DUCTILITY STUDIES OF PARAMETRICALLY EXCITED SYSTEMS

by

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SYNOPSIS

The method proposed for studying ductility and excursion ratios of inelastic frameworks subject to parametric earthquake motions of one horizontal and vertical components employs a mathematical formulation based on elastic and dissipated strain energy. The numerical results derived for various structures when compared with those obtained by using other conventional methods, illustrate the advantages of the method and show the significant effect that coupling earthquake motions has on ductility requirements.

INTRODUCTION

In a seismic design, economic considerations generally require that some of the energy put into a structural system during strong earthquake motions must be dissipated by large inelastic deformations. It is common to express the maximum required inelastic deformations in terms of a ductility ratio. When a structure is subject to one horizontal and vertical earthquake motions, the ductility requirement is greater than that for a system that is subject to horizontal motion only (1, 2). Because conventional methods, however, are inadequate for culaculating ductility ratios of systems subject to coupling motions, a method is presented herein which is based on dissipated strain energy. Numerical examples of the method are compared with those for which other methods were used.

CONVENTIONAL DUCTILITY FORMULATIONS

As in Ref. (5), the ductility ratio may be defined as the maximum absolute joint rotation divided by the yield rotation. By using the idealized moment-rotation relationship shown in Fig. 1, one can express the ductility ratio as

$$\mu = |\theta|_{max}/\theta_y = 1 + (\alpha/\theta_y) \tag{1}$$

in which α = plastic rotation of a joint, $\theta_y = M_pL/6EI$ as the yield rotation resulting from the symmetric plastic moment M_p, L = member length, and EI = flexural rigidity of the member. The excursion ratio is the total plastic rotation of a joint during an earthquake divided by the yield rotation as

$$\epsilon = \sum_{i=1}^{N\mu} (\alpha_i/\theta_y) = \sum_{i=1}^{N\mu} (\mu_i - 1) \tag{2}$$

in which N_μ is the total number of times in which the joint suffers the plastic deformation during the earthquake. Actually, the magnitude of θ_y depends entirely on the magnitude and direction of the existing moment at either end; for unsymmetric bending, $\theta_y = M_pL/2EI$. Thus Eqs. (1) and (2) cannot realistically indicate the ductility requirement at each joint.

Another ductility formulation (1) may be derived from Eq. (1) or Fig. 1 in an

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alternate form as

$$\mu = 1 + (M_{max} - M_p)/pM_p \tag{3}$$

The excursion ratio associated with Eq. (3) is

$$\epsilon = \sum_{i=1}^{N\mu} \frac{(M_{max})_i - M_p}{pM_p} = \sum_{i=1}^{N\mu} (\mu_i - 1). \tag{4}$$

It is apparent that Eq. (3) eliminates the short comings presented by Eq. 1 in that the ductility ratio includes the boundary conditions expressed in terms of moments. By considering the Bauschinger effect (4) by limiting the stress range to $2M_p$ during an elastic unloading as shown in Fig. 2, one may observe that the half cycle below the axis yields small θ_{y2}. The use of either Eq. (1) or (3) will give an unreasonable ductility ratio.

DISSIPATED ENERGY METHOD

The actual moment-rotation relationship of a structural joint resulting from a coupling earthquake motion is nonlinear and may be typically shown in Fig. 3. It is obviously difficult to determine the necessary information for Eqs. (1) and (3). The proposed method of calculating the ductility ratio of a joint is based on the relationship between the dissipated strain energy at that end and the elastic strain energy of the member as

$$\mu = 1 + (E_d/E_{es}) \tag{5}$$

The definition of E_d and E_{es} is illustrated in Fig. 2. Apparently the dissipated strain energy is directly related to the plastic rotation; any increase in the plastic rotation results in an increase in E_d and a corresponding increase in μ. The elastic strain energy of a total member is used so that the boundary conditions of moments and rotations of the member are considered. For a member under symmetric bending, Eq. (5) becomes

$$\mu = 1 + (M_p\alpha/(2)\ (\tfrac{1}{2})\ \theta_y M_p) = 1 + (\alpha/\theta_y) \tag{6}$$

which is identical to Eq. (1).

