ESTIMATION OF MAXIMUM HYSTERETIC RESPONSE TO NON-WHITE RANDOM EXCITATION

by

H. Goto, H. Kameda and H. Iemura(I)

SYNOPSIS

Random response of single-degree-of-freedom bilinear hysteretic structures are analyzed to furnish useful imformation for aseismic structural design with emphasis on ductility requirements. Stationary and nonstationary random excitations with a single peak frequency are employed to clarify the effects of non-white frequency components of earthquake ground motion. The response of bilinear structures is predicted by means of equivalent linearization techniques. Using these results, probability distribution of the maximum response is predicted through pure-birth and envelope methods. Monte Carlo simulation performed on a digital computer verifies applicability of the analytical methods.

1. INTRODUCTION

During strong earthquakes, deformation of most structures would exceed the yield limit. In this sence, numerous studies have been devoted to earthquake response analysis of structures with nonlinear hysteretic restoring force. In most of these studies, deterministic methods to calculate response to certain specified excitations have been used to verify structural safety against strong ground motions. Some of them deal with nondeterministic analysis which discusses prediction of root-mean-square (rms) response of hysteretic structures subjected to stationary white-noise excitation.

For assessing structural reliability during earthquakes, it is desirable to estimate the probability distribution of maximum response of hysteretic structures, since it represents the load effect beyond the yield limit. It is also required in simulating future earthquake motions to take account of non-white property of frequency and nonstationary characteristics of amplitude.

In this study, linearization techniques are adopted to analyze random inelastic response. First passage problems are discussed to predict the probability distribution of maximum hysteretic response subjected to stationary and nonstationary random excitation with a single peak frequency. Monte Carlo simulation is performed on a digital computer to check the accuracy of the theoretical analysis.

2. METHOD OF ANALYSIS

2-1 Equivalent Linearization

The equation of motion of single-degree-of-freedom structures with hysteretic restoring force $q(n,\mu,\dot{\mu},t)$ is written as

$$\ddot{\mu}(t)+\beta_0\dot{\mu}(t)+\omega_0^2 q(n,\mu,\dot{\mu},t) = -r_s\psi(t)f(\omega_f,h_f,t) \qquad \text{.................(1)}$$

where $\mu(t)$= ductility factor response; β_0 and ω_0= damping coefficient and natural frequency, respectively, in infinitesimal vibration; r_s= intensity parameter of excitation; $f(\omega_f,h_f,t)$= stationary random process of which power spectrum density is defined by Eq.(5); $\psi(t)$= nonstationary envelope function.

Hysteretic property in Eq.(1) will be replaced by equivalent linear dam-

(I) Respectively, Professor, Associate Professor and Research Associate, Department of Transportation Engineering, Kyoto University, Kyoto, JAPAN

ing coefficient β_{eq} and natural frequency ω_{eq} as

$$\ddot{\mu}(t)+\beta_{eq}\dot{\mu}(t)+\omega_{eq}^2\mu(t)= -r_s\psi(t)f(\omega_f,h_f,t) \qquad \text{.................(2)}$$

In this study, typical two linearization techniques are employed. One is the least mean-square error method[1] which minimizes the mean-square error due to linearization, and the other is the energy balance method[2] which equates hