A DAMPING MODEL FOR RESPONSE ANALYSIS OF MULTISTOREYED BUILDINGS

by

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SYNOPSIS

The investigation presented in the paper makes an experimental study on the damping pattern of building models during free vibrations and forced vibrations and an analytical study on a flexible mathematical method for incorporating any chosen intermodal damping in response analysis. A case study by computer calculations regarding the influence of damping matrix on the response of multistoreyed buildings with and without joint rotations, on rigid and on flexible foundations is also done.

INTRODUCTION

The response forces of a building subject to an earthquake are controlled by its damping. While it is possible to systematically determine the inertia and stiffness properties of a building, it is not possible to so define damping. It is usual to define damping by an equivalent viscous damping coefficient for each normal mode. The response values are highly sensitive to damping and hence the damping values ought to be very carefully chosen in order to obtain meaningful response values. An approximate damping idealisation would be incongruous with the present day sophisticated techniques of response analysis. In order that the well known equation of motion, $\underline{M}\,\ddot{\underline{x}} + \underline{C}\,\dot{\underline{x}} + \underline{K}\,\underline{x} = -\underline{M}\,\ddot{\underline{z}}$ ...(1), can be uncoupled for solution, the damping matrix should be defined such that it has orthogonal properties. This can be achieved in many ways such as the adoption of 'mass proportional', 'stiffness proportional', 'mass cum stiffness proportional' or 'equal interfloor' damping definitions or to adopt Caugheys' equation. While The Caughey series method involves in serious computational errors, the mass cum stiffness damping matrix tends to give moderated response envelopes close to equal intermodal damping [1] . Experimental and test results reported in literature (e.g. vide Ref. 2,4,5) indicate that the coefficient of critical damping of buildings is sensitive to the amplitude of vibration and the natural frequencies of the modes. While there is general consistency in the reports that damping increases with the force level and the amplitude, results are not conclusive regarding its pattern of dependence on the intermodal natural frequencies. The reports that damping ratios consistently increase with force levels suggest that damping would be greater for wind and earthquake induced motions than for ambient and forced vibrations with low force levels realised during tests on structures.

EXPERIMENT

Experiments were conducted in the present work on building models of steel for a study of damping patterns. The 5C frame of Housner and Brady [3] which is a five storeyed steel building shear frame was chosen and a single bay was idealised as a building with floor dimensions 6.096 m x 6.096 m on four columns. Adopting a scale factor of 20, the dynamic scale model had its properties: 753.6, 645.9, 538.3, 430.6 and 323.0 Kg/cm for

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stiffness of each column in storey 1,2,3,4 and 5 respectively and mass of each floor 0.0023108 $kgsec^2/cm$. Mode models of the building were idealised to have an effective mass (M_e) and effective stiffness (K_e) for each mode, where $M_e = (\underline{A}^T \underline{M}\,\underline{1})^2 / (\underline{A}^T \underline{M}\,\underline{A})$ and $K_e = p^2 M_e$ ...(2,3). $\underline{A}$ is the vector of mode shape and p the natural frequency of the mode. Apart from the above set of mode models which corresponded to the idealisation of the building as a shear frame, a second set of mode models for the first four modes of the building model taking into account the joint rotations of the actual steel beam sections used was also fabricated. The building model and a set of mode models are indicated in Fig.1. The models were subject to nearly sinusoidal lateral excitations in a vibration bed excited by an unbalanced double mass exciter whose force level could be regulated. The response displacements of the models were picked up at the floor levels by LVDT pick ups and recorded in 5 channels of a Phillips Oszilloscript. The first set of models were excited with a force level corresponding to setting 10/10 of the exciter which was double the force level of the setting 5/5 used to excite the second set of models. The displacement response vs. forcing frequency curves are given in Figs.2 and 3. The coefficient of critical damping, 'zeta', (z) for all the mode models (denoted by ESDS 1-4 for set I and ESDS 1R-4R for set II) and the building model was analysed by the method of "logarithmic decrement" and also by the method of "band width between half power points" and the results have been plotted in Fig.5 to a logarithmic scale. The corresponding amplitudes have also been plotted to a normal vertical scale with their origins on the respective zeta values. The zeta values of all the models by free vibration lies between .14 and 1.1% (average value: 0.45%) and by forced vibration, between 1.3% and 22.8% (average value: 7.02%). Considering forced vibrations alone, the average zeta values for set I and set II experiments are 12.94% and 2.58% respectively. The corresponding average values of amplitudes are 1.28 mm and 0.602 mm. It is observed that the forced vibration values of zeta are far in excess of free vibration values and that zeta is influenced by the amplitude of vibration and the force level. The base shear of a building is a good indication of the response forces and displacements resulting from the input energy of the forcing function and consequently the ESDS force(equivalent single degree structure force) itself and in turn, the absolute acceleration of the ESDS emerges as a parameter controlling the variation of zeta during forced vibrations.

