ANALYSIS OF SHEARWALL-FRAME SYSTEMS

by

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SYNOPSIS

This work presents that a shearwall-frame system could be substituted by a virtual system which consists of a shearwall and infinitely rigid floor beams, jointed to the shearwall at its axis on one ends with springs defined by their bending stiffness and supported by sliding supports on the other ends. The internal forces of this virtual system could be calculated by using a set of equations written for the moments on the shearwall sections just above the floor beams. This set of equations is similar to the three moments equations. The internal forces of the real system could be found afterwards easily.

ANALYSIS

A shearwall-frame system may consists of a) shearwalls, b) internal frames and end frames, c) the tie beams joining frames to shearwalls and shearwalls to each other (Fig.1) It is known that frames could be substituted, wich an acceptable aproximation, by shearcolumns with story shear stiffness

$\Delta X_i D_i = \Delta X_i \; \Sigma \; (12 \; E \; I_{cij} \; a_{ij}/\Delta X_i^3)$ [1] . Such a system being cut with horizontal planes just above the floor beams, is shown in Fig.2 with its internal forces. M^{ℓ}_{ki} and M^{r}_{ki} are bending moments applied to the shearwall on its axis by the floor tie beams.

The moment equation of equilibrium for the i th portion of the shearwall, divided by ΔX_i is

$$(M_{pi+1} - M_{pi})/\Delta X_i = (\Delta M_p/\Delta X)_i = - (M^{\ell}_{k+1} + M^{r}_{ki+1})/ \; \Delta X_i - Q_{pi+1} \qquad 1.$$

By writing the same equation for the (i-1) th portion of the shearwall, subtracting it from 1 and eliminating the

$(Q_{pi+1} - Q_{pi})$ difference by using the

$$Q_{pi+1} - Q_{pi} = - W_{i+1} - (Q^{\ell}_{di+1} + Q^{r}_{di+1}) + (Q^{\ell}_{di} + Q^{r}_{di})$$

horizontal projection equation of the i th portion of the system, the following relation could be obtained :

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$$\left(\frac{\Delta M_p}{\Delta x}\right)_i - \left(\frac{\Delta M_p}{\Delta x}\right)_{i-1} = -\left(\frac{M_{ki+1}}{\Delta x_i} - \frac{M_{ki}}{\Delta x_{i-1}}\right) - Q_{di+1} + Q_{di} \qquad 2.$$

M_{ki} and Q_{di} are : $M_{ki} = M^{\ell}_{ki} + M^{r}_{ki}$, $Q_{di} = Q^{\ell}_{di} + Q^{r}_{di}$.

By using the known relations of deformations - internal forces for the y horizontal displacements of the system,

$$Q^t_{di} = \Delta x_i D^t_i \left(\frac{\Delta y}{\Delta x}\right)_i , \quad M^t_{ki} = k^t_i \theta = k^t_i \left(\frac{\Delta y}{\Delta x}\right)_i \qquad t = r,\ell \qquad 3.$$

and assuming for simplicity that k^t_i , D^t_i and Δx_i are constant, could be obtained from 2,

$$(\Delta M_p/\Delta x)_i - (\Delta M_p/\Delta x)_{i-1} = -\{k+\Delta x.D.\Delta x\}(\Delta^2/\Delta x^2)_i - W_{i+1} \qquad 4.$$

Here is $k = k^{\ell} + k^{r}$, $D = D^{\ell} + D^{r}$. k^t_i could be obtained by using k given in [2] and changing I_s into I^t_k (Fig.1).

Now a system could be assumed consisting of the shearwall with the moment of inertia I_p and the infinitely rigid floor beams jointed to the shearwall at its axis by springs with bending stiffness R

$$R = k + \Delta x \cdot D \cdot \Delta x \qquad 5.$$

on their one ends and with sliding supports on their other ends as shown in Fig.3. It is obvious that equation 4 gives the relation between $(\Delta M_p/\Delta x)_i$ and the horizontal y displacements of this system under Wi forces. Therefore the system shown in Fig.1 could be transfered into a virtual system shown in Fig.3a. It could be stated that this transformation is also valid in the case in which the rigidities of shearwalls and frames differ from floor to floor. However in this case R changes from floor to floor. R_i should be taken as $R_i = k_i + (\Delta x_i D_i)_m \Delta x_{im}$. Δx_{im} and $(\Delta x_i D_i)_m$ are averages of the corresponding values of immediate upper and lower floors of the i th floor beam. For the uppermost floor is $R_{n+1} = k_{n+1} + (\Delta x_n D_n)\Delta x_n$.

