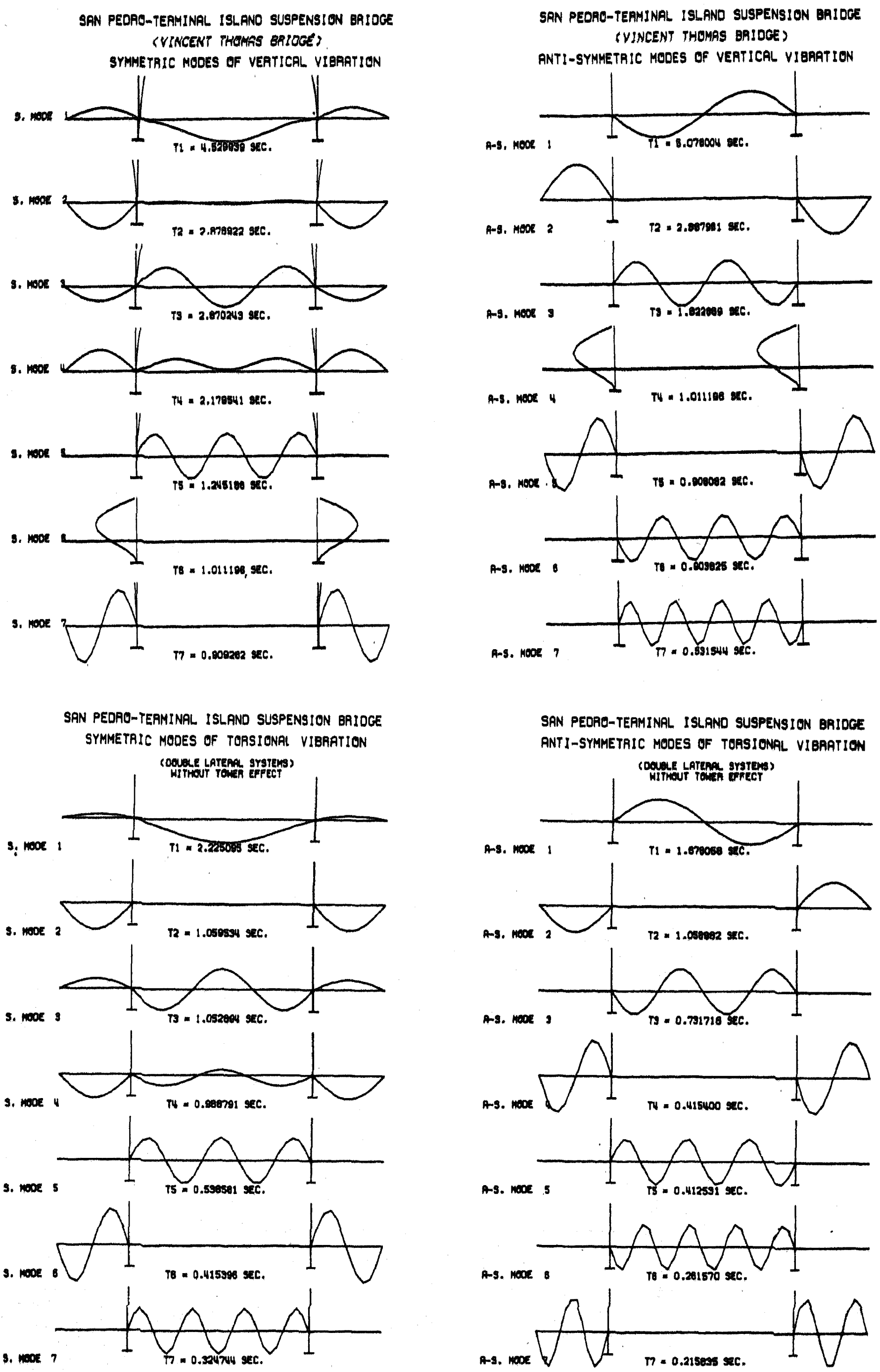

SAN PEDRO-TERMINAL ISLAND SUSPENSION BRIDGE
(VINCENT THOMAS BRIDGE)
SYMMETRIC MODES OF VERTICAL VIBRATION
S. MODE 1
T1 = 4.529939 SEC.
S. MODE 2
T2 = 2.878922 SEC.
S. MODE 3
T3 = 2.870243 SEC.
S. MODE 4
T4 = 2.178541 SEC.
S. MODE 5
T5 = 1.245186 SEC.
S. MODE 6
T6 = 1.011198 SEC.
S. MODE 7
T7 = 0.909262 SEC.
SAN PEDRO-TERMINAL ISLAND SUSPENSION BRIDGE
(VINCENT THOMAS BRIDGE)
ANTI-SYMMETRIC MODES OF VERTICAL VIBRATION
A-S. MODE 1
T1 = 5.078004 SEC.
A-S. MODE 2
T2 = 2.887981 SEC.
A-S. MODE 3
T3 = 1.822089 SEC.
A-S. MODE 4
T4 = 1.011198 SEC.
A-S. MODE 5
T5 = 0.908082 SEC.
A-S. MODE 6
T6 = 0.803825 SEC.
A-S. MODE 7
T7 = 0.831544 SEC.
SAN PEDRO-TERMINAL ISLAND SUSPENSION BRIDGE
SYMMETRIC MODES OF TORSIONAL VIBRATION
(DOUBLE LATERAL SYSTEMS)
WITHOUT TOWER EFFECT
S. MODE 1
T1 = 2.225095 SEC.
