TRANSIENT RESPONSE OF COOLING TOWERS TO PROPAGATING BOUNDARY EXCITATION

S. F. Masri[I], V. I. Weingarten[II]

Introduction

Many axisymmetric shell structures such as cooling towers and nuclear power plant containment structures can be subjected to traveling seismic waves. The characteristic times defining the passage of the wave front across the structure, may have significant effects on the response of the cooling tower. At the present time these shell structures are analyzed by subjecting the structure to seismic base motion. This type of motion using linear theory induces a beam type response of the cooling tower and does not permit other types of response in the structure. A traveling wave solution physically more closely represents the actual problem and allows the structure to respond in all of its natural modes. The cooling tower problem is considered in the present investigation. The cooling tower is analyzed as a shell of revolution by the finite element method. The ground motion is assumed to be a traveling wave and is decomposed into Fourier components for each time interval. The normal mode method in conjunction with the Wilson-Theta integration technique is used to determine the response of the cooling tower.

Analysis Procedure

The finite element method was used to determine the dynamic response of a hyperboiloidal shell to a traveling wave. The problem to be solved is essentially the determination of the response of a shell of revolution subjected to a dynamic support displacement loading. The solution of the problem will be carried out in the following manner:

(a) Determination of the modes and frequencies of a general shell of revolution.

(b) Determination of the dynamic response of the shell by using a modified form of the normal mode method.

(c) Studying the effect on the dynamic stresses in a typical cooling tower shell design as the wave speed and wave shape are varied.

Free Vibration for a Discrete Dynamic System

Let the discrete dynamic system of n degrees of freedom be described by the generalized coordinates $q_1, q_2, \ldots q_n$, or written as a column vector $\{q\}$. It follows from the classical dynamics that the kinetic energy T and the potential energy V of the system can be expressed respectively as, for small oscillations,

$$T = \frac{1}{2} \{\dot{q}\}^T [M] \{\dot{q}\} \tag{1}$$

$$V = \frac{1}{2} \{q\}^T [K]\{q\} \tag{2}$$

I Associate Professor, Civil Engineering Department , University of Southern California, Los Angeles, California

II Professor, Civil Engineering Department, University of Southern California, Los Angeles, California.

where [M] and [K] are symmetric mass and stiffness matrices. Hamilton's principle states

$$\delta \int_{t_1}^{t_2} L\, dt = \delta \int_{t_1}^{t_2} (T - V)\, dt = 0 \tag{2}$$

Substitution of equation (1) and (2) into the preceding expression yields the vibration equation

$$[M]\{\ddot{q}\} + [K]\{q\} = 0 \tag{3}$$

Assuming that simple harmonic motion takes place, equation (3) can be written in the form

$$[K]\{q\} = \omega^2 [M]\{q\} \tag{4}$$

where ω denotes the natural frequency and [K] and [M] are the stiffness and mass matrices which result from a finite element formulation.

Finite Element Matrix Equation for Shell

The shell is discretized as shown in Fig. 1, and a typical conical element is shown in Fig. 2. The generalized displacements assumed for a nodal circle are the three orthogonal displacements and one rotation of the normal to the shell middle surface about the circumferential direction. The conical shell element has two nodal circles and therefore eight generalized displacements which are expressed, for the n th harmonic, as $\{q_L\}$ in local coordinates and as $\{q_e\}$ in global coordinates. The q's are related by the transformation equation as follows:

$$\{q_L\} = [\lambda]\{q_e\} \tag{5}$$

The meridional dependence of the displacement field is approximated by the following shape function for the conical element.

$$\begin{aligned} U_n(s,t) &= \alpha_{1n} + \alpha_{2n} s \\ V_n(s,t) &= \alpha_{3n} + \alpha_{4n} s \\ W_n(s,t) &= \alpha_{5n} + \alpha_{6n} s + \alpha_{7n} s^2 + \alpha_{8n} s^3 \end{aligned} \tag{6}$$

