SEISMIC ANALYSIS OF ASYMMETRICAL STRUCTURES SUBJECTED TO ORTHOGONAL COMPONENTS OF GROUND ACCELERATION

by

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SYNOPSIS

A procedure to compute the seismic responses of asymmetrical structures subjected to two orthogonal components of ground motion is presented. It is an extension of the response spectrum technique for structures under unidirectional excitation. This method is applied to a realistically proportioned asymmetrical L shaped building subjected to two horizontal components of the 1940 El Centro earthquake records. Its accuracy is checked with results obtained using time history dynamic analysis.

INTRODUCTION

The use of response spectrum technique to obtain the seismic response of symmetrical structures is well established. Its use is enhanced by the publication of response spectrum data for many earthquake records [1]. The use of this technique is less common for asymmetrical buildings. In asymmetrical buildings, the response of the structure in two directions is coupled. The response in one direction is affected not only by ground motions in that direction, but also by ground motions perpendicular to that direction. Therefore, to obtain the true response of the structure, it is necessary to consider both horizontal components of ground motions acting on the structure simultaneously. In the present paper, the problem of bidirectional excitation of asymmetrical structures is examined. In particular, the problem relating to the application of the response spectrum technique to such a problem is discussed. An L shaped flat slab shear wall multi-storey building of representative dimensions is used as an example for illustration.

THEORY

Consider an asymmetrical N mass linear dynamical system subjected to two orthogonal components of ground accelerations $g_x(t)$ and $g_y(t)$ in the X and Y directions. This system has 3N degrees of freedom and can be described by a displacement vector $\{\Delta\}$ defined by

$$\{\Delta\} \equiv \text{col.}\ (x_1, x_2 \ldots\ldots x_n, y_1, y_2 \ldots\ldots y_n, \theta_1, \theta_2 \ldots \theta_n) \qquad (1)$$

where x_j, y_j refer to the displacements of a reference point O on the j^{th} mass in the X and Y directions respectively, and θ_j is the rotation of the j^{th} mass. The equation of motion about O can be written as

$$[M]\ \{\ddot{\Delta}\} + [C]\ \{\dot{\Delta}\} + [K]\ \{\Delta\} = [M]\begin{Bmatrix} g_x \\ g_y \\ 0 \end{Bmatrix} \qquad (1)$$

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where [M], [C] and [K] are the mass, damping, and stiffness matrix respectively. Let [ϕ] be the modal matrix of the problem. Making the normal mode transformation

$$\{\Delta\} = [\phi] \{q\} \qquad (2)$$

the uncoupled equation of motion becomes

$$[M^*] \{\ddot{q}\} + [C^*] \{\dot{q}\} + [K^*] \{q\} = [\phi]^T [M] \begin{Bmatrix} g_x \\ g_y \\ 0 \end{Bmatrix} \qquad (3)$$

where [M*] and [K*] are generalized mass and stiffness diagonal matrices respectively and [C*] is the damping matrix, assumed to be diagonal. The response of mode i is given by

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q} + \omega_i^2 q_i = P_{xi}\, g_x(t) + P_{yi}\, g_y(t) \qquad (4)$$

where ω_i and ζ_i is the natural frequency and fractional critical damping for mode i. P_{xi} and P_{yi} are modal participation factors for mode i in the X and Y directions respectively.

The key difference in an uni-directional excitation and a bi-directional excitation problem is shown in eqn. (4). In the case of unidirectional excitation in the X direction for example, $g_y(t)$ is zero. Then, the maximum response for q_i can be obtained directly if the spectral values for $g_x(t)$ is available. However, for a bi-directional excitation, it is necessary to obtain the spectral values for a time series $G_i(t)$ given by

$$G_i(t) \equiv P_{xi}\, g_x(t) + P_{yi}\, g_y(t) \qquad (5)$$

before the maximum response of q_i ($i = 1, 2 \ldots$) and hence the spectral response calculations for the system can be carried out. For simplicity in computation, it is desirable to by-pass the spectral value calculation of $G_i(t)$ and make use of the already tabulated spectral values of $g_x(t)$ and $g_y(t)$ directly.

In general, the time at which the maximum response of q_i occurs due to the X direction excitation alone will be different from the time at which q_i becomes maximum due to the Y direction excitation alone. Therefore, the response from bi-directional excitation can be approximated by combining the responses of the system subjected to individual unidirectional excitation in X and Y directions in a root sum square manner. In other words, the response from bi-directional excitation R* is given by

$$R^* = \sqrt{(R)_x^2 + (R)_y^2} \qquad (6)$$

where $(R)_x$ and $(R)_y$ are the responses of the asymmetrical structure due to unidirectional excitation in the X and Y direction respectively. $(R)_x$ and $(R)_y$ can be obtained by the usual response spectrum technique, using the tabulated spectral values of $g_x(t)$ and $g_y(t)$.

EXAMPLE

In order to check the validity of eqn. (6) let us consider an asymmetrical L-shaped shear wall building subjected to the 1940 El Centro ground records. The proportions of the building are chosen realistically as follows. The overall dimensions are 152' by 140'. A typical floor plan arrangement is shown in Fig. 1. The building is 195' in height with an average floor height of 9.5 feet. The floor slab is taken to be 6 in. and the average wall thickness is 10 in. The coupling effect of the slabs is taken into account by assigning connecting beams with a moment of inertia of .1 ft^4 between appropriate pairs of shear walls. A value of 5.21 x 10^5 kip per square ft. is used for the modulus of elasticity. The structure is modelled by a five mass system with masses located at 40 ft., 81 ft., 129 ft., 167 ft., and 195 ft. from the ground. The first four masses weigh 6990 kip each and each mass has a polar moment of inertia of 22.3 x 10^6 kip-ft. about its mass center. The top mass has half the weight and half the moment of inertia. Point 0 is the reference point and all displacement and force quantities are referred to a vertical axis passing through 0. The mass center of each mass is assumed to be located at a distance 52 feet in the X direction and 58 feet in the Y direction from 0. The stiffness matrix about 0 is computed by inverting a flexibility matrix. The flexibility matrix is obtained from using a method presented previously by the authors [2]. 5% critical damping is used for the first three modes and an average of 10% or more damping value is used for higher modes. Four cases of excitation are studied. In Case 1, the building is subjected to a unidirectional excitation consisting of the N-S component of the El Centro record acting along the X direction. In Case 2, the E-W component of El Centro record is assumed to act in the Y direction only. In Case 3, both the N-S component and the E-W component are assumed to act simultaneously, along the X, and Y directions respectively. In Case 4, the excitation is similar to Case 3 except that the E-W component record used here is assumed to be 180 degrees out of phase of the E-W record use in Case 3.

DISCUSSION and CONCLUSION

The first six natural periods and the modal participation factors P_{xi} and P_{yi} of the structures are shown in Table I. The periods are reasonably well separated. Therefore, it is expected that the total response can be estimated accurately by a root sum square combination of modal responses. The mode shapes are shown in Fig. 2 and 3. Modes 2 and 5 are mainly lateral modes while modes 1, 3, 4 and 6 are coupled lateral-torsional modes.

The base shears, base torque, top deflections and top rotation for the four cases considered are computed by the response spectrum technique and also by the method of direct integration of the equations of motion. The results are shown in Table II. Since cases 1 and 2 are unidirectional excitation cases, the spectrum technique can be applied directly and the values labelled RSS are the root sum square of the first six modal responses in each of the cases. Mode 2 provides the major contribution to the base shears while mode 3 contributes most to the base torque. The ratio between the values based on dynamic analysis and spectrum technique in each case is listed and they will serve as a measure of the accuracy of the spectrum technique. The values labelled (RSS)* in cases 3 and 4 are obtained as the root sum square of the spectral responses of cases 1 and 2, as suggested by equation (6). A comparison of the response ratios in all four cases shows that the procedure suggested provides the same order of accuracy to estimate the seismic response of a bi-directionally excited structure as the standard response spectrum procedure to to estimate the response of a uni-directional excited stru

Another observation is the sensitivity of the response parameters considered to the phase relationship between the two components of ground excitation. By reversing the phase of the E-W component record, as is done in Case 4, a change of 20% in base shears and 30% in base torque occurs in the present calculation.

In summary, a method based on the spectrum technique is presented to estimate the seismic responses of asymmetrical structures subjected to bi-directional ground excitations. The method is applied to a realistically proportioned asymmetrical multi-storey building. Comparison with results based on dynamic analysis indicates that the proposed procedure gives reasonable estimates of the seismic responses. However, in view of the complexity of the behavior of asymmetrical structures subjected to bi-directional excitation, more verification of the present proposed procedure should be carried out before it can be accepted.

ACKNOWLEDGMENT

The investigation is supported by a grant from National Research Council of Canada and the authors wish to acknowledge the help of Dr. Tzu-I Hsu in carrying out the spectral response computations.