The internal forces of this virtual system could be easily determined by solving the set of equations

$$\delta_{i,i-1} X_{i-1} + \delta_{i,i} X_i + \delta_{i,i+1} X_{i+1} + \delta_{i,o} = 0, \quad i = 1,2, \ldots n,$$

which is written by choosing M_{pi} values of Fig.2 as X_i unknowns of statically indeterminate system as it is shown in Fig.3b. The moments of the springs with the bending stiffness R_i are shown as M_{Ri}.

M_{pi} values of the real system are found as X_i. The sum of the bending moments of the tie floor beams on a floor at shearwall axis is $M_{ki} = M_{Ri}k_i/R_i$ and the moment of any tie floor beam is $M_{ki}^t = M_{ki}k_i^t/k_i$. Shear forces of the shearwalls Q_{pi} could be obtained from Eq.1. The force $Q_{di} = Q_{di}^{\ell} + Q_{di}^{r}$ could be found from horizontal equilibrium for the portion of the system above i th floor : $Q_{di} + Q_{pi} = T_{o,1}$ (Fig.1 and 2). Q_{di}^{ℓ} and Q_{di}^{r} values could be determined by assuming that they are proportinal with D_i^{ℓ} and D_i^{r} according to Eq.3. The bending moments M_{Ai} of the tie floor beams at the ends joining to the frames are found by using the results given in 2 with the notations in Fig.1 as a function of M_{ki} as

$$M_{Ai}^t = M_{ki}^t \left[(1+\frac{b}{2a} - A)/\left\{ 1+\frac{b}{2a}\right)(1+\frac{b}{a}) - (1+\frac{3b}{2a})\frac{A}{2}\}\right].$$

Here A is $A = \{ K_{AC}(1+b/2a) + K_{AF} \}/\Sigma K$. M_{Ai}^t moments could be assumed as couples $F_i^t = M_{Ai}^t/(\Delta x_i + \Delta x_{i-1})$ acting on the frames at the floor levels (Fig.4). Internal forces of the frame under F_i^t and Q_{di}^t could be determined by using the method given in [1]

If the system contains several shearwalls with moments of inertias $I_p(1)$, $I_p(2)$,..,$I_p(m)$, changing in the same manner according to x, they could be substituted by a single şhearwall with a moment of inertia $I_p(s) = \sum_1^m I_p(z)$

$z = 1,2,..m.$

REFERENCES

[1] Muto, K. ; WCEE 1956.

[2] Cardan,B. ; ACI journal 1961, September.

