

# EARTHQUAKE RESPONSE OF TALL CHIMNEYS

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## SYNOPSIS

The seismic response of four tall chimneys, 120m to 180m in height has been worked out taking the soil-structure interaction into account. The earthquake force is taken in the form of average acceleration response spectra. The soil stiffness is varied corresponding to shear wave velocity of 150m/s to 1200m/s. Rock base is also considered. The distribution of shear forces and bending moments along the height of the chimneys has been studied which show considerable departure from that recommended by Housner and adopted in Indian Standards IS:1893. Alternative distributions are proposed.

## PROBLEM STATEMENT

Tall chimneys of more than 100m height are now required in several locations to avoid pollution. The aim of this paper is to study their seismic response. The Indian Standard Earthquake Code IS:1893-1975(6) specifies normalised distribution of seismic moments and shears along the height of stack like structures, the expressions being in accordance with Housner's (4), the values at base being calculated using the fundamental period and the standard response spectra.

It is considered worth investigating whether this normalised distribution will be applicable to very tall chimneys as well. For this purpose four tall chimneys in India - Bhadarpur 150m, Patratu 120m, Obra 130m and Bokaro 180m - have been taken as examples. Their relevant data is given in Table 1. Since base rotation could change the frequency and mode shapes appreciably, the soil-structure interaction has been considered as an important parameter. The soil foundation system consists of circular raft resting on i) rock, ii) soil with properties (3) shown in Table 2 and iii) piles passing through soft soil with coefficient of subgrade reaction  $n_h = 1000t/m^3$ . The soil below the raft has been replaced by equivalent elastic translational and rotational springs. The earthquake force is taken in the form of average acceleration response spectra as proposed by Blume normalised to one g peak ground acceleration (2), Fig. 1.

## RESPONSE EVALUATION

The chimney structure is idealised by a multimass model, Fig.2, the mass being lumped at every 5m height or near about. The transfer matrix method is used for dynamic analysis in which deformations due to bending and shear, and the rotatory inertia have been considered.

The soil below the raft foundation has been replaced by equivalent elastic translational and rotational springs and dashpots as proposed by Richart et.al. (8) and Novak (7) which depend upon the shear modulus of soil  $G$ , Poisson's ratio  $\nu$ , radius of raft  $r_0$  and mass density of soil  $\rho$ . Additional damping to be added to the

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damping of the structure occurs due to energy feed back into the soil which depends on the modal displacements and slopes of the structure besides the above soil constants (7). The calculated values of spring stiffnesses and damping coefficients for Bokaro chimney alone are given in Table 2. For the pile foundation equivalent translational spring (1) only has been considered as follows, the rotation at the base being assumed as negligible:

$$K_{xp} = \frac{NEI}{1.2 T^3} + n_h \frac{d^2}{2} \quad (1)$$

where N = number of piles, EI = flexural rigidity of one pile,  $n_h$  = coefficient of subgrade reaction, d = thickness of pile cap or raft, T = characteristic length of pile given by  $(EI/n_h)^{1/5}$ . The damping in soil in this case is taken as 10% of critical and concept of weighted damping (9) is used for getting the damping in each mode.

The large openings near the base of the chimneys have been taken into account and the analysis has been carried out about both the principal axes as shown in Fig. 3. Six modes are considered and the modal superposition as proposed by Husid et.al. (5), has been used to evaluate the total response wherein the total response is found as the average of the 'modulus superposition' and the 'quadratic superposition'.

## RESULTS AND DISCUSSION

1) Periods: The variation of time periods with ratio  $h/r_b$  is given in table 3 where h = height of chimney and  $r_b$  the radius of gyration of its section at base. It is seen that as  $h/r_b$  increases, the period also increases. For these chimneys the first four periods could be calculated by the following expression approximately:

$$T_r = C_r \frac{h}{r_b} \sqrt{\frac{W_t h}{E_s A g}} \quad \text{with } C_1 = 2.80, C_2 = 0.670 \\ C_3 = 0.28, C_4 = 0.156 \quad (2)$$

The time periods decreases as the soil stiffness increases but there is no significant change beyond the shear wave velocity of 600m/s, that is for larger values of  $V_s$ , the foundation could be taken as fixed against sliding and rotation.

2) Base Moments and Shears: The coefficients for base moment ( $M_b/W_t h$ ) and base shear ( $V_b/W_t$ ) are tabulated in Table 4 wherein  $M_b$  = total base moments,  $W_t$  = total weight of chimney, h = height of centre of gravity of mass of chimney above raft and  $V_b$  = total base shear. It is seen that moment and shear coefficients, increases with the decrease of  $h/r_b$  ratio. Also they increase as  $V_s$  increases upto 600m/s beyond which they tend to decrease as the soil becomes stiffer.

