# SEISMIC RESPONSE ANALYSIS OF LONG-SPAN SUSPENSION BRIDGE TOWER AND PIER SYSTEM

by

Yoshikazu YAMADA[I], Hirokazu TAKEMIYA[II] and Kenji KAWANO[III]

## SYNOPSIS

Following the previous studies [1-3], this paper reports the dynamic response characteristics of the tower and pier system of long-span suspension bridge. In this paper particular interent is placed upon the foundation effect on the behavior of superstructure. A frequency-independent model is developed for the foundation reactions which are originally of frequency-dependent nature. From the practical design point of view the equivalent viscous damping factor is also discussed for the foundation.

## INTRODUCTION

For the earthquake-resistant design of long-span suspension bridges special attention must be paid to the tower and pier parts [1]. This system as shown in Fig. 1, comprising of a very flexible tower shaft and a rigid massive pier, makes a complicated vibrational system much affected by the interaction with the surrounding foundation. In the previous studies [2,3] the foundation effect was investigated by assuming constant stiffness springs and constant coefficient dampers to represent foundation. However, considering their significance upon the superstructure the more elaborate modelling is desired. Herein, the authors incorporated the results from the continuum mechanics approach. The effort is made to devise frequency-independent model in order to make it possible the direct time domain response analysis since they are of frequency-dependent nature. The response analysis of the total interaction system is made through complex mode method.

Also considered is the approximate response analysis for the practical design purpose. Emphasis is placed on the assessment of the radiation damping in the foundation so that the material damping is not included in it.

## FORMULATION

<u>Foundation Model</u> - For the modelling of the foundation reaction to the pier, when it is horizontally translated as well as rotated about its gravity center (G.C.), the findings from the continuum mechanics approach are adopted. Since the pier which is dealt with herein is embeded to some depth, reactions are taken into account both from the pier base and sides. The previous studies have shown the effect from the base is much greater than that from the sides. The cross-section of the pier is actually shaped rectangular but it is replaced by an equivalent circular cross-section on the equal area basis.

---

I Professor, Department of Civil Engineering, Kyoto University, Kyoto, Japan.

II Assistant Professor, Structural Engineering Division, Asian Institute of Technology, Bangkok, Thailand (on leave from Kyoto University).

III Assistant, Department of Civil Engineering, Kyoto University.

The model for the base reaction is based on the Veletsos and Verbic work [4]. They presented a closed form of frequency-dependent impedance functions for the elastic half-space upon which a massless thin rigid disc is placed. The authors developed an equivalent frequency-independent model which can produce the identical flexibility functions with those [6]. These are given as shown in Figs. 2 whose system characteristics are

$$k_{bx} = K_x, \quad c_{bx} = b_1(\frac{r}{V_s})k_x; \quad m_\theta = b_3(\frac{r}{V_s})^2 K_\theta, \quad k_{\theta 1} = K_\theta, \quad k_{\theta 2} = -b_1 K_\theta$$

$$c_{\theta 1} = -c_{\theta 2} = b_1 b_2 (\frac{r}{V_s}) K_\theta \tag{1}$$

in which $K_x$ = static translational stiffness, $K_\theta$ = static rotational stiffness, $b_i$ = constants which depend on the Poisson's ratio $\nu$, as in Table 2, r = radius of the disc, $V_s$ = shear wave velocity given by $\sqrt{G/\rho}$, G = shear modulus and $\rho$ = mass density for the base foundation.

The modelling of the sides reaction is referred to the Beredugo and Novak's work [5]. Adopting the approximations, one can get

$$k_{sx} \simeq G_s \bar{S}_{u1}, \; c_{sx} \simeq \sqrt{\rho_s G_s}\, r\bar{S}_{u2}, \; k_{s\theta} \simeq G_s r^2 \bar{S}_{\theta 1}, \; c_{s\theta} \simeq \sqrt{\rho_s G_s}\, r^3 \bar{S}_{\theta 2} \tag{2}$$

in which $G_s$ is the shear rigidity and $\rho_s$ is the mass density of the side foundation; and $\bar{S}_{u1}$, $\bar{S}_{u2}$, $\bar{S}_{\theta 1}$, and $\bar{S}_{\theta 2}$ are constants listed in Table 3.