NUMERICAL INTEGRATION FOR VARIABLE DAMPING

The value of zeta (z_{fr}) at the commencement of excitation and the upper limit (z_{fce}) when the absolute acceleration reaches a specified value (f) are assessed and utilised in the numerical integration procedure instead of adopting the usual constant zeta value throughout the time history. During each time step zeta value is interpolated by the rule $z = z_{fr} + (z_{fce} - z_{fr})\, a_e / f$ ...(4), where a_e is the absolute acceleration pertaining to the time step. When equal intermodal damping is desired, zeta value for mode-1 is calculated by Eqn.(4) and this value adopted for all the modes. For the development of the procedure and for case study, the Wilson-Clough method of numberical integration with constant damping matrix (vide reference 1) was adopted. The response acceleration in this method is of the form

$\ddot{\underline{x}} = \underline{F}^{-1}\,\underline{R}$ ...(5), where $\underline{F}$ is a square matrix $= \underline{M} + \frac{dt}{2}\underline{C} + \frac{(dt)^2}{6}\underline{K}$ and $\underline{R}$ is a column vector dependent on the known velocities and acceleration of the previous time step (t) and the ground acceleration of the present step (t + dt). With a constant damping matrix, $\underline{F}$ need be inversed only once at the beginning of the procedure but with variable damping $\underline{F}$ should be inversed at every time step which results in uneconomical computer effort. This disadvantage is overcome in the present work by making use of effective mass ($\underline{M}_e$), effective damping ($\underline{C}_e$) and effective stiffness ($\underline{K}_e$) matrices which are all diagonal matrices. The damping matrix elements C_e take the values: $2z\ M_e\ p$ of the corresponding modes. In this procedure the matrix $\underline{F}$ in Eqn.(5) is diagonal and its inversion is simple and is achieved by assigning reciprocal values to the diagonal elements. The response values of displacements, velocities and accelerations in this method will be that of the ESD structures ($\underline{x}_e$, $\dot{\underline{x}}_e$, $\ddot{\underline{x}}_e$). These are converted to the response of the real structure (x_i, $\dot{x}_i$, $\ddot{x}_i$ of the mass at level 'i') by the relationship:

$(x_i / x_e) = (\dot{x}_i / \dot{x}_e) = (\ddot{x}_i + \ddot{z}) / (\ddot{x}_e + \ddot{z}) = (A_i / R_e)$ ...(6) where A_i is the mode shape vector at level 'i' and R_e is the radius of participation of the mode, taking the value: $(\underline{A}^T\ \underline{M}\ \underline{A}) / (\underline{A}^T\ \underline{M}\ \underline{1})$ ...(7). The method was compared with the standard procedure of Wilson and Clough which adopts the regular mass, damping and stiffness matrices, by response analysis of Housner and Brady's 5A frame with joint rotation and 5C shear frame [3] subject to Koyna T. E.Q. with a number of combinations of zero damping 'mass proportional', 'stiffness proportional' and 'mass cum stiffness proportional' damping. Analysis was also done with soil interaction for the 5C frame but with zero damping for comparison of the methods. The zeta values adopted were .025 in the fundamental mode for the mass proportional and stiffness proportional damping and .025 in the first and second modes for the mass cum stiffness proportional damping matrix. The corresponding intermodal damping values are: mass proportional, frame 5A, zeta (1) .025 (2) .00813 (3) .00452 (4) .00301 (5) 0.00221; frame 5C, zeta (1) .025 (2) 0.00957 (3) 0.00615 (4) 0.00471 (5) .00381; stiffness proportional, frame 5A, zeta (1) .025 (2) .0769 (3) .1383 (4) .2074 (5) . 2834; frame 5C, zeta (1) .025 (2) .0653 (3) .1017 (4) .1325 (5).1642; mass cum stiffness proportional, frame 5A, zeta (1) .025 (2) .025 (3) .0373 (4) .0532 (5) .0712; frame 5C, zeta (1) .025 (2) .025 (3) .0326 (4) .0401 (5) .04819. There was exact agreement in the response values by the Wilson-Clough method and the ESDS method. The computer time required for analysing structures with more than six storeys, by the ESDS method is less than that by the Wilson- Clough method. As a typical example of the response values by the various damping assumptions, the first floor maximum absolute acceleration of 5A frame normalised by dividing by gravitational acceleration, for the Koyna T. E.Q. with 5% damping are: 1.03748 (no damping) , 0.90846 ($a\ \underline{M}$) , - 0.22089($b\ \underline{K}$) and 0.32318 ($a\ \underline{M} + b\ \underline{K}$). With equal intermodal damping of 2.5%, the corresponding value is 0.46407. The intermodal zeta values reveal that the mass proportional and stiffness proportional damping matrices respectively emphasise and suppress the higher modes. The mass cum stiffness matrix is an improvement but that also suppresses higher modes to some extent. Equal intermodal damping regulates evenly the intermodal response contributions.

RESPONSE ANALYSIS

Response dependent damping in accordance with Eqn.4 of the patterns (z_{fr} = 0.0, z_{fce} = 0.15, zeta increment step, 'zinc' = 0.01, f = 300) , (0.02, 0.2, 0.01, 300) , (0.03, 0.15, 0.01, 300) and (0.05, 0.1, 0.01, 300) were adopted to analyse the five storey building frame and the Incometax building subject to Hiroo EW, Koyna L., and Koyna T. earthquakes. The TV tower was analysed for Hiroo EW E.Q. with the damping pattern (0.002, 0.1205, .01, 1400.) derived from the experimental results of the steel building frame. Typical results of the maximum response forces and the response dependent damping pattern during the time history have been plotted in Figs. 4,6 and 7. Each earthquake-structure combination has its own time dependent damping pattern even for identical damping parameters. The variable damping acts like a centrifugal governor mechanism and moderates excessive and deficient displacements during the time history. The response values with response dependent damping are less than that with a constant damping: ξ_{f}. The pattern of interfloor maximum response forces varies from earthquake to earthquake. The floor displacements are somewhat insensitive to higher mode contributions over fundamental mode while the response accelerations and floor forces are quite sensitive. The importance of the higher mode contributions depends on the pattern of earthquake and the first mode alone is a poor representation of the total effects of the storey forces.

CONCLUSIONS

i) Damping during forced vibrations is far in excess of the free vibration value. For numerical integration, a stage by stage alteration of damping with the increase in absolute response acceleration is recommended and a method for it presented.

ii) Equal intermodal damping is an improvement over the mass cum stiffness proportional damping matrix which suppresses higher modes to some extent.

iii) Maximum response values with response-dependent damping are considerably less than that obtained by a constant damping matrix. Further experimental research on the response-dependent damping pattern of structures is recommended.