3) Distribution of Moments and Shears along Height: Figure 4 shows the normalised distributions of moment and shear along height of the tallest chimney at Bokaro for different shear wave velocities. The distribution of moments shows higher values for softer soil conditions than  $V_s = 600\text{m/s}$ . For higher values of  $V_s$  than 600m/s, the distribution curves practically coincide with the fixed base distri-

bution curve, which implies that the foundation could be treated as fixed for  $V_S = 600\text{m/s}$ . The shear distribution curves show little variation for different shear wave velocities.

Figure 5 shows the normalised distribution of moment along the height of all the chimneys; (a) is for rock and stiff soil, (b) for soft soil with  $V_S = 150\text{m/s}$  and (c) for pile foundation. It is found that the distribution curves for all the chimneys are much lower for rock as well as for foundation resting on piles as compared with Housner's curve. The distribution of moment in chimneys resting on soft soil does match with that of Housner.

Figure 6 similarly shows the distribution of shear along height for various cases. It is seen that in all the cases, the distribution of shear is more or less alike. Housner's distribution very much overestimates the shear in a large portion of the height from  $0.2h$  to  $0.95h$  measured from top and underestimates it substantially in the top  $0.2h$ .

4) Contribution of I Mode in Total Response: The contribution of I Mode response to the total response of Bokaro Chimney for deflection, moment and shear expressed as ratio is presented in Table 5. It is seen that the higher modes contribute significantly and consideration of only the first mode will be inadequate and unsafe. It was also found that the contribution of higher modes beyond the fourth mode is generally small and could be neglected.

#### PROPOSED DISTRIBUTIONS

The following distributions are proposed for moment and shear in tall chimneys for different soil foundation conditions. These are plotted in Figs. 4, 5 and 6 for comparison with calculated results and are seen to more or less envelope the same:

#### DISTRIBUTION OF MOMENT AND SHEAR IN TALL CHIMNEYS

Soil-Foundation	$M/M_b$	$V/V_b$
Raft on Hard Soil $V_S = 600\text{m/s}$	$0.4\left(\frac{x}{h}\right)^{1/2} + 0.6\left(\frac{x}{h}\right)^4$	$1.1\left(\frac{x}{h}\right)^{1/2} - 0.75\left(\frac{x}{h}\right) + 0.9\left(\frac{x}{h}\right)^4$ but 1.0
Raft on Soft Soil $V_S = 150\text{m/s}$	$0.6\left(\frac{x}{h}\right)^{1/2} + 0.4\left(\frac{x}{h}\right)^4$	$1.1\left(\frac{x}{h}\right)^{1/2} - 0.75\left(\frac{x}{h}\right) + 0.65\left(\frac{x}{h}\right)^4$
Pile Foundation	$0.5\left(\frac{x}{h}\right)^{1/2} + 0.5\left(\frac{x}{h}\right)^4$	$0.66\left(\frac{x}{h}\right)^{1/2} - 0.2\left(\frac{x}{h}\right) + 0.54\left(\frac{x}{h}\right)^4$

#### CONCLUSIONS

From this study the following conclusions may be drawn for earthquake response of tall chimneys:

- 1) Base of chimney may be taken as fixed for soils with  $V_S = 600\text{m/s}$  for all practical purposes. For softer soils, its flexibility must be considered.

- 2) For computing total response at various sections, at least first four modes of vibration must be considered for sufficient accuracy. Consideration of first mode only will be grossly inadequate.
- 3) The normalised distribution of moments and shears along the height of tall chimneys is generally much lower than that shown by Housner's curves. New expressions are suggested which take soil stiffness also into account.

#### ACKNOWLEDGEMENTS

The assistance of Sri Nem Kumar in computation work and useful discussion with Dr. S.K. Thakkar in the initial stages of the study are thankfully acknowledged. The data on the tall chimneys was courteously provided by the Central Water and Power Commission, India.

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TABLE 1 - PROPERTIES OF CHIMNEYS