Note the foregoing polynomials allow the displacements and rotation continuity at the two nodal circles. Upon evaluating equation (6) at the two nod nodal circles, yields

$$\{q_L\} = [L]\{\alpha_n\} \tag{7}$$

Eliminating $\{q_L\}$ from equations (5) and (7) gives

$$\{\alpha_n\} = [L]^{-1}[\lambda]\{q_e\} = [T]\{q_e\} \tag{8}$$

Let $\{q^M\}$ = Col. $\{\frac{\partial U_n}{\partial s}, \frac{\partial V_n}{\partial s}, \frac{U_n}{r}, \frac{V_n}{r}, \frac{W_n}{r}\}$, then

$$\{q^M\} = [S^M]\{\alpha_n\} \tag{9}$$

Similarly, let $\{q^F\}$ = Col. $\{\frac{\partial^2 W_n}{\partial s^2}, \frac{1}{r}\frac{\partial V_n}{\partial s}, \frac{1}{r}\frac{\partial W_n}{\partial s}, \frac{U_n}{r^2}, \frac{V_n}{r^2}, \frac{W_n}{r^2}$ then

$$\{q^F\} = [S^F]\{\alpha_n\} \tag{10}$$

Using equations (9) and (10) and eliminating $\{\alpha_n\}$ with the aid of equation (8) gives the strain energy expression, for the nth harmonic, as

$$V_n = \frac{1}{2}\{q_e\}^T [T]^T [I] [T] \{q_e\} \tag{11}$$

where

$$[I] = [I^M] + [I^F] \tag{12}$$

and

$$[I^M] = hH\int_o^L [S^M]^T [\varphi^M] [S^M] \, r\,ds \tag{13}$$

$$[I^F] = hH\int_o^L [S^F]^T [\varphi^F] [S^F] \, r\,ds \tag{14}$$

Referring to Fig. 2, the following relation is obtained to relate the variables 4 and s in the integrals.

$$r = \frac{r_j - r_i}{L} s + r_i = \frac{a}{L} s + r_i \tag{15}$$

The integrations, in equations (13) and (14), are evaluated numerically in the computer program using a ten-point Gauss quadrature integration procedure.

Comparing equations (11) with (2) yields the element stiffness matrix in global coordinates as

$$[K_e] = [T]^T [I] [T] \tag{16}$$

where $[K_e]$ is symmetric because $[\Phi^M]$ and $[\Phi^F]$ are symmetric.

The mass matrix is obtained by a similar procedure.

The kinetic energy, and the potential energy, for the entire shell can be expressed as summations of the respective quantities over all the conical elements composing the shell. Thus the mass matrix, and the stiffness matrix for the entire shell may be assembled from the element matrices.

The modes and frequencies of the shell of revolution can now be solved by substituting the mass and the stiffness matrices in equation (4) and solving for the eigenvalues and eigenvectors of the resulting matrix equation.

Equations of Motion with Support Displacement Excitation

The finite element matrix equation of motion for a shell of revolution which includes viscous damping forces is given by

$$[M]\{q\} + [C]\{\dot{q}\} + [K]\{x\} = \{P(t)\} \quad (17)$$

The matrix [C] is a viscous damping matrix which is assumed to be proportional to the mass and stiffness matrix

$$[C] = \alpha[M] + \beta[K] \quad (18)$$

The external forcing vector $\{P(t)\}$ is assumed to be zero for the ground motion displacement loading. An equivalent forcing vector $\{\overline{P}(t)\}$ will be subsequently derived for determining the response of the shell to ground motion excitation. The equations of motion for the shell subject to support excitation are derived in a manner similar to reference (1). The total displacement vector $\{q\}^c$ can be partitioned into boundary displacements $\{q\}^b$ and the displacement of the other nodes

$$\{q\}^c = \left\{\begin{matrix} q \\ \cdots \\ q^b \end{matrix}\right\} \quad (19)$$

The total displacements of the nodes are the quasi-static displacement $\{q_s\}$ and the dynamic displacements $\{q\}$. In partitioned form this can be written as

$$\left\{\begin{matrix} q_t \\ \cdots \\ q_t^b \end{matrix}\right\} = \left\{\begin{matrix} q_s \\ \cdots \\ q_s^b \end{matrix}\right\} + \left\{\begin{matrix} q \\ \cdots \\ q^b \end{matrix}\right\} \quad (20)$$

Substitution of equation (20) into the equations of motion yields

$$\begin{bmatrix} M & \vdots & M_b \\ \cdots & & \cdots \\ M_b^t & \vdots & M_{bb} \end{bmatrix} \left\{\begin{matrix} \ddot{q}_t \\ \cdots \\ \ddot{q}_t^b \end{matrix}\right\} + \begin{bmatrix} C & \vdots & C_b \\ \cdots & & \cdots \\ C_b^t & \vdots & C_{bb} \end{bmatrix} \left\{\begin{matrix} \dot{q}_t \\ \cdots \\ \dot{q}_t^b \end{matrix}\right\} + \begin{bmatrix} K & \vdots & K_b \\ \cdots & & \cdots \\ K_b^t & \vdots & K_{bb} \end{bmatrix} \left\{\begin{matrix} q_t \\ \cdots \\ q_t^b \end{matrix}\right\} = \{0\} \quad (21)$$

where $[K_{bb}]$, $[C_{bb}]$ and $[K_{bb}]$ denote forces at the support nodes due to unit accelerations, velocities and displacement at the support nodes. The matrices $[K_b]$, $[C_b]$ and $[M_b]$ are coupling matrices between the support nodes and the non-support nodes. The [K], [C] and [M] represent the stiffness, damping, and mass matrices of all non-support nodes.

Using the quasi-static equilibrium equation

$$[K]\{q_s\} + [K_b]\{q_s^b\} = 0 \quad (22)$$

one obtains

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{P\} \quad (23)$$

where

$$\{P\} = -[M \vdots M_b]\left\{\begin{matrix} \ddot{q}_s \\ \cdots \\ \ddot{q}_s^b \end{matrix}\right\} - [C \vdots C^b]\left\{\begin{matrix} \dot{q}_s \\ \cdots \\ \dot{q}_s^b \end{matrix}\right\}$$

since the damping forces are generally considerably smaller than the inertia terms, the effective force vector is given by

$$\{\overline{P}\} = [MK^{-1}K_b - M_b]\{\ddot{q}_s^b\} \quad \text{or} \quad \{\overline{P}\} = [\overline{M}]\{\ddot{q}_s^b\} \quad (24)$$

where the effective mass $[\overline{M}]$ is defined by

$$[\overline{M}] = [MK^{-1}K_b - M_b] \quad (25)$$

Assuming that the horizontal ground acceleration excitation shape $\ddot{q}_H(t)$ and the propagating wave speed is known, the value of the base acceleration as a function of time and circumferential angle θ can be determined. The base acceleration components in global coordinates can then be expanded in terms of Fourier components

$$\ddot{q}_1 = 0 \qquad \ddot{q}_2 = \sum_{n=1}^{N} a_n(t) \sin n\theta$$
$$\ddot{q}_3 = \sum_{n=0}^{N} b_n(t) \cos n\theta \qquad \ddot{q}_4 = 0 \tag{26}$$

Once $a_n(t)$ and $b_n(t)$ are determined, the effective force $\{\bar{P}\}^n$ is known for each Fourier componenet. The equations of motion for each Fourier component is given by

$$[M]^n\{\ddot{q}\}^n + [C]^n\{\dot{q}\}^n = [K]^n\{q\}^n = \{\bar{P}\}^n \tag{27}$$

The standard normal mode analysis is used to solve equation (27) for $\{q\}^n$. Superposition of the displacement modes for a particular Fourier component given by the equation (27) and then the superposition displacement modes of all the Fourier components will finally give the dynamic response of the cooloing tower. These calculations are still in progress and results will be reported at the conference.