<u>Governing Equations</u> - Referring to the deformed configuration in Fig. 3, one can establish the equation of motion as

$$\begin{bmatrix} M_p & 0 & 0 \\ 0 & J_p + m_\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{x}_o \\ \ddot{\theta} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} c_{bx} + c_{sx} D & -c_{bx}\frac{H}{2} + c_{sx}\alpha & 0 \\ -c_{bx}\frac{H}{2} + c_{sx}\alpha & c_{\theta 1} + c_{bx}(\frac{H}{2})^2 + c_{s\theta} + c_{sx}\beta & 0 \\ 0 & 0 & c_{\theta 2} \end{bmatrix} \begin{Bmatrix} \dot{x}_o \\ \dot{\theta} \\ \dot{\phi} \end{Bmatrix}$$

$$+ \begin{bmatrix} k_{bx} + k_{sx} D & -k_{bx}\frac{H}{2} + k_{sx}\alpha & 0 \\ -b_{bx}\frac{H}{2} + k_{sx}\alpha & k_{\theta 1} + k_{\theta 2} + k_{bx}(\frac{H}{2})^2 + k_{s\theta} + k_{sx}\beta & -k_{\theta 2} \\ 0 & -k_{\theta 2} & k_{\theta 2} \end{bmatrix} \begin{Bmatrix} x_o \\ \theta \\ \phi \end{Bmatrix}$$

$$= \begin{Bmatrix} Q(t) \\ M(t) + Q(t)\frac{H}{2} \\ 0 \end{Bmatrix} \tag{3}$$

in which $M_p$ = mass of pier, $J_p$ = mass moment of inertia about a horizontal axis passing through the pier's G.C., M(t) = moment acting at the pier top, Q(t) = shear force at the same cross-section, H = height of pier, D = embedment depth; $\alpha = D(D-H)/2$, $\beta = D(D^2/3 - DH/2 - DH^2/4)$.

The governing equation for the superstructure is expressed as [1-3]

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{x\} = \{0\} \tag{4}$$

so that the internal forces at the tower base (or pier top) is provided with

$$Q(t) = \{1\}[M]\{\ddot{y}\} \tag{5}$$

$$M(t) = \{h_i - \frac{H}{2}\}^T[M]\{\ddot{y}\} \tag{6}$$

in which $\{y\}$ = total horizontal displacement vector at each mass with respect to a fixed vertical axis, $\{x\}$ = horizontal deformation vector relative to the tower base, $\{h\}$ = vector of each mass height. Combining Eqs. 3 through 6, one can get the following governing equation of motion for the total interaction system.

$$[\tilde{M}]\{\ddot{\tilde{x}}\} + [\tilde{C}]\{\dot{\tilde{x}}\} + [\tilde{K}]\{\tilde{x}\} = \{\tilde{F}\}\ddot{z} \tag{7}$$

Complex Mode Analysis - As the inertia matrix $[\tilde{M}]$ in Eq. 7 turns out singular with respect to $\ddot{\phi}$, the solution is obtained by the following steps. First, convert this 2nd order differential equations into 1st order ones by use of the state vector $\{u\}^T = \{\{\dot{\tilde{x}}\}^T\{\tilde{x}\}^T\}$. Then delete $\ddot{\phi}$ from this so that the associated row and column vectors from $[\tilde{M}]$, $[\tilde{C}]$ and $[\tilde{K}]$. Furthermore, presuming the tower to be decomposed into classical normal modes due to its rather homogeneous property, one can reasonably reduce the number of degrees of freedom of interaction system with the knowledge that higher modes less contribute to the response. Hence, one can get

$$[A]\{\dot{u}\} + [B]\{u\} = \{P\}\ddot{z} \tag{8}$$

in which $[A] = \begin{bmatrix} [0] & [\bar{M}]^T \\ [\bar{M}] & [C^*] \end{bmatrix} \quad [B] = \begin{bmatrix} -[\bar{M}'] & [0] \\ [0] & [M^*] \end{bmatrix} \quad \{P\} = \begin{Bmatrix} \{0\} \\ [T]^T\{\tilde{F}\} \end{Bmatrix}$