Sl. No.	DESCRIPTION	BHADAR PUR				PATRATU				OBRA				BOKARO			
		(m)	(m)	(m)	(m)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)	(mm)
1.	Height	150	120	120	180	180	120	120	180	180	120	120	180	180	120	120	180
2.	Internal dia, at top , above flue duct , below flue duct	6.48	5.10	5.10	6.00	6.00	5.10	5.10	6.00	6.00	5.10	5.10	6.00	6.00	5.10	5.10	6.00
		11.50	7.80	8.97	15.77	15.77	7.80	8.97	15.77	15.77	7.80	8.97	15.77	15.77	7.80	8.97	15.77
		11.60	9.01	10.32	16.82	16.82	9.01	10.32	16.82	16.82	9.01	10.32	16.82	16.82	9.01	10.32	16.82
3.	Shell Thickness, at top , above flue duct , below flue duct	160	180	180	180	180	180	180	180	180	180	180	180	180	180	180	180
		300	450	400	800	800	450	400	800	800	450	400	800	800	450	400	800
		600	750	750	800	800	750	750	800	800	750	750	800	800	750	750	800
4.	Lining Thickness, at top , above duct	115	114	114	112	112	114	114	112	112	114	114	112	112	114	114	112
		230	350	350	540	540	350	350	540	540	350	350	540	540	350	350	540
5.	Size of air gap	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
6.	Raft foundation, dia , thickness	20	19	20	35	35	19	20	35	35	19	20	35	35	19	20	35
		3.50	4.0	3.7	3.0	3.0	4.0	3.7	3.0	3.0	4.0	3.7	3.0	3.0	4.0	3.7	3.0
7.	Concrete shell, E	1.56	1.47	1.80	1.80	1.80	1.47	1.80	1.80	1.80	1.47	1.80	1.80	1.80	1.47	1.80	1.80
		(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )	(10 <sup>6</sup> t/m <sup>2</sup> )
8.	Weight of chimney above raft, Wt	4614	3469	4227	11160	11160	3469	4227	11160	11160	3469	4227	11160	11160	3469	4227	11160
9.	Flue ducts, width height	8.7	8.3	10.7	8.5	8.5	8.3	10.7	8.5	8.5	8.3	10.7	8.5	8.5	8.3	10.7	8.5
		(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)	(m)
10.	Pile Foundation, 600 dia, 15m deep piles, Number Thickness of pile cap	100	50	50	120	120	50	50	120	120	50	50	120	120	50	50	120
		2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0

TABLE 2 - SOIL PROPERTIES

S. No.	SHEAR MODULUS G (t/m <sup>2</sup> ) (kg/cm <sup>2</sup> )	UNIT WEIGHT γ (t/m <sup>3</sup> )	POISSON'S RATIO	STIFFNESS AND DAMPING CONSTANTS FOR BOKARO			
				K <sub>x</sub> x 10 <sup>6</sup> Kg x 10 <sup>9</sup>	C <sub>x</sub> x 10 <sup>5</sup>	t sec/m	t-sec
1.	3670	1.60	0.50	0.343	0.105	0.299	0.197
2.	18350	2.00	0.45	1.662	0.477	0.575	0.314
3.	32200	2.24	0.35	7.124	1.807	1.216	0.665
4.	375780	2.56	0.30	32.020	7.672	2.600	1.422

TABLE 3 - VARIATION OF FUNDAMENTAL TIME PERIOD WITH h/t<sub>B</sub> RATIO

S. No.	SHEAR WAVE VELOCITY (m/sec)	BHADARPUR	PATRATU	OBRA	BOKARO
		K = 34.2	K = 33.0	K = 29.6	K = 28.4
1.	1200	4.098	3.070	2.544	2.516
2.	600	4.136	3.095	2.573	2.543
3.	150	4.718	3.481	3.018	2.974

K = h/t<sub>B</sub> = slenderness ratio

TABLE 4 - BASE MOMENT AND SHEAR COEFFICIENTS

SHEAR WAVE VELOCITY (m/s)	BASE MOMENT COEFFICIENT				BASE SHEAR COEFFICIENT V <sub>B</sub> /Wt			
	BHADAR PUR	PATRATU	OBRA	BOKARO	BHADAR PUR	PATRATU	OBRA	BOKARO
	K=34.2	K=33.0	K=29.6	K=28.4	K=34.2	K=33.0	K=29.6	K=28.0
FIXED	0.332	0.405	0.440	0.456	0.481	0.570	0.602	0.671
1200	-	-	-	0.458	-	-	-	0.695
600	0.333	0.409	0.447	0.458	0.523	0.660	0.696	0.783
300	-	-	-	0.430	-	-	-	0.745
150	0.284	0.350	0.351	0.341	0.513	0.600	0.585	0.535

TABLE 5 - THE RATIO OF FIRST NODE RESPONSE TO THE TOTAL RESPONSE

S. No.	K/h	Raft Foundation Resting on Rock				Raft Foundation resting on Soft Soil			
		Deflection	Moment	Shear	Shear	Deflection	Moment	Shear	Shear
1.	0.25	0.953	0.209	0.304	0.944	0.189	0.332	0.332	
2.	0.50	0.790	0.467	0.365	0.851	0.475	0.407	0.407	
3.	0.75	0.567	0.710	0.295	0.670	0.721	0.391	0.391	
4.	1.0	-	0.403	0.176	-	0.685	0.253	0.253	

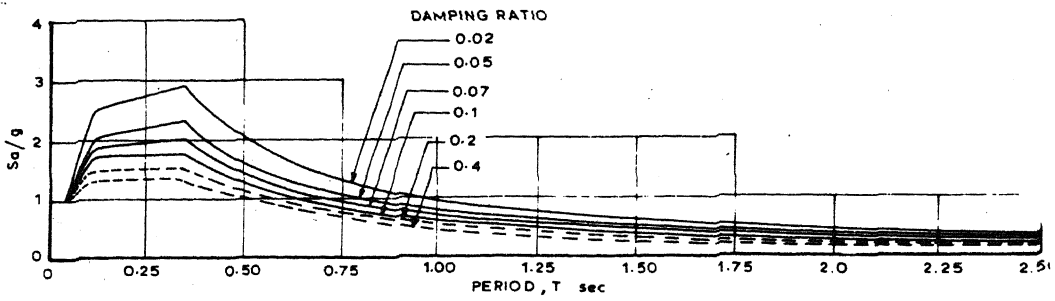


FIG. 1 - AVERAGE ACCELERATION RESPONSE SPECTRA

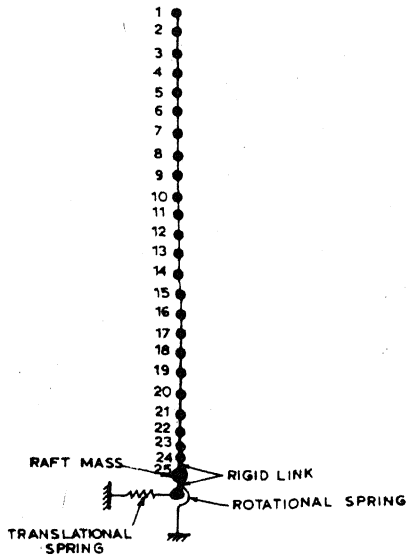


FIG. 2 - LUMPED MASS MODEL OF A CHIMNEY

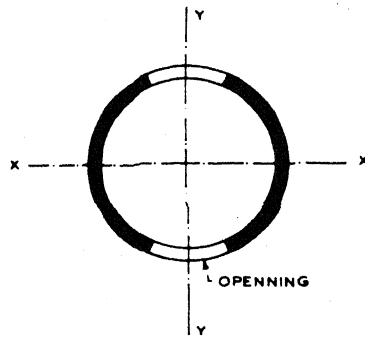


FIG. 3 - CROSS SECTION AT FLUE DUCT

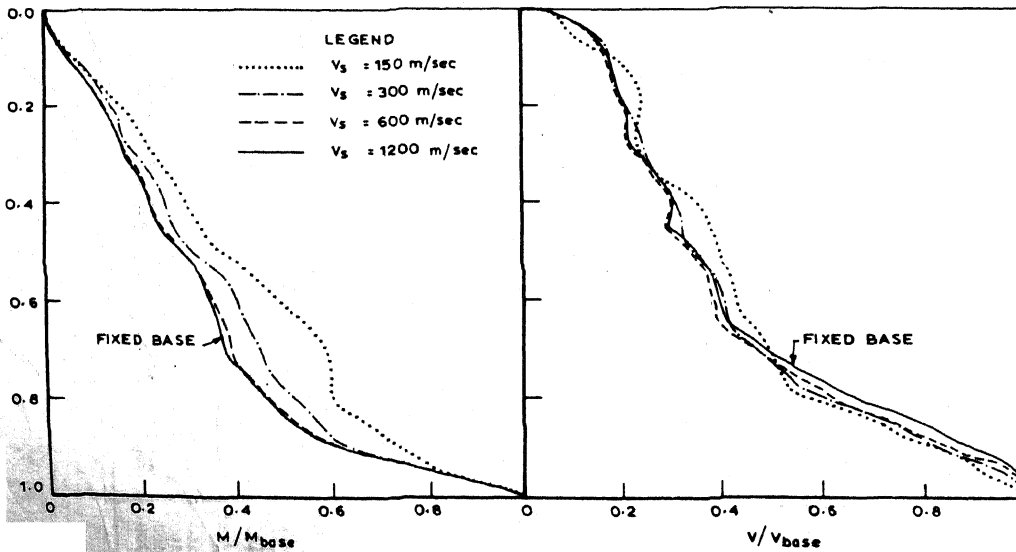


FIG. 4 - NORMALISED DISTRIBUTIONS OF MOMENT AND SHEAR FOR DIFFERENT SHEAR WAVE VELOCITIES (BORARO-CHIMNEY)

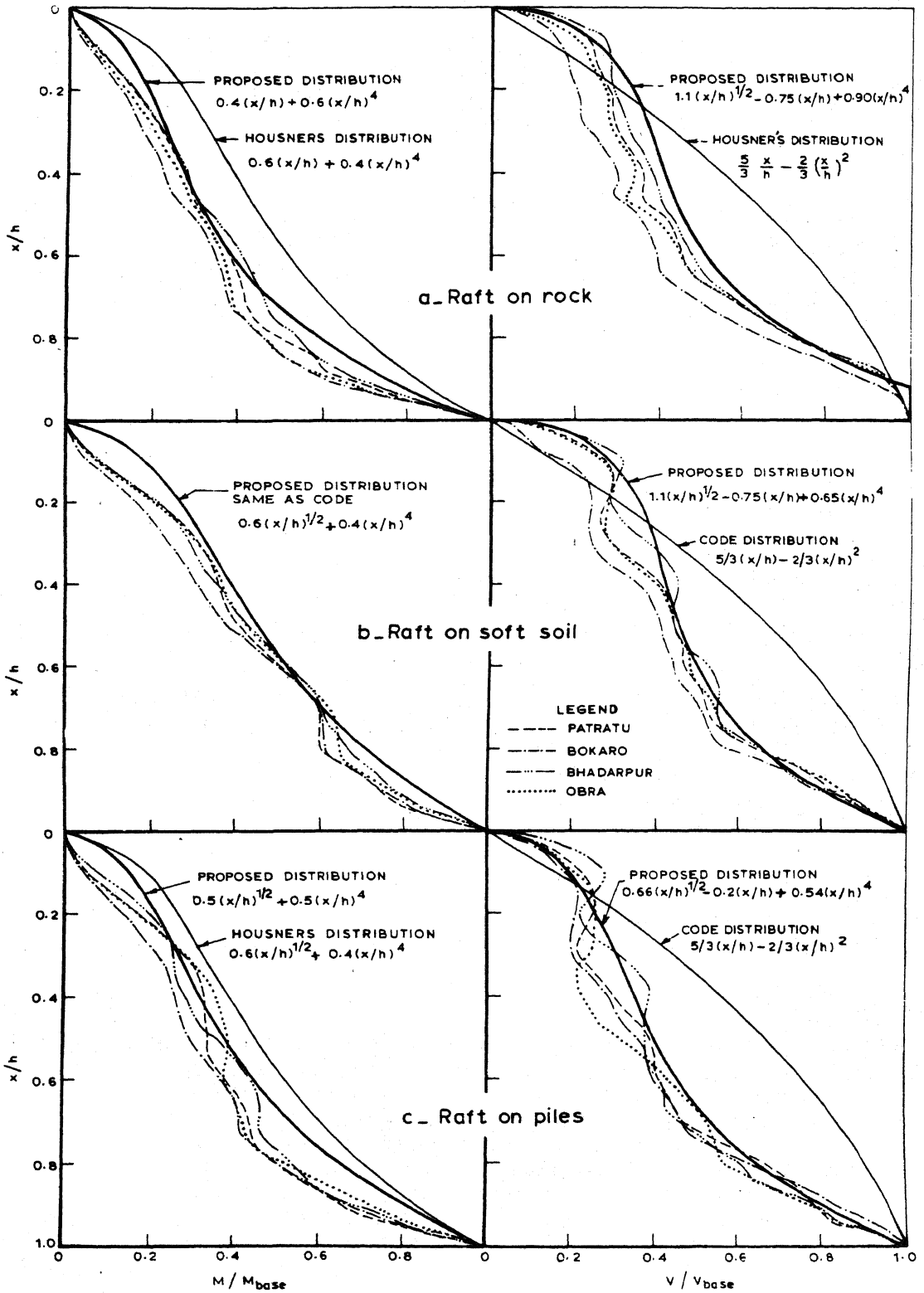


FIG. 5 - NORMALISED DISTRIBUTION OF MOMENT ALONG HEIGHT OF CHIMNEY

FIG. 6 - NORMALISED DISTRIBUTION OF SHEAR ALONG HEIGHT OF CHIMNEY

## DISCUSSION

### A.R. Chandrasekaran (India)

A comparison has been made of the results obtained by the authors with those based on the method of the discussor as reported in Ref. 1. Table 1 gives the base shear and base moment coefficients for the four chimneys. Figs. 1 and 2 show the shape of bending moment and shear force diagrams.

It is seen that author's values for base shear and base moment are rather large. This may be due to the method of combining modes. The discussor had analysed a number of problems using time wise combinations which might be more realistic. Referring to Table 5 of the author's paper, it is seen that the first mode contribution is small as compared to total response and particularly so in case of shear. This conclusion may also be due to the assumption of combination of modes.

Regarding the shape of shear and moment diagrams, the author's values are rather large in the top 20% of the height as compared to that of the discussor.

Even though the values (in the form of ratio) of the author are somewhat smaller in the bottom half of the structure, because the base values are larger, the actual values of moment and shear are generally large for most of the case.

A comparison of bending moment and shear at a height of 0.1 L from the top for author's and discussor's proposal is given in Table 2. In the table, the equivalent uniform seismic coefficient is expressed in terms of spectral acceleration in the first mode of vibration. It is seen once again that the authors values are too large.

### Reference

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### Author's Closure

Husid and Ronberg (5) proposed average of modulus and quadratic superposition of modal response for evaluating the total response of a chimney. The same method has been adopted in evaluating the shear and moment response. From the table 8 it is seen that the base shear as well as base moment increase as the slenderness ratio increases where as the discussor's results show marginal decrease in base shear coefficient, the base moment coefficient remaining almost constant. The same table shows the base moment and



base shear coefficients as worked out by timewise analysis for El-Centro May 1940 earthquake, normalised to 1.0g peak ground acceleration. The coefficients based on timewise analysis compare well with the authors' values whereas the discussor's values are low.

It can be seen from the discussor's paper (1) that as the time period and slenderness ratio increase and the taper ratio decreases, the contribution of first mode to the total response decreases. The discussor has analysed chimneys having time period upto 2.0 sec whereas the authors have analysed taller chimneys having time periods greater than 2.5 seconds. Table 7 gives the values of  $\gamma S_d$ , i.e. the participation factor multiplied by spectral displacement value, for Bokaro Chimney. It is seen that higher modes do contribute to the response. Fig. 7 shows the contribution of first mode response to the total response along the height of chimney for displacement, moment and shear respectively for different shear wave velocities for Bokaro Chimney only. It is seen that first mode response contribution to deflection is large proportion of the total whereas it is much smaller proportion in the case of shear. It is also seen that the contribution of first mode response is not uniform all along the height of the chimney. Therefore the response based on first mode only will grossly underestimate the total response.

The authors have analysed the four chimneys constructed at various part of the country for recommending the moment and shear distribution along the height. In the analysis, the following elements have been considered, which, in the author's opinion, will contribute to the moment and shear distribution along the height.

- i) Lining and its variation of thickness along the height
- ii) Flue duct opening
- iii) Variation of shell thickness along the height
- iv) Change of taper ratio above the flue duct

The authors have considered only tall chimneys greater than 120 m height in which actual area and radius of gyration have been taken where as the discussor has considered linear variation of both in his study. The above factors will contribute to different moment and shear distributions.

Figures 8 and 9 show the moment and shear distribution derived from timewise superposition of modes which are compared with the author's recommended distribution curves. It is seen that the recommended distributions fit these curves also quite well.

Actually the value of moment in top 20% height of chimney is quite a bit less than the authors' recommended shape (See Fig. 5a). Since this increase in moment in the top 20% height will be taken without any increase in the normal section, the authors have tried to give one expression for the distribution of moment and shear along the height of chimney for convenience. The above argument will also explain the higher equivalent uniform seismic coefficient at one tenth height from top.

TABLE - 1

Base Shear and Base Moment

$$V_B = C_v \cdot W \cdot S_a^{(1)} / g$$

$$M_B = C_m \cdot W \cdot H \cdot S_a^{(1)} / g$$

	Badarpur		Patratu		Obra		Bokaro	
	C <sub>v</sub>	C <sub>m</sub>	C <sub>v</sub>	C <sub>m</sub>	C <sub>v</sub>	C <sub>m</sub>	C <sub>v</sub>	C <sub>m</sub>
Authors	1.836	1.267	1.993	1.416	2.013	1.472	2.237	1.520
Discussor	1.384	1.000	1.374	1.000	1.346	1.000	1.334	1.000

TABLE - 2

Shear and Moment at 0.1H from Top

$$V_{0.1} = a_v \cdot W_{0.1} \cdot S_a^{(1)} / g$$

$$M_{0.1} = a_m \cdot W_{0.1} \cdot H_{0.1} \cdot S_a^{(1)} / g$$

	Badarpur	
	a <sub>v</sub>	a <sub>m</sub>
Author	9.17	21.02
Discussor	4.54	5.56

TABLE - 6

COMPARISON OF BASE MOMENT AND SHEAR  
COEFFICIENTS FOR FIXED BASE CONDITIONS

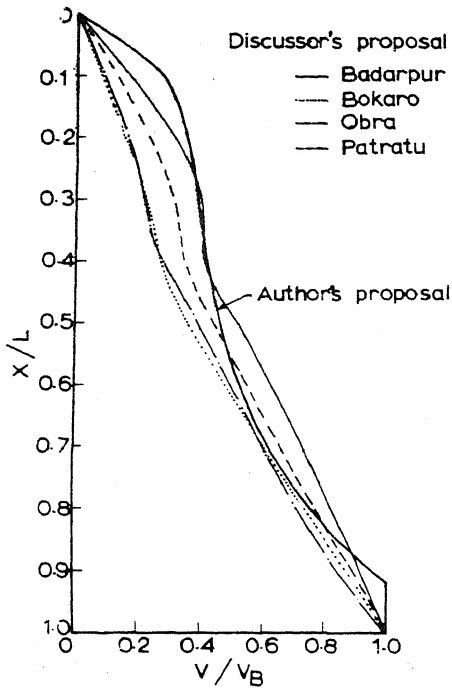
	BASE MOMENT COEFFICIENT				BASE SHEAR COEFFICIENTS			
	$M_b/W_t h$				$V_b/W_t$			
BADAR- PUR K=	PATRATU K=33.0	OBRA K =	BOKARO K = 28	BADAR- PUR K=	PAT- RATU K =	OBRA K =	BOKA- RO K=	
34.2		29.6		34.2	33.0	29.6	28.0	
Authors (Based on response spectra normalised w.r.t lg)	0.332	0.405	0.440	0.456	0.481	0.570	0.602	0.671
Based on El- Centro, May 1940 Earth- quake norma- lised to 1.0g peak ground acceleration	0.260	0.382	0.484	0.417	0.363	0.474	0.600	0.699
Discussor's	0.262	0.286	0.299	0.300	0.363	0.393	0.402	0.402