NUMERICAL EXAMPLES

Example 1. The frame shown in Fig. 4 is analyzed for two times horizontal (N69°W) and three times vertical components of the 1952 Taft earthquake with 5% damping. Each of the floor masses of $W_1 = 185.9^k$, $W_2 = 173.8^k$, and $W_3 = 87^k$ is lumped half at the girder center and one-fourth on the top of the column. The ductility and excursion ratios of the elastoplastic case are shown in Figs. 4 and 5, and those of the bilinear case (p=0.05) are given in Figs. 6 and 7. The analyses for the horizontal earthquake component only (H) include the P-Δ effect of the gravity load. The symbol, H + V, represents the analyses associated with the horizontal and vertical components of the earthquake in addition to the P-Δ effect of the gravity load and vertical inertial force. The structure is designed for strong columns that do not have any plastic rotation during the response period.

Example 2. The four-story-three-bay rigid frame shown in Fig. 8 is analyzed for the N-S and vertical components of the 1940 El Centro. The analyses are similar to Example 1 except that the columns have plastic deformations for which the columns' plastic moments are reduced according to the AISC recommendations (3). The comparisons of ductility and excursion ratios based on Eqs. 1 and 5 are shown in Figs. 8 and 9.

Example 3. The ten-story-single-bay rigid frame shown in Fig. 10 is analyzed with the same conditions given in Example 2. Except that all the plastic moments are increased by 2.5 for the purpose of preventing an early collapse of the system. The yielding stress is assumed to be 36 ksi.

OBSERVATIONS AND CONCLUSIONS

1. The higher values of ductility ratio show a larger deviation among all three formulations; however, the ductility ratio based on the energy formulation has less deviation for each girder than that obtained by the other two methods.

2. The ductility ratio obtained by using Eq. (1) is always larger than that obtained by using Eqs. (3) and (5).

3. The formulation based on energy includes the boundary conditions of a member and the plastic deformation at the joint.

4. No problem results for the method based energy when moment-rotation relationship is nonlinear in elastic range for a structure subject to coupling motions.

5. Energy formulation is more adaptable when considering a bilinear response with a Bauschinger effect.

6. The inclusion of the vertical ground motion results in almost all cases for an increase in the ductility and excursion ratios.

7. For the three-story building, the vertical earthquake motion demands a larger ductility for girders on the upper floors.

8. For the ten-story building, the vertical earthquake motion causes larger ductility and excursion ratio in the girders located approximately at the quarter points of the structural height.

9. The ductility requirement related to the location of a system is somewhat different from the general conclusions given in Ref. 1.

ACKNOWLEDGEMENTS

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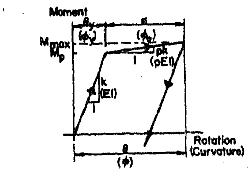