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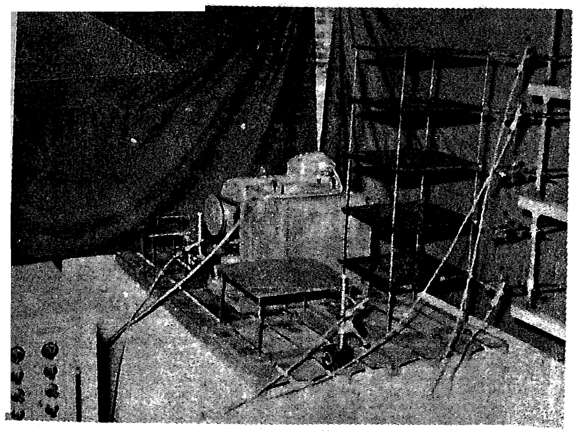

FIG.1. Building and mode models and experimental set.up

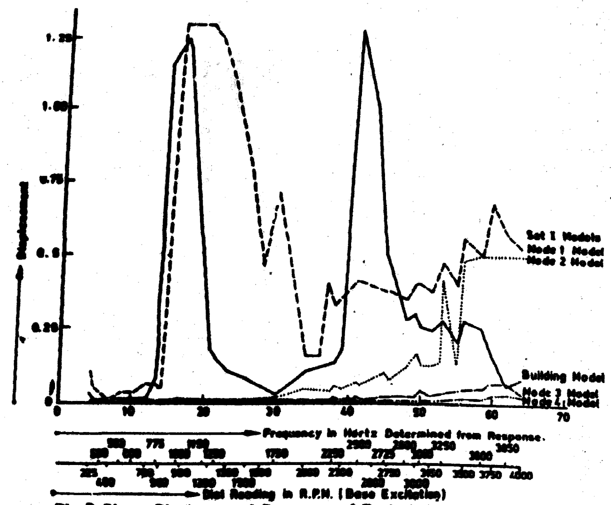

Fig.3. Storey Displacement Response of Test Models Subject to Nearly Sinusoidal Base Excitation (Exciter Setting 10/10)

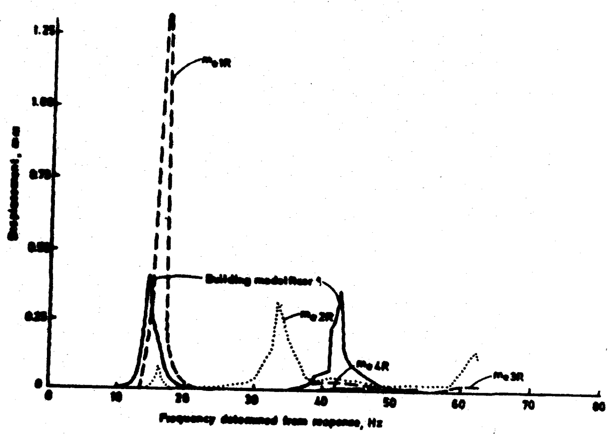

FIG. 2 . Displacement response of second set of models (exciter setting 5/5)

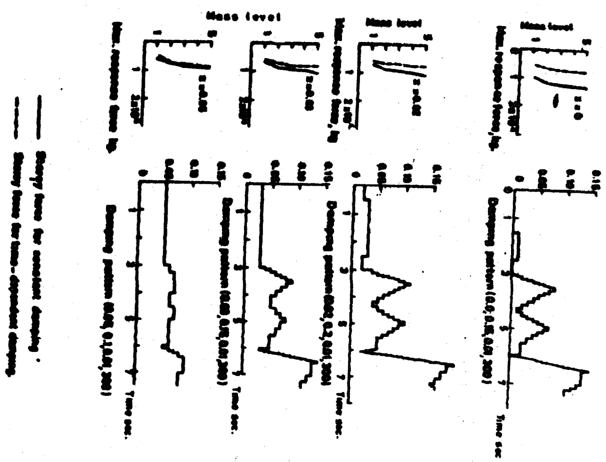

FIG. 4 . Response of five-storey building frame to Hiroo E.W (1970) earth quake and corresponding time dependent damping patterns.

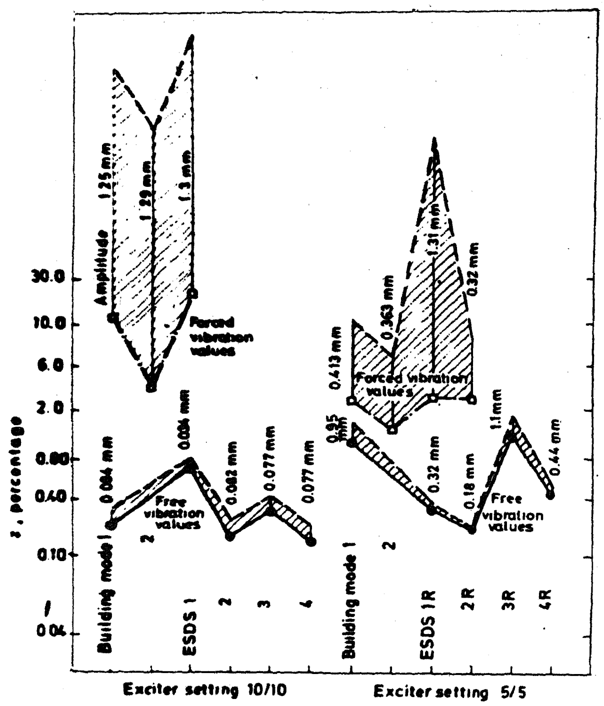

FIG.5 .Coefficients of critical damping from experiments of set I and set II models

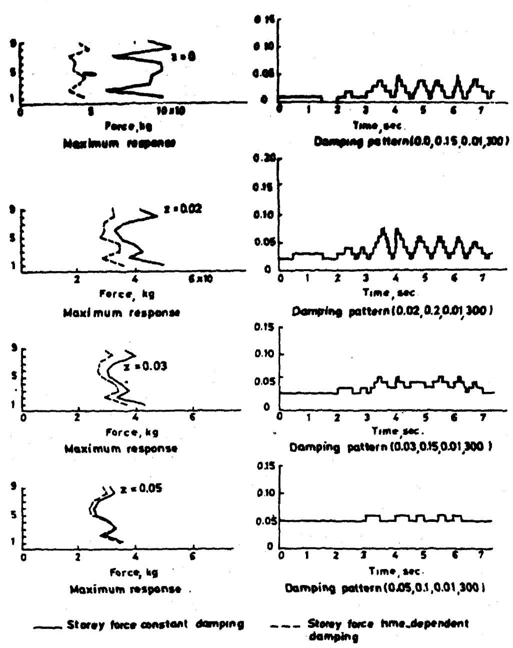

FIG. 6 Responses of I.T. building to Koyna transverse earthquake and corresponding time dependent damping pattern.

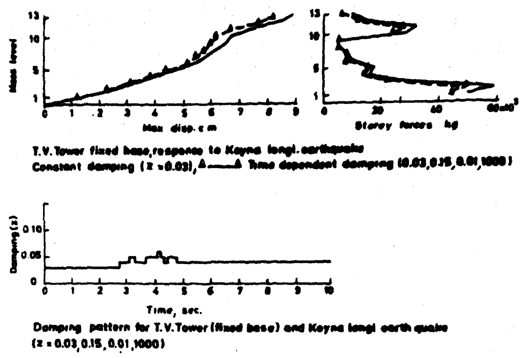

FIG. 7 .Time dependent damped response of T.V.Tower, to Koyna longitudinal earthquake