$K = l/r_b$ , Slenderness ratio

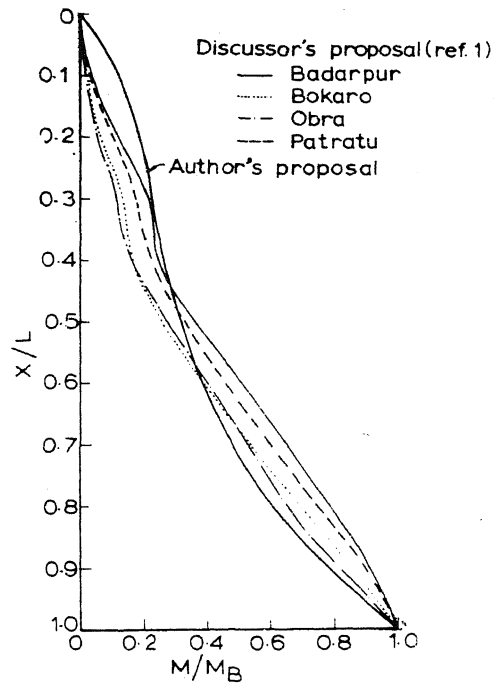
TABLE - 7

BOKARO CHIMNEY

MODE NO	TIME PERIODS SEC	DAMPING RATIO	PARTICI- PATION FACTOR	SPECTRAL DISPLACE- MENT $S_d$ (cm)	$\gamma S_d$
1	2.516	0.05	2.165	47.2	102.1
2	0.759	0.05	-2.067	15.5	32.1
3	0.346	0.05	1.586	6.8	10.8
4	0.201	0.05	-1.218	2.1	2.6



SHEAR DIAGRAM  
FIG. 1



MOMENT DIAGRAM  
FIG. 2

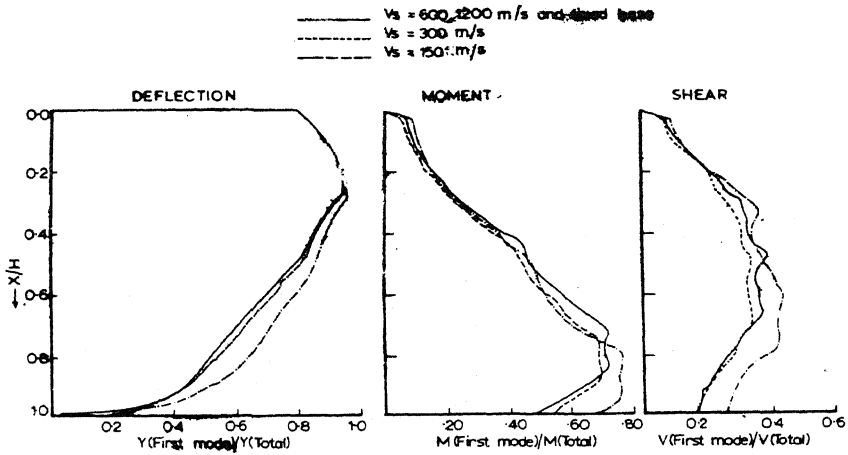


FIG. 7. CONTRIBUTION OF FIRST MODE RESPONSE TO THE TOTAL RESPONSE.

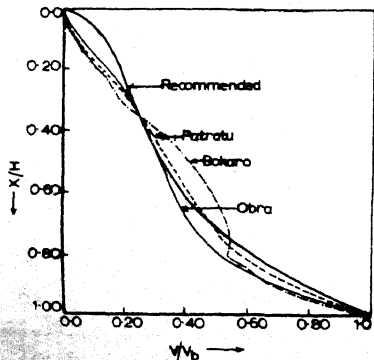


FIG. 8. MOMENT DISTRIBUTION CURVES BASED ON TIME HISTORY ANALYSIS.