Fig. 1. Moment - Deformation Relationship

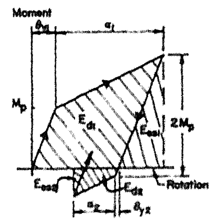

Fig. 2. Bauschinger Effect on Moment

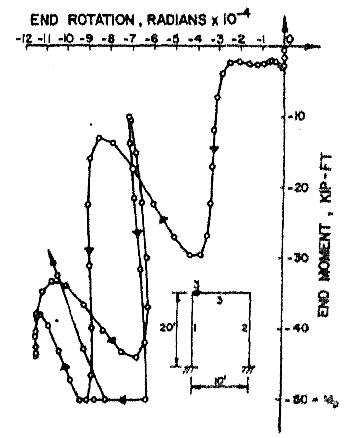

Fig. 3. Moment-Rotation at Node 3 Due to Coupling Earthquake Motion

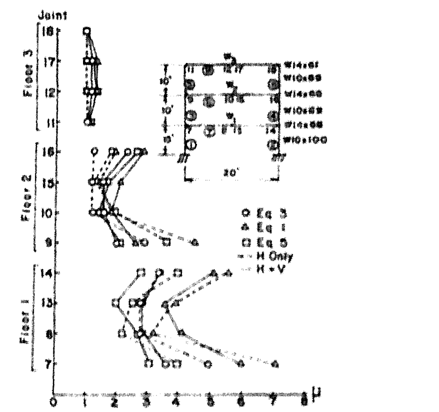

Fig. 4. Elasto-Plastic Ductility Ratios of Girders, 1952 Taft, 5% Damping

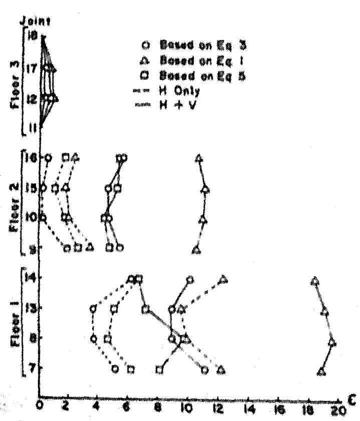

Fig. 5. Elasto-Plastic Excursion Ratios of Girders of Fig. 4

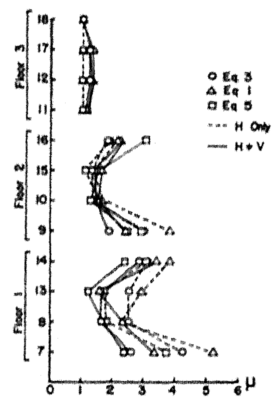

Fig. 6. Bilinear Ductility Ratios of Girders of Fig. 4

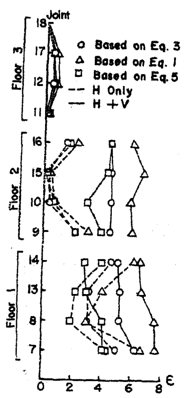

Fig. 7. Bilinear Excursion Ratios of Girders of Fig. 4

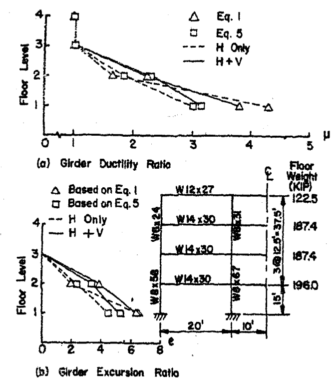

Fig. 8. Elasto-Plastic Girder Ductility and Excursion Ratios, 1940 El Centro, Undamped

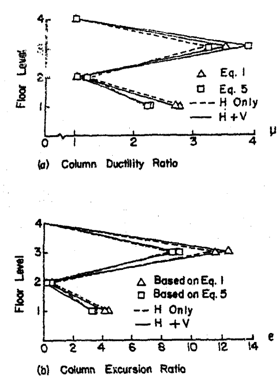

Fig. 9. Elasto-Plastic Column Ductility and Excursion Ratios of Fig. 8

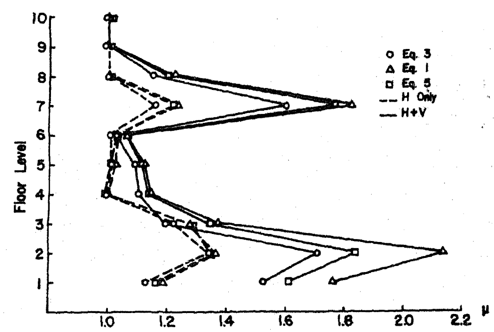

Fig. 9. Girder Excursion Ratios of Undamped Elasto-Plastic 10-Story Frame, 1940 El Centro

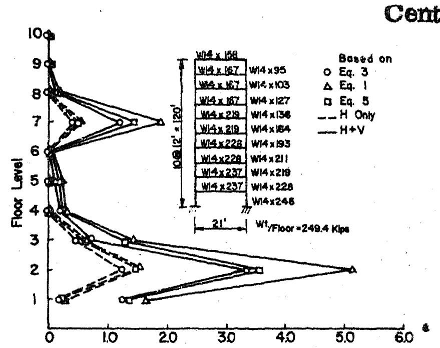

Fig. 11. Girder Ductility Ratios of Fig. 10