Conclusions

The method presented in this paper can be used to solve any shell of revolution structure subjected to ground motion. Although final numerical results are not available, preliminary results indicate that the cooling tower dynamic response to a traveling wave is quite different than a standing wave excitation. A parametric study of the effect of different wave shapes and wave velocities will be presented at the conference.

References

1. Clough, R. W., and Penzien, J., Dynamics of Structures, McGraw-Hill, 1975.

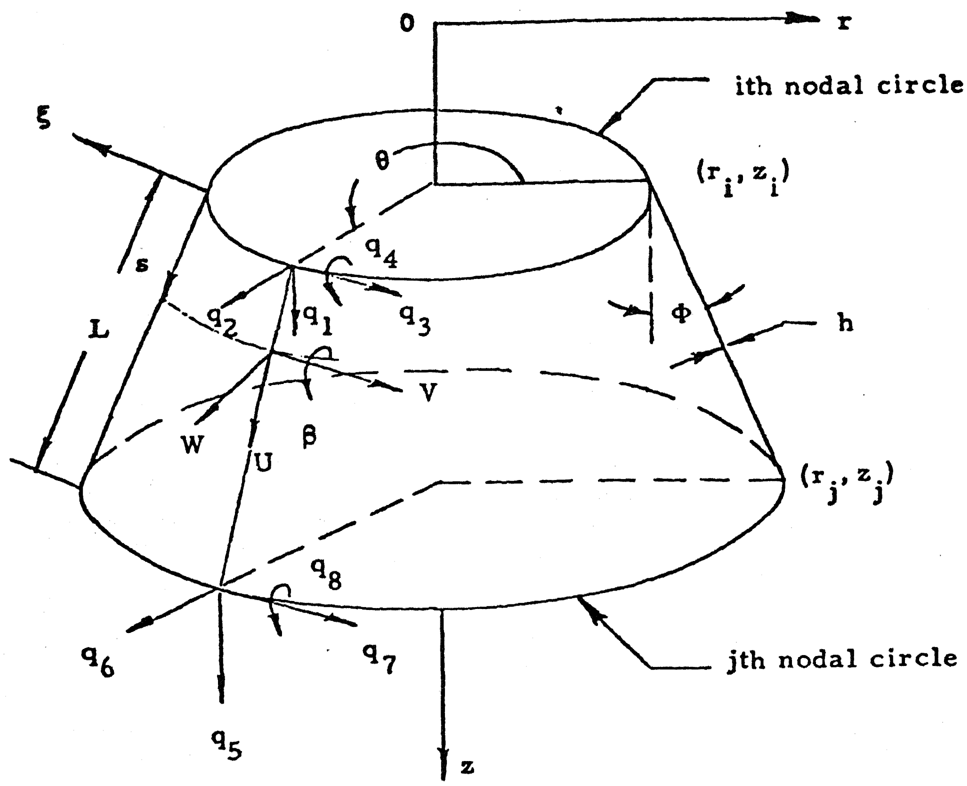

Figure 2 A Conical Shell Element

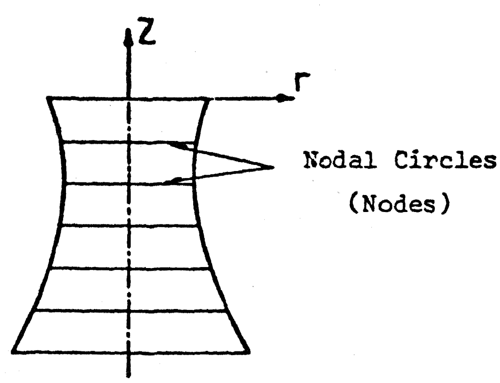

Figure 1

Discretized Shell of Revolution