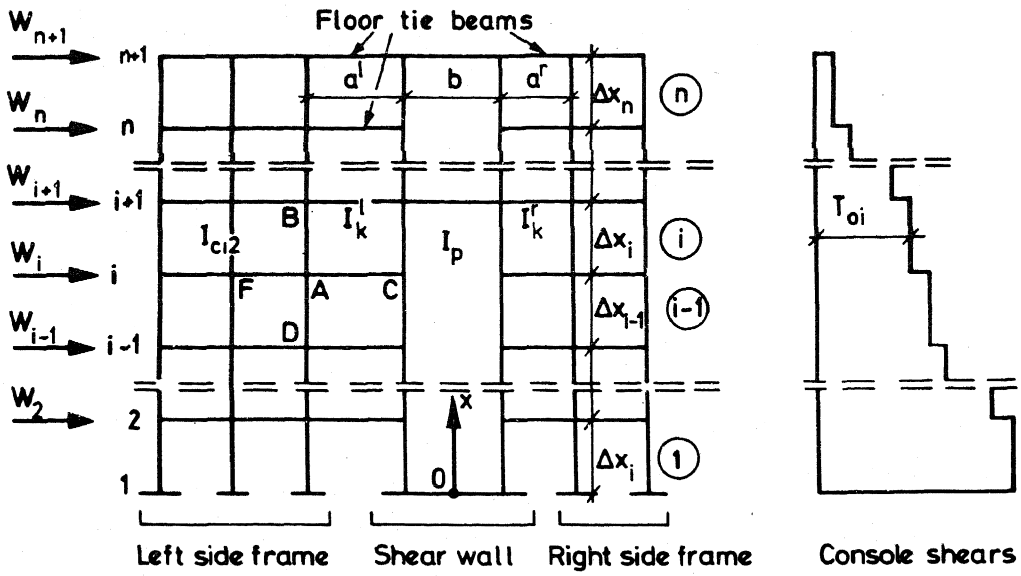

Fig 1

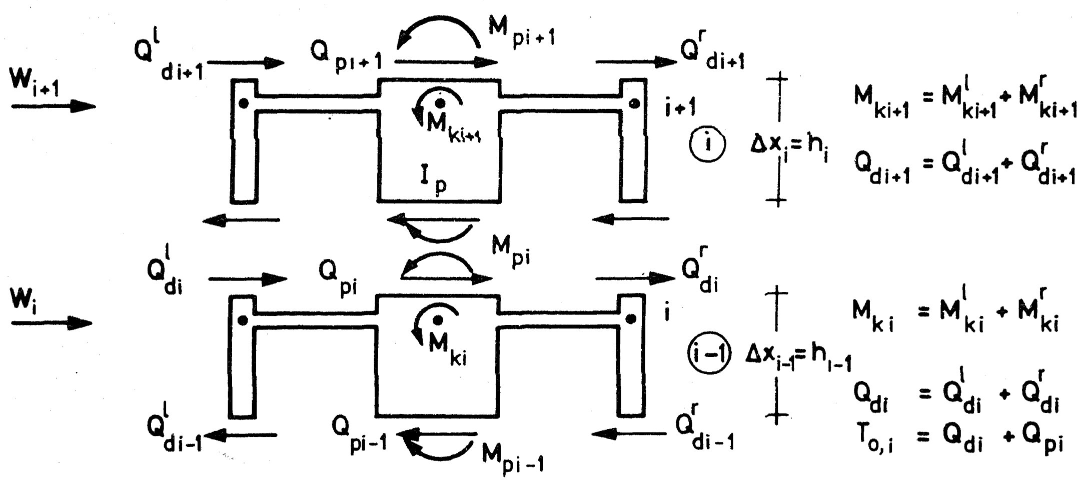

Fig 2

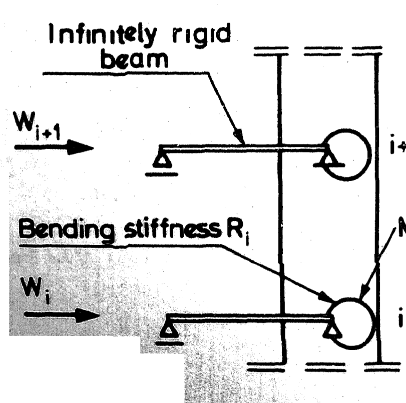

Fig 3a

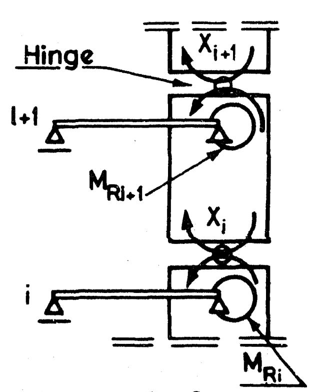

Fig 3b

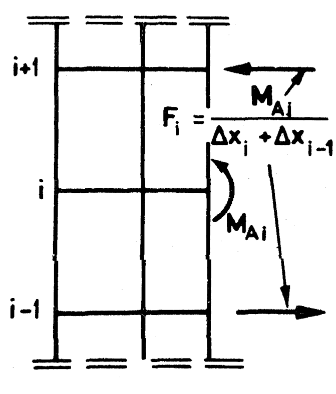

Fig 4