DYNAMIC ANALYSIS OF CABLE STRUCTURES USING LARGE DEFLECTION THEORY

by

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SYNOPSIS

This paper describes the application of a finite element approach in analyzing the dynamic response of geometrically nonlinear cable structures during periods of stron g ground shaking. Cable structures referred to in this paper are those whose overall structural stiffness is primarily governed by cable elements i.e., elements which are highly sensitive to changes in geometry. Elements of this type violate the basic assumptions set forth in small deflection theory. Thus, large deflection theory must be used to formulate element stiffnesses in a structure of this type. A discussion of cable, truss, and beam-column element formulation is included. In large deflection theory, an element stiffness is a function of its deformation. Given the geometry of a cable structure under gravity loading condition, a "reference state" is obtained by means of an iterative procedure. An outline of the algorithm for this procedure is included. A "state" is defined when all the forces and deformations in the various elements of the structure are known. Once the gravity load state is established the structure can be solved for any time varying load function e.g., a ground acceleration due to an earthquake. The time-dependent load function can be approximated as a series of step functions. A description of two analytical step-by-step methods follow in which a reference state is established at the end of each time step and used to obtain a solution for the next time step. An approximate method using modal analysis after establishing the gravity load state is compared with the step-by-step analysis. The paper concludes with reference to a three dimensional seismic dynamic analysis of a pipeline suspension bridge using the methods outlined in the paper.

BASIC CONCEPTS IN NONLINEAR ANALYSIS

The force displacement relation can be expressed as $p = ku$ ...(1)

where, p = element load vector in local coordinates

u = element displacement vector in local coordinates

k = element stiffness matrix

Figure (1) illustrates a nonlinear relation between p and u. It is an abstract idealization of Eq. (1) since both p and u are vectors.

The force displacement relation at state r is $p^{(r)} = k_{or}\, u^{(r)}$ ...(2)

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In general, k_{or} is a stiffness matrix which is a function of $u^{(r)}$ and the material properties of the element. In elastic analysis the material properties are considered to be constant, in which case, k_{or} is a function of u_r only. In the analysis that follows, material nonlinearity is not considered.

k_{or} can be decomposed into two components as follows[1],

$$k_{or} = k_o^T + k_{or}^G \qquad \ldots(3)$$

where, k_o^T is the linear elastic stiffness of the element at the unstressed state o, while the geometric stiffness k_{or}^G is a function of $u^{(r)}$.

If the forces and displacements at an intermediate state i are known, then the forces $p^{(r)}$ can be expressed in terms of $p^{(i)}$ and $u^{(i)}$ as follows,

$$p^{(r)} = p^{(i)} + k_{ir} \quad u_{ir} = p^{(i)} + \left(k_i^T + k_{ir}^G \right) \quad u_{ir} \qquad \ldots(4)$$

where, k_i^T is a function of the known displacements $u^{(i)}$ and is called the tangent stiffness of the element at state i, k_{ir}^G is a nonlinear stiffness in u_{ir}.

The matrix k_i^T can be obtained from the linear elastic matrix k_o^T by means of coordinate transformation[1], [5].

METHODS OF ANALYSIS

Consider the force deformation relation of the structure as idealized in Figure (2). Assume a state i is completely defined. A state is defined once the geometry of the structure as well as the forces and deformations in all its elements are known under a certain loading condition. The tangent stiffness K_i^T of the structure can be assembled from the tangent stiffness of the elements using the direct stiffness method[4]. State i can be used as a reference state to obtain the solution for a state j by using either of the following two methods:

Method (1).

Referring to Figure (2) solution is

a) Solve the equation of equilibrium $\Delta P_o = P^{(j)} - P^{(i)} = P^{(j)} - F^{(i)} = K_i^T \; U_{i1}$

b) $U^{(1)} = U^{(i)} + U_{i1}$

c) Transform global displacements $U^{(1)}$ into element deformation $u^{(1)}$.

d) Transform elements coordinates to conform with new geometry.

e) Evaluate element stiffnesses k_1^T and element forces $p^{(1)}$.

f) Transform element forces into global coordinates and obtain $F^{(1)}$.

g) Check if $F^{(1)} = P^{(j)}$ then state j is defined.

h) If above condition is not satisfied then obtain K_1^T and repeat steps (a) through (f) until condition (g) is satisfied.

The number of cycles required to obtain covergence depends upon the degree of nonlinearity of the structure, the size of the loading step, the machine precision and the degree of tolerance allowed.

Method (2)

Solution is carried out in small step-by-step loading increments as illustrated by the schematic sketch shown in Figure (3). At the end of each loading increment the tangent stiffness and element forces are evaluated. The unbalanced joint loads R, which equal the difference between the applied loads P and element forces F, are added to the next loading increment.

APPLICATION TO A PIPELINE SUSPENSION BRIDGE

The suspension bridge analyzed consists of two main cables (1 5/8" Ø Galvanized Bridge Strand) supporting a 30" diameter pipe in a vertical plane. Two wind cables (1 3/8" Ø Galvanized Bridge Strand) symmetrically located with respect to the longitudinal axis of the pipeline and lying in a nearly horizontal plane, comprise the lateral supporting system. The four cables are anchored to a reinforced concrete abutment at each end of the bridge. Two steel towers 59' and 72' high at the north and south ends respectively and 400' apart support the main cables. Each tower supports a pair of steel outriggers, each 31' long, hinged at the sides of the tower. The outriggers project laterally, normally the vertical plane of the pipeline, and support the two horizontal wind cables. The towers are hinged at their bases, and are supported by reinforced concrete piers founded on rock.

Figure (4) shows a sketch of the idealized computer model. Cable elements were used to simulate the main cables, wind cables, main stays and wind stays. Geometrically nonlinear beam-column elements were used to simulate the towers' legs as well as the top and bottom chords of the outriggers. Diagonals and verticals for towers and outriggers were simulated by geometrically nonlinear truss elements.

GRAVITY LOAD SOLUTION

In order to analyze the bridge for dynamic loading, the gravity load "reference state" had to be established. The final geometry and the prestressing on the main cables as well as the wind cable were already known for such loading condition. A solution scheme based on Method (1), as previously described, was adopted. In such a scheme while the geometry of the structure was kept fixed, the deformations and forces in the elements, other than those of the main cables and wind cables, were allowed to change to maintain equilibrium and compatibility. After a gravity load reference state was obtained, the modal shapes and frequencies were obtained based on the tangent stiffness of the structure. The fundamental periods of vibration for lateral and vertical excitation are 1.72 seconds and 1.33 seconds respectively.

SOLUTION FOR DYNAMIC EARTHQUAKE LOADING

The equations of motion at time t may be written as

$$M\ddot{U}^{(t)} + C\dot{U}^{(t)} + F^{(t)} = P_o + P^{(t)} \qquad \ldots(5)$$

where, $U, \dot{U}, \ddot{U}$ are the global displacements, velocities, and accelerations respectively

M, C are the mass and damping matrices and are assumed to be constant in this analysis

$F^{(t)}$ is the internal resisting elements forces vector in global coordinates

P_o is the external gravity forces vector

$P^{(t)}$ is the external dynamic forces vector

Since M and C are assumed to be constant an incremental form for Eq. (5) is

$$M\Delta \ddot{U} + C\Delta \dot{U} + \Delta F = \Delta P \qquad \text{...(6)}$$

where Δ denotes a change in the variable from time t to a time $t + \Delta t$.

Using the approximate relation $\Delta F = K_i^T \Delta U$, Eq. (6) then becomes

$$M\Delta \ddot{U} + C\,\Delta \dot{U} + \Delta F = \Delta P \qquad \text{...(7)}$$

Based on the linear acceleration method, the change in velocity and acceleration can be expressed in terms of the change in displacement as well as the velocity and acceleration at time t. Eq. (7) can then be rewritten as

$$\bar{K}\,\Delta U = \Delta \bar{P} \qquad \text{(8-a)}$$

$$\text{where, } \bar{K} = K_t^T + \frac{6}{t} M + \frac{3}{t} C \qquad \text{(8-b)}$$

$$\Delta \bar{P} = \Delta P + \frac{3}{\Delta t} M \left\{ 2\dot{U}^{(t)} + \ddot{U}^{(t)} \Delta t \right\} + \frac{1}{2} C \left\{ 6\dot{U}^{(t)} + \ddot{U}^{t} \Delta t \right\} \qquad \text{(8-c)}$$

The system of Eqs. (8) can be solved as in the incremental load method[2], explained previously. It should be noted that at the end of each time step unbalanced (or residual) forces will exist. The magnitude of such loads depend on the size of the time step chosen. In this analysis whenever the residual joint loads exceeded the maximum of either 0.01 of the incremental joint loads, or, 0.1 kips, then, iteration, as previously explained in Method (1) , was carried out. Another method of solution usir modal superposition was also used. The method assumes that the stiffnes does not significantly change after the gravity load state. Comparison between the results for the two methods is made in the succeeding section.

For the comparison to be valid, the damping matrix C used in the direct integration method had to be equivalent to that used in the modal analysis. The damping matrix was based on the assumption of constant modal damping equal to 0.02 of critical. The damping matrix C was obtained by transforming from modal space to real space[7].

The structure was subjected to ground shaking equal to 1.4 x El Centro E/Q of May 1940 for a 15.0 second duration. In the modal analysis solution, eight modes were considered adequate to simulate the dynamic behavior of the structure. In the direct integration method the E/Q input record was filtered to eliminate frequencies higher than that of the eighth mode. This was done to avoid unnecessary numerical errors as well as to obtain a valid comparison between the two methods. The integration was carried out in 600 time steps 0.025 seconds each. The time step was less than 1/5 of the 8th mode period.

ANALYSIS OF RESULTS

Table 1 summarizes several of the pertinent maximum response parameters. It can be seen that the lateral displacements obtained by the direct integration and the modal analyses are in good agreement. This may suggest that the lateral stiffness of the structure, as provided by the wind cables, remained essentially unchanged for the direction of the response. As the pipeline deflects laterally, the loss of stiffness of the cable on the deflected side is effectively balanced by an increasing stiffness on the other side. The longitudinal displacement at the tip of the outriggers also show good agreement. The vertical displacements computed by direct integration are about twice those indicated by the modal analysis method. The longitudinal displacements computed by the direct integration method at the top of the towers are 35% greater than the corresponding modal analysis displacements.

CONCLUSION

The above comparison, points out the inadequacy of using a modal analysis for cable structures showing a high degree of geometric nonlinearity. However, for cable structures not exhibiting significant geometric nonlinear behavior, a modal analysis may be used to approximate a dynamic response, thereby reducing computational expense.

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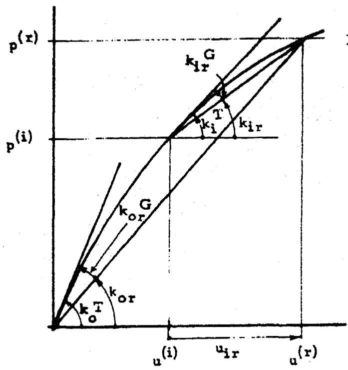

Figure (1)

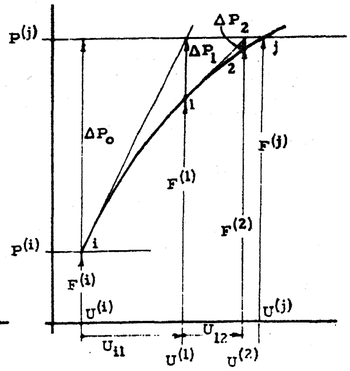

Figure (2)

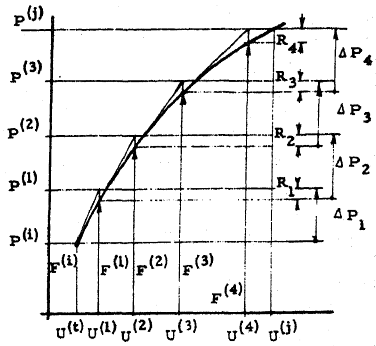

Figure (3)

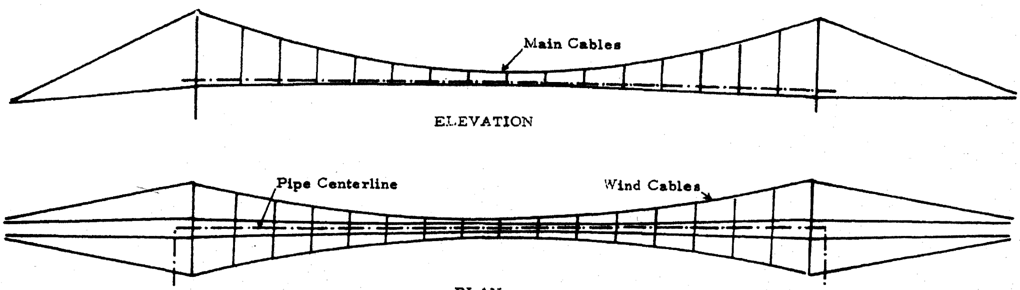

Figure (4) IDEALIZED COMPUTER MODEL

TABLE 1 - SUMMARY OF MAXIMUM RESPONSES	Gravity Loading	1.4 x Imperial Valley E/Q, May 1940 El Centro — Direct Integration	1.4 x Imperial Valley E/Q, May 1940 El Centro — Modal Analysis
I - Displacements (inches)*			
1) At pipe mid span			
a) transverse	0.	11.1	10.3
b) vertical	0.	4.9	2.6
2) At top of tower			
a) longitudinal	0.	0.51	0.38
b) transverse	0.	0.22	0.19
3) At tip of outrigger			
a) longitudinal	0.	0.39	0.32
b) vertical	0.	0.17	0.14
II - Bending in Pipeline @ Tower (kip ft.)			
a) vertical	8.2	119.0	69.3
b) horizontal	0.3	221.0	192.0
III - Horizontal Component of Cable Tension (kips)			
a) Main Cables	50.0	138.0	57.2
b) Wind Cables	20.0	69.0	34.8

*** Displacements are considered with reference to gravity load state.**

DISCUSSION

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Only a limited amount of work has been done on the dynamics of cable structures and the authors have presented a useful piece of research. It would be of interest to know the number of modes that were superimposed to obtain the results with the modal super-position technique. The writer's experience with cable structures has shown that often a large number of modes are to be considered for obtaining the total response. This was also stated by Prof. G.W. Housner in presenting his paper which appears on p. 3-303 of the conference proceedings. It has been stated in the paper that when the pipe deflects laterally, the loss of stiffness of one wind cable is compensated by the gain in the other. Writer's work with cable systems of the type shown in Fig. 1 has indicated that this may be true only for very small deflections.

Author's Closure

Not received.

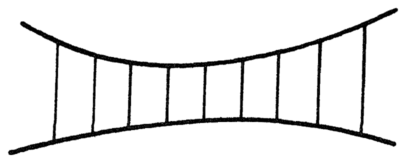

FIG. 1