$$[T] = \begin{bmatrix} [V] & & & \\ & 1/\sqrt{m_t} & & \\ & & 1/\sqrt{J} & \\ & & & 1 \end{bmatrix} \qquad \begin{aligned} \{u\}^T &= \{\{\dot{q}\}^T \ \dot{x}_o \ \dot{\theta} \ ; \ \{q\}^T \ x_o \ \theta \ \phi\} \\ m_t &= M_p + \{1\}^T[M]\{1\} \\ J &= J_G + m_\theta + \{h\}^T[M]\{h\} \end{aligned}$$

and $[\bar{M}]$, $[\bar{M}']$ are respectively matrices after the zero column vector being deleted and the zero-row vector in addition to this from $[M^*]$. $[M^*]$, $[C^*]$ and $[K^*]$ are given by

$$[T]^T[\tilde{M}][T] = [M^*] \ , \ [T]^T[\tilde{C}][T] = [C^*] \ , \ [T]^T[\tilde{K}][T] = [K^*] \tag{9}$$

in which $[V]$ = classical normal modes matrix for the tower part only.

Equation 8 is solved through the complex eigenvalues and the associated complex eigenvectors. The response analysis to the deterministic input motion is carried out by taking the linear acceleration technique proposed in Ref. 6.

## NUMERICAL RESULTS AND DISCUSSION

Numerical computation is carried out for the data in Fig. 1 and listed in Tables 1 and 3. The results are summarized as follows:

Modal Frequencies - Figure 4 shows the variation of modal frequencies of first five modes obtained from the imaginary parts of complex eigenvalues of Eq. 8 with the shear velocity of base foundation in which a pair of conjugates are counted as one. Note that the adjacent modal frequencies can get closer each other for a certain foundation condition due to the dynamic interaction, which was already observed in the previous papers [2,3] for the conventional constant characteristic system. The horizontal dotted lines come from the natural frequencies of the rigidly supported tower, and the dashed lines from the pier part only.

Modal Damping Factors - The quotients of the real to the imaginary parts of the complex eigenvalues are defined as approximate modal damping factors herein. Figures 5 give their variation with the shear velocity for the first five modes with and without sides reaction. Note that the damping amount varies for each mode with how much being affected by the motion of the pier. For the modes rather independent to it, the amount remains as near that imposed to the tower part. The values greater than this are explained as the radiation damping of the foundation. By referring to the mode shapes one can assess the damping amounts for pier's rocking and sway motion.

Earthquake Response - Figure 6 indicates the maximum displacement response due to EL CENTRO 1940, NS adjusted to have the maximum acceleration of 200 gal. One can observe that the interaction effect particularly due to the base foundation is very significant. Figure 7 compares the maximum displacements obtained by the present method with those from the conventional foundation modal of constant stiffness springs from shear wave velocity and damping coefficients as to yield the same amount of damping both for pier's 1st and 2nd vibration modes. Their quite well agreement is duly explained from the damping amount in Fig. 5(a). Figures 8 are the comparison of their response time histories, indicating the difference between the present and the conventional analysis. It may be concluded that for a certain foundation condition one can get reasonably accurate approximate response even from the latter method but there exists a situation that needs the present method as far as the time history matching is wanted.