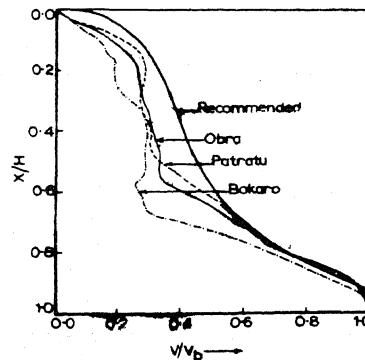
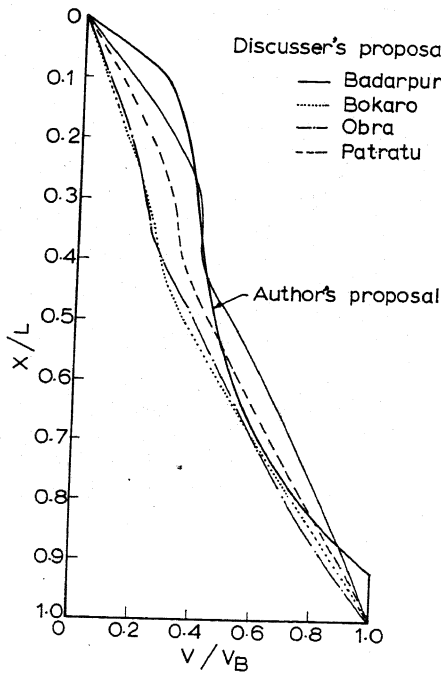
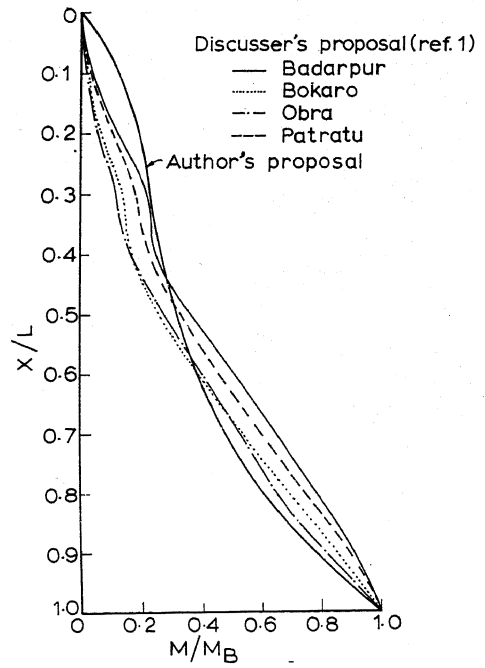


FIG. 9. SHEAR DISTRIBUTION CURVES BASED ON TIME HISTORY ANALYSIS.



SHEAR DIAGRAM  
FIG. 1



MOMENT DIAGRAM  
FIG. 2

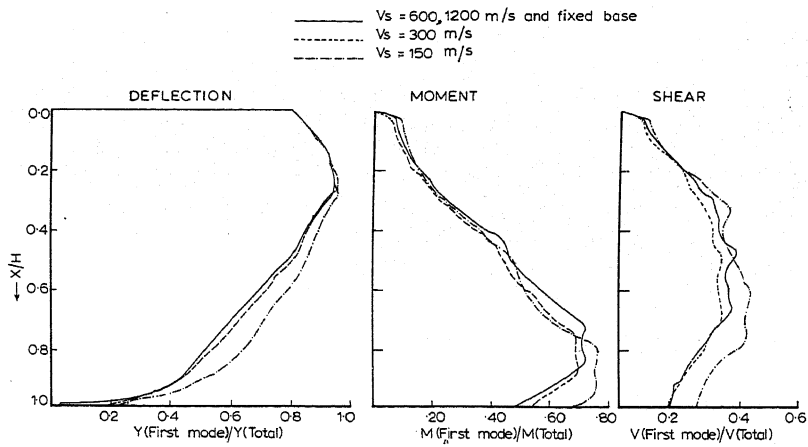


FIG. 7. CONTRIBUTION OF FIRST MODE RESPONSE TO THE TOTAL RESPONSE.

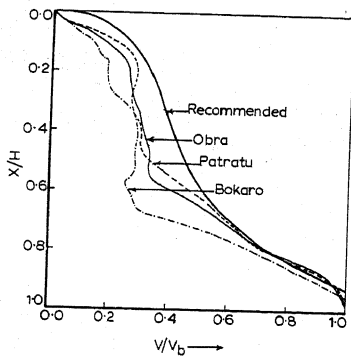


FIG. 9. SHEAR DISTRIBUTION CURVES BASED ON TIME HISTORY ANALYSIS.

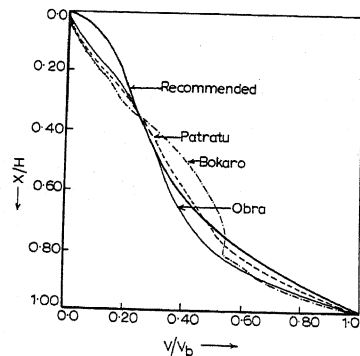


FIG. 8. MOMENT DISTRIBUTION CURVES BASED ON TIME HISTORY ANALYSIS.