S. MODE 2
T2 = 1.059534 SEC.
S. MODE 3
T3 = 1.052804 SEC.
S. MODE 4
T4 = 0.988791 SEC.
S. MODE 5
T5 = 0.538581 SEC.
S. MODE 6
T6 = 0.415396 SEC.
S. MODE 7
T7 = 0.324744 SEC.
SAN PEDRO-TERMINAL ISLAND SUSPENSION BRIDGE
ANTI-SYMMETRIC MODES OF TORSIONAL VIBRATION
(DOUBLE LATERAL SYSTEMS)
WITHOUT TOWER EFFECT
A-S. MODE 1
T1 = 1.678068 SEC.
A-S. MODE 2
T2 = 1.058882 SEC.
A-S. MODE 3
T3 = 0.731718 SEC.
A-S. MODE 4
T4 = 0.415400 SEC.
A-S. MODE 5
T5 = 0.412531 SEC.
A-S. MODE 6
T6 = 0.281570 SEC.
A-S. MODE 7
T7 = 0.215835 SEC.

FIGURE 8

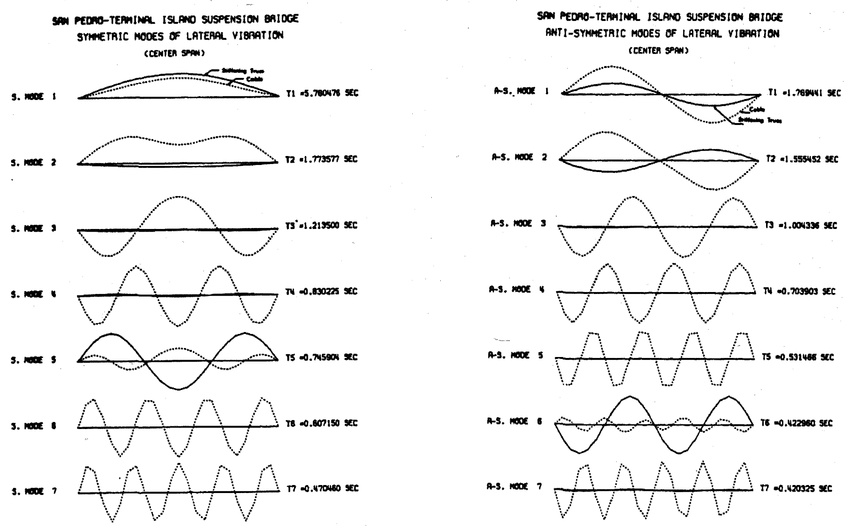
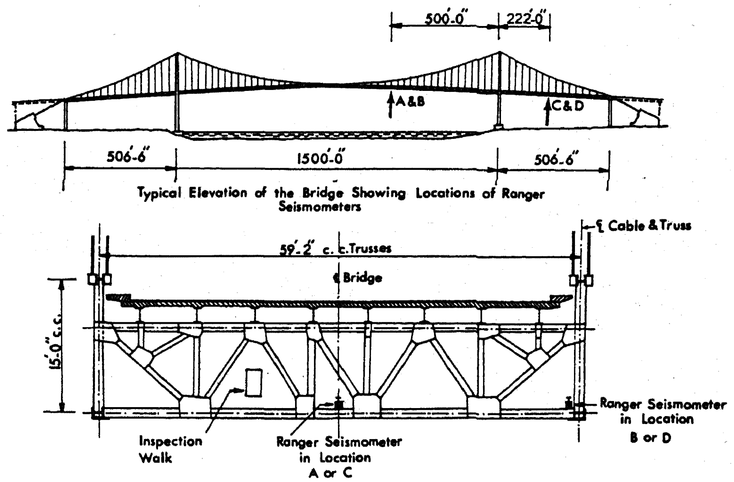


FIGURE 6

FIGURE 7



Typical Elevation of the Bridge Showing Locations of Ranger Seismometers

Typical Cross Section of the Bridge Showing Locations of Ranger Seismometers

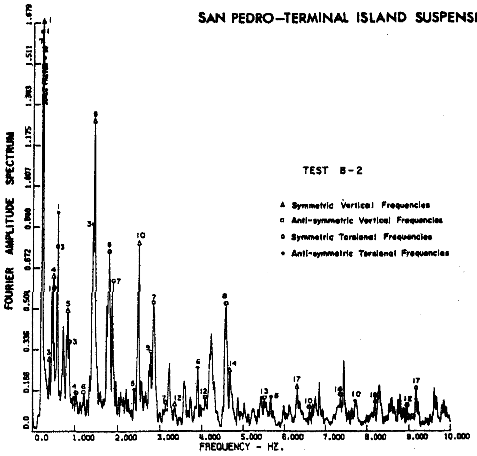


FIGURE 8