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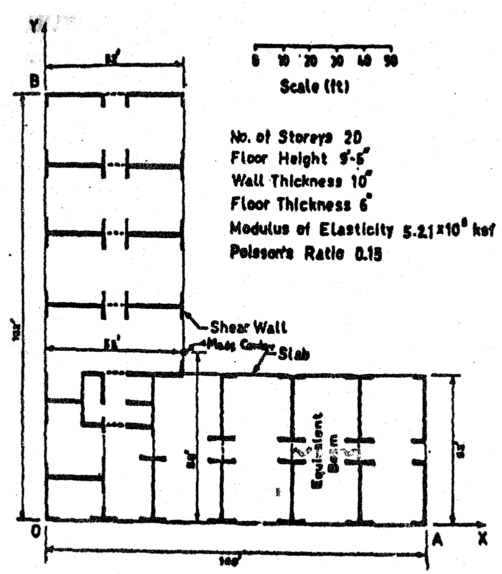
Table I: Natural Periods and Modal Participation Factors

MODE	1	2	3	4	5	6
Natural Period (seconds)	1.59	1.09	0.80	0.33	0.22	0.17
P_x	2.04	-1.34	-0.52	-1.07	-0.70	-0.27
P_y	-1.81	-1.54	0.43	0.93	-0.81	0.22

TABLE II

	Base Shear (Kip)		Base Torque (Kip-ft.) $\times 10^{3}$	Top Deflection (Ft.)		Top Rotation (rad) $\times 10^{-3}$
	X-Direction	Y-Direction		X-Direction	Y-Direction	
Case 1						
$(RSS)_1$	4830	5060	488	0.53	0.55	4.70
Dynamic	5730	5030	540	0.52	0.60	5.90
Dynamic/(RSS)	1.19	0.99	1.11	0.98	1.09	1.26
Case 2						
$(RSS)_2$	3640	3910	306	0.39	0.37	3.25
Dynamic	3590	5260	366	0.40	0.40	3.23
Dynamic/(RSS)	0.99	1.35	1.20	1.03	1.08	0.99
Case 3						
RSS*	6050	6390	576	0.66	0.66	5.71
Dynamic	6660	6540	728	0.71	0.83	7.96
Dynamic/RSS*	1.10	1.02	1.26	1.08	1.26	1.39
Case 4						
RSS*	6050	6390	576	0.66	0.66	5.71
Dynamic	7520	8090	525	0.65	0.58	5.40
Dynamic/RSS*	1.24	1.27	0.91	0.98	0.88	0.95



STRUCTURAL PLAN OF A L SHAPED APARTMENT BUILDING

Fig. 1

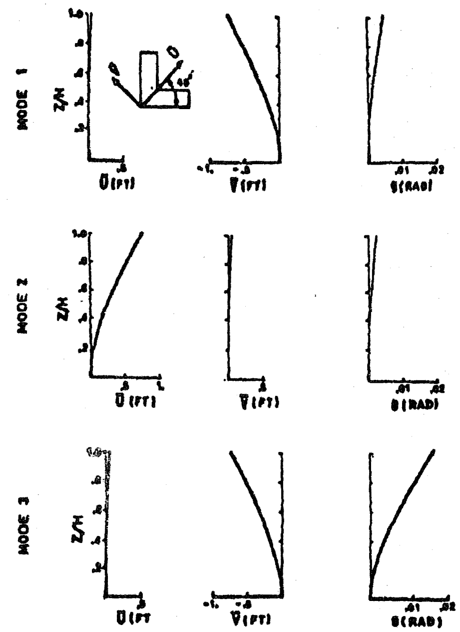


Fig. 2

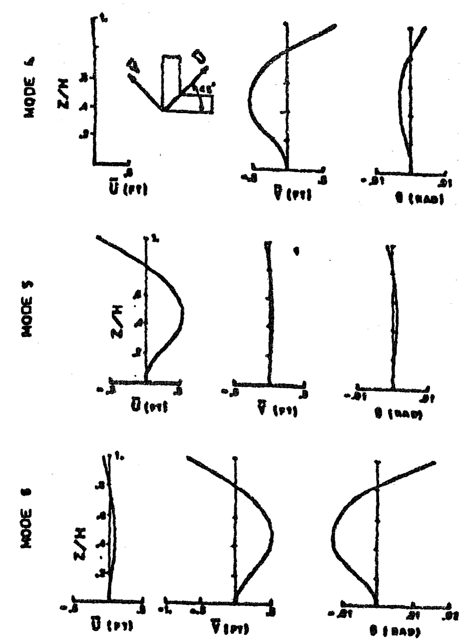


Fig. 3