## BIBLIOGRAPHY


1. Konishi, I., and Yamada, Y., "Earthquake Response and Earthquake Resistant Design of Long Span Suspension Bridges", Proc. of 3rd WCEE, Vol. III IV-312, 1965.
2. Konishi, I., and Yamada, Y., "Studies on the Earthquake Resistant Design of Suspension Bridge Tower and Pier Systems", Proc. of 4th WCEE, Vol. I B-4, 1969, pp. 107-118.
3. Yamada, Y., Goto, Y., and Takemiya, H., "Studies on the Earthquake-Resistant Design of Suspension Bridge Tower and Pier System", Proc. of 5th WCEE, Vol. I, 1974, pp. 1329-1338.
4. Veletsos, A.S., and Verbic, B., "Basic Response Functions for Elastic Foundations", J. Eng. Mech. Div., ASCE, Vol. 100, No. EM2, 1974, pp. 189-20
5. Beredugo, Y.O., and Novak, M., "Coupled Horizontal and Rocking Vibration of Embedded Footings", Canadian Geotechnical J. Vol. 9, 1972, pp. 477-497.
6. Takemiya, H., "Frequency-Independent Model for the Building-Foundation Interaction", Submitted to J. Eng. Mech. Div., ASCE.

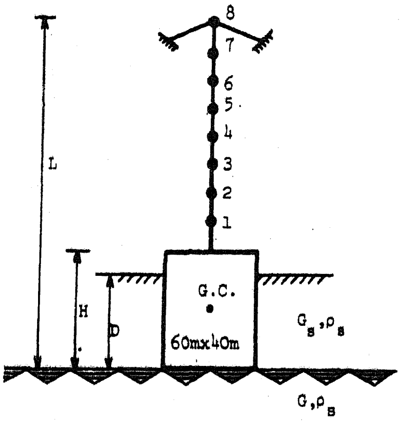



Fig. 1 Tower and Pier System

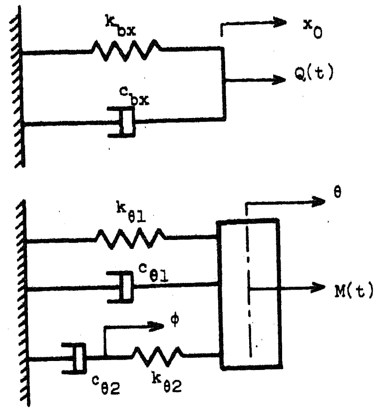



Fig. 2 Foundation Model

Table 1

| Node | A(m$^2$) | I(m$^2$) | W(ton) |
|---|---|---|---|
| 1 | 4.95 | 70.0 | 1476 |
| 2 | 4.50 | 51.44 | 1266 |
| 3 | 4.07 | 36.85 | 1071 |
| 4 | 3.60 | 25.60 | 893 |
| 5 | 3.15 | 17.15 | 731 |
| 6 | 2.70 | 10.99 | 585 |
| 7 | 2.25 | 6.65 | 455 |
| 8 | 1.62 | 3.75 | 177 |

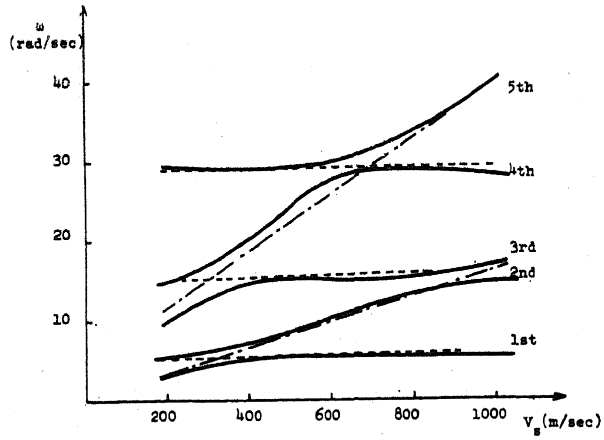



Fig. 4 Modal Frequencies

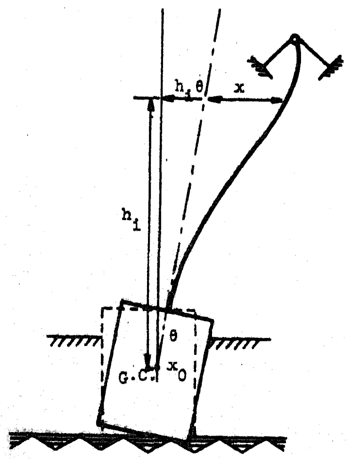



Fig. 3 Schematical Model

Table 2 (After Ref. 4)

| | $b_1$ | $b_2$ | $b_3$ |
|---|---|---|---|
| Rotation | 0.45 | 0.8 | 0.023 |
| Translation | 0.6 | - | - |

(ν=0.45)

Table 3 (After Ref.5)

| $\bar{S}_{u1}$ | $\bar{S}_{u2}$ | $\bar{S}_{\theta 1}$ | $\bar{S}_{\theta 2}$ |
|---|---|---|---|
| 4.10 | 10.6 | 2.5 | 1.80 |

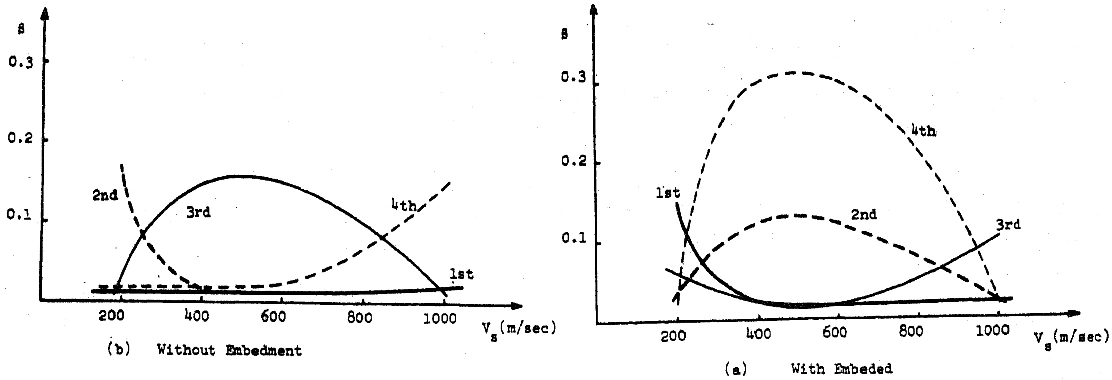
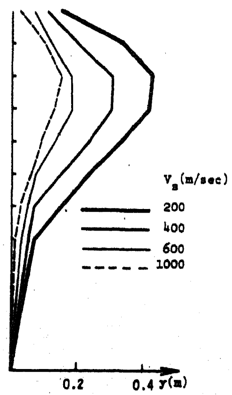



Fig. 5 Modal Damping Factors



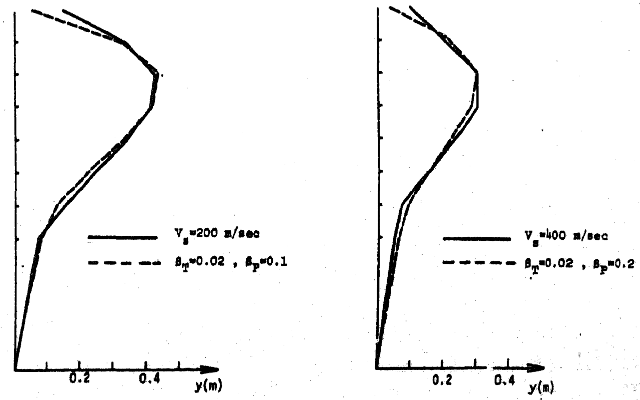




Fig. 7 Maximum Displacement Comparison

Fig. 6 Maximum Displacement
EL CENTRO 1940,NS(200gal)

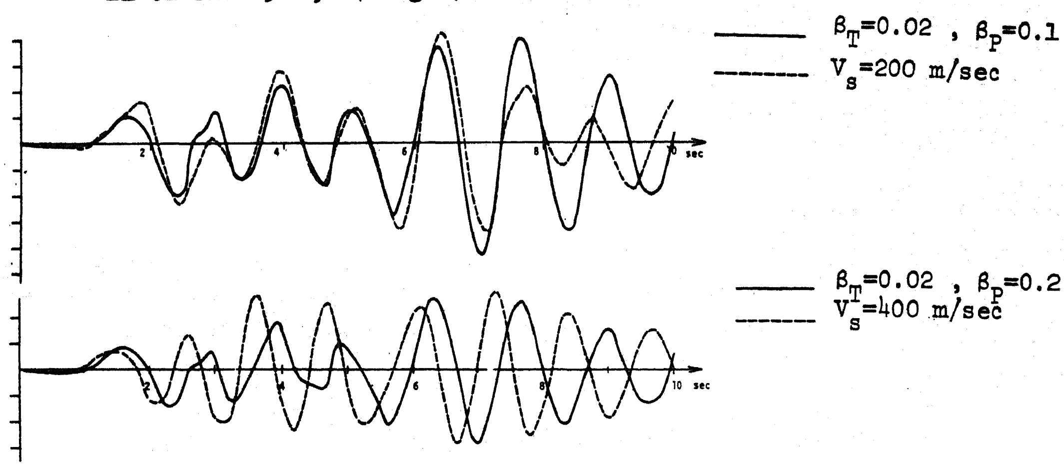



Fig. 8 Response Time Histories(Node 6)

## DISCUSSION

### B. Balwant Rao (India)

The study of the response of suspension bridge tower and pier system by the authors in very revealing.

We in India are building a number of long span prestressed concrete cantilever type of bridges with hinges or suspended spans. Some of these are proposed in seismic belts.

We are concerned with the out of phase movements of the pier and the cantilever under seismic conditions. These piers being tall and resting on well foundations will suffer considerable deformation during an earthquake. In order to provide for these deformation during an earthquake as required in CES-FIP recommendation to account for such out of phase movements there are two options open as:

i) To cover the wide opening caused at the expansion joint by an adequately designed system of joints, or

ii) To tie up the tips of cantilevers to prevent the out of phase movement and to provide a damping at the junction during its inward movement.

The first design requires the estimation of the magnitude of this opening and the joints will be very expensive. The second solution needs to know the critieria and the force to which these dampers have to be designed.

Japan with considerable experience in such designs might have developed rules for the design of such features.

Will the authors kindly throw light on the design of these or suggest a better or more suitable solution in such situations.

### V.K. Gupta (India)

Authors have presented useful information regarding the foundation effect on the behaviour of super-structure. In the process of complex mode analysis, the tower is assumed to be of homogeneous property. It is, therefore, assumed to be decomposed into classical normal modes. It is stated that higher modes contribute less to the response thereby reducing the degree of freedom of the system. The writer would like to know as to how many modes are considered in finding the response of the system. Writer's experience with cable roof system show that higher modes may also have a significant contribution towards the response. It would therefore be of great interest to know the number of modes considered by the authors.

## Author's Closure

With regard to the question of Mr. Balwant Rao, we wish to state that the check against falling down of the girders from the supporting pier is one of the important subjects in earthquake resistant design of bridges. The specification of JAPAN ROAD ASSOCIATION states that:

1. Enough length should be left between the shoe and the edge of the supporting pier by following the formula

   $S = 20 + 0.5\ell$ for $\ell$ (span length) less than 100 m.
   $S = 30 + 0.4\ell$ for $\ell$ more than 100 m.
   and S should be more than 35 cm for important bridges.

2. Enough length should be left between edges of the adjacent girders at the internal hinge. This should be more than 60 cm. 70 cm should be taken for bridges at the soft alluvium.

3. The tie-up of the adjacent girders by steel plates or steel bars is also recommended to prevent an excessive openning between girders by slip.

With regard to the question of Mr. Gupta, we wish to state that the effective reduction of the numbers of degrees of freedom was made in reference to the modal frequencies variation of the substructure with the soil condition (or the shear velocity $V_s$) and the modal frequencies of the rigidly supported superstructure. As stated in the paper the modal frequencies of the total structure originated from the superstructure remain almost constant, while those attributed to the substructure increase with $V_s$ (See Fig. 4). Hence, it may be enough to incorporate the superstructural modes below the higher substructural mode provided that those in this range dominate the superstructural response. Otherwise, one may determine those to incorporate by considering the superstructure only. For the practical range of soil condition, say $V_s = 200 \sim 800$ m/sec, the appropriate number for this end will be to account up to the third mode. However, in this study the first four modes have been used.