OPTIMUM TUNING OF THE DYNAMIC DAMPER TO CONTROL RESPONSE OF STRUCTURES TO EARTHQUAKE GROUND MOTION

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SYNOPSIS

A method is presented for tuning the dynamic damper so that the mean square acceleration of the anti-earthquake structure to which the damper is attached can be made minimum. The acceleration power spectral density of earthquake ground motion at the base under the ground layer is assumed to be constant for a certain frequency range. The natural frequency and the damping of the dynamic damper which give optimum tuning are shown in the form of chart.

INTRODUCTION

The dynamic damper or the dynamic vibration absorber is an effective means to control mechanical vibrations, and the method of tuning it is treated in most text-books of mechanical vibration. It can also be used to control structural vibrations induced by earthquake ground motion. When it is used to this end, however, the optimum tuning condition will be more or less different from that written in the text-books because the spectral density of earthquake ground motion needs to be taken into account in this case. This paper discusses the method to tune the dynamic damper so that it can minimize the mean square acceleration of the structure excited by the earthquake ground motion. The maximum acceleration of the structure which is of primary concern in designing anti-earthquake structures will be made minimum by making the mean square acceleration minimum.

BASIC IDEA OF OPTIMUM TUNING

The vibrating system to be discussed in this paper is shown in Fig.1, where the structure whose vibrations are to be controlled is represented by an undamped single degree of freedom system. In Fig.1, m_1 is the mass of the structure, k_1 is the stiffness of the members which restrain the mass, m_2 is the mass of the dynamic damper, c is the damping coefficient, k_2 is the stiffness, $z(t)$ is the absolute displacement of the ground surface, $x_1(t)$ is the displacement of the mass m_1 relative to the ground surface, and $x_2(t)$ is the displacement of the mass m_2 relative to the mass m_1.

Let $P(\omega)$ be the acceleration power spectral density at the base, $F(j\omega)$ be the complex frequency response of acceleration from the base to the ground surface, and $G(j\omega)$ be the complex frequency response of acceleration from the ground surface to the structure. Then the acceleration power spectral density of the structure is given by $|G(j\omega)|^2|F(j\omega)|^2P(\omega)$, and

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the mean square acceleration of the structure is given by

$$\overline{\ddot{x}_1(t)^2} = \int_0^\infty |G(j\omega)|^2|F(j\omega)|^2 P(\omega)d\omega \qquad (1)$$

It will be reasonable to assume that $P(\omega)=P_0$ for $\omega_1 \le \omega \le \omega_2$ and $P(\omega)=0$ for $\omega<\omega_1$ or $\omega_2<\omega$. Eq.(1) will become

$$\overline{\ddot{x}_1(t)^2} = P_0 \int_{\omega_1}^{\omega_2} |G(j\omega)|^2|F(j\omega)|^2 d\omega \qquad (2)$$

Then the problem is to find the integrand $|G(j\omega)|^2|F(j\omega)|^2$ which gives minimum integral $\overline{\ddot{x}_1(t)^2}$.

When the transmissibility $|G(j\omega)|$ of the two degree of freedom system is plotted with ω taken as abscissa, all curves of $|F(j\omega)|$ pass through two fixed points whatever the damping may be. Since the transmissibility $|F(j\omega)|$ is independent of the system parameters which govern $|G(j\omega)|$, all curves of the integrand $|G(j\omega)|^2|F(j\omega)|^2$ also intersect at two fixed points. If the mass m_2 and the stiffness k_2 are chosen so that the ordinates of these two points will be made equal and, further, if the damping coefficient c is selected so that the integrand $|G(j\omega)|^2|F(j\omega)|^2$ will take the maximum at these two points, the integrand $|G(j\omega)|^2|F(j\omega)|^2$ will be made as samll as possible. It is mathematically not exact to consider that this as small as possible integrand gives minimum integral, but the integral given by this integrand will be very near the minimum. This is the basic idea of optimum tuning.

DERIVATION OF OPTIMUM TUNING CONDITION

Equations of motion of the two degree of freedom system shown in Fig.1 are

$$\begin{aligned} m_1(\ddot{z}+\ddot{x}_1) + k_1x_1 - c\dot{x}_2 - k_2x_2 &= 0 \\ m_2(\ddot{z}+\ddot{x}_1+\ddot{x}_2) + c\dot{x}_2 + k_2x_2 &= 0 \end{aligned} \qquad (3)$$

Let $X_1(s)$, $X_2(s)$ and $Z(s)$ be Laplace transforms of $x_1(t)$, $x_2(t)$ and $z(t)$. Then Eqs(3) become

$$\begin{aligned} (m_1s^2+k_1)X_1(s) - (cs+k_2)X_2(s) &= -m_1s^2Z(s) \\ m_2s^2X_1(s) + (m_2s^2+cs+k_2)X_2(s) &= -m_2s^2Z(s) \end{aligned} \qquad (4)$$

The acceleration transfer function from the ground surface to the structure is obtained from Eqs(4) as

$$G(s) = -\frac{m_1m_2s^4+(m_1+m_2)cs^3+(m_1+m_2)k_2s^2}{m_1m_2s^4+(m_1+m_2)cs^3+(m_2k_1+m_1k_2+m_2k_2)s^2+k_1cs+k_1k_2} \qquad (5)$$

Substituting $s=j\omega$ $(j=\sqrt{-1})$, $\omega_{01}=\sqrt{k_1/m_1}$, $\omega_{02}=\sqrt{k_2/m_2}$, $\nu=\omega_{02}/\omega_{01}$, $R=m_2/m_1$, $\zeta=c/2\sqrt{m_2k_2}$ and $\lambda=\omega/\omega_{01}$ into Eq.(5) gives the acceleration transmissibility

$$|G(j\omega)| = g(\lambda) = \left[\frac{\{(1+R)\nu^2\lambda^2-\lambda^4\}^2+\{2\zeta(1+R)\nu\lambda^3\}^2}{\{\lambda^4-\lambda^2(1+R)\nu^2\lambda^2+\nu^2\}^2+\{2\zeta\nu\lambda(1-(1+R)\lambda^2)\}^2}\right]^{1/2} \qquad (6)$$

The dimensionless frequencies for which all $g(\lambda)$ curves intersect are given by equating the transmissibilities Eq.(6) for two extreme cases, $\zeta=0$ and $\zeta=\infty$. Then

$$\frac{(1+R)\nu^2\lambda^2-\lambda^4}{\lambda^4-\lambda^2-(1+R)\nu^2\lambda^2+\nu^2} = -\frac{(1+R)\nu\lambda^3}{\nu\lambda\{1-(1+R)\lambda^2\}} \qquad (7)$$

Eq.(7) becomes

$$2(1+R)\lambda^4 - \{2(1+R)^2\nu^2+(2+R)\}\lambda^2 + 2(1+R)\nu^2 = 0 \qquad (8)$$

Eq.(8) has two positive roots, $\lambda_P{}^2$ and $\lambda_Q{}^2$, which give the abscissas at which all $g(\lambda)$ curves intersect. It is seen from Eq.(8) that the roots depend only on the mass ratio R and the natural frequency ratio ν.

When the ground layer has single predominant period of ground motion, the acceleration transmissibility $|F(j\omega)|$ is written as

$$|F(j\omega)| = [\frac{p^4+(2hp\omega)^2}{(p^2-\omega^2)^2+(2hp\omega)^2}]^{1/2} \qquad (9)$$

where $2\pi/p$ is the predominant period and h is the damping ratio of the ground layer. Substituting $\lambda=\omega/\omega_{0\,1}$ and $\rho=p/\omega_{0\,1}$ into Eq.(9) gives

$$|F(j\omega)| = f(\lambda) = [\frac{\rho^4+(2h\rho\lambda)^2}{(\rho^2-\lambda^2)^2+(2h\rho\lambda)^2}]^{1/2} \qquad (10)$$

To find the natural frequency ratio ν for which the ordinates of the intersection points become equal, it is more convenient to substitute $\zeta=0$ into Eq.(6) because the ordinates of the intersection points are independent of ζ. Then

$$\{\frac{(1+R)\lambda_P{}^2}{1-(1+R)\lambda_P{}^2} f(\lambda_P)\}^2 = \{-\frac{(1+R)\lambda_Q{}^2}{1-(1+R)\lambda_Q{}^2} f(\lambda_Q)\}^2 \qquad (11)$$

By substituting Eq.(10) into Eq.(11) and making use of the relation between the coefficients and the roots of Eq.(6), Eq.(11) becomes

$$a_0\nu^6 + a_2\nu^4 + a_4\nu^2 + a_6 = 0 \qquad (12)$$

where $a_0 \sim a_6$ are all polynomial expressions of R, ρ and h. Eq.(12) has a single positive root, and for this ν the ordinates of the intersection points are made equal. In Fig.2 the natural frequency ratio ν thus obtained is plotted with ρ taken as abscissa, and R as the parameter.

Next step is to find out ζ for which the integrand $|G(j\omega)|^2|F(j\omega)|^2$ takes the maximum at λ_P and λ_Q. This is done simply by equating the derivative of the integrand with λ to zero

$$\partial[g(\lambda)^2 f(\lambda)^2]/\partial\lambda = 0 \qquad (13)$$

Eq.(13) gives

$$b_0\zeta^4 + b_2\zeta^2 + b_4 = 0 \qquad (14)$$

where $b_0 \sim b_4$ are all polinomial expressions of λ, ρ and ν. Two ζs, ζ_P for λ_P and ζ_Q for λ_Q, are obtained from Eq.(14). As these ζs are not much different to each other, the average of these will give a damping near the optimum. The arithmetic average is shown in Fig.3 with ρ taken as abscissa, and R as the parameter.

It is important to check the magnitude of the force which results from the relative motion between the structure and the dynamic damper. Too large force will break the structure, though the damper itself does not necessarily need to survive the earthquake. The transfer function from the ground acceleration to the force between the structure and the dynamic damper is given by

$$H(s) = \frac{(cs+k_2)X_2(s)}{s^2Z(s)} = -\frac{m_2k_1(cs+k_2)}{m_1m_2s^4+(m_1+m_2)cs^3+(m_2k_1+m_1k_2+m_2k_2)s^2+k_1cs+k_1k_1} \quad (15)$$

Then the transmissibility is

$$|H(j\omega)| = m_2h(\lambda) = m_2\left[\frac{\nu^4 + (2\zeta\nu\lambda)^2}{\{\lambda^4-\lambda^2-(1+R)\nu^2\lambda^2+\nu^2\}^2+\{2\zeta\nu\lambda(1-(1+R)\lambda^2)\}^2}\right]^{1/2} \quad (16)$$

The mean square damping force is thus expressed as

$$\overline{(c\dot{x}_2+k_2x_2)^2} = P_0\int_{\omega_1}^{\omega_2}|H(j\omega)|^2|F(j\omega)|^2d\omega \quad (17)$$

If a single dynamic damper is replaced by smaller ones of which the total effect is equivalent to that of a single one, the damping force which each smaller damper applies to the structure can be decreased.

TUNING PROCEDURE

A dynamic damper is designed in the following way by making use of Figs 2 and 3. The natural frequency of the structure ω_{01} and the predominant period of ground motion $2\pi/p$ have to be known first. The frequency ratio ρ is then determined by them. If the mass ratio R is given by design considerations, the natural frequency ratio ν and the damping ratio ζ are given by Figs 2 and 3. Although it does not seem easy to make an exact estimation of the equivalent mass of the structure, a misestimation will not much affect the result obtained as easily seen from the figures. The dynamic damper shown in Fig.1 takes the form of a mass-damper-spring system for the convenience of discussion, but it will be understood that the dynamic damper in any other forms can be used to control structural vibrations.

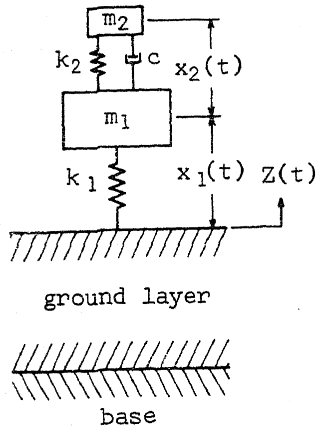

Fig.1

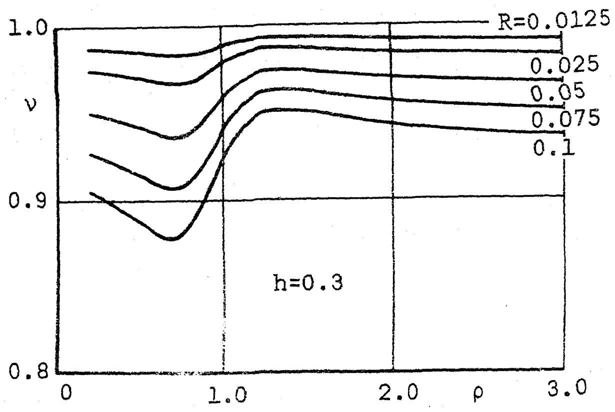

Fig.2 Optimum natural frequency ratio ν

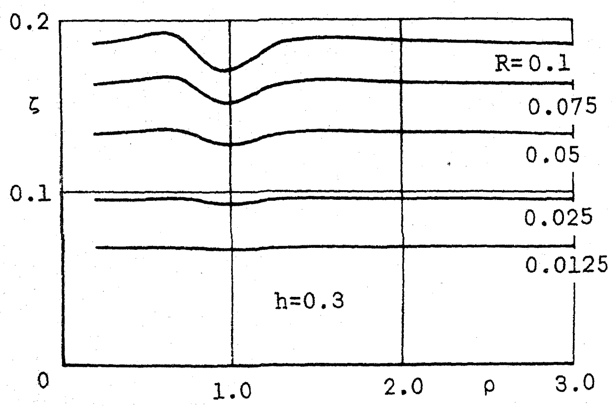

Fig.3 Optimum damping ratio ζ

In Fig.2 and 3, h is the damping ratio of the ground layer, $R=m_2/m_1$ and $\rho=p/\omega_{01}$, where $\omega_{01}=\sqrt{k_1/m_1}$ and $2\pi/p$ is the predominant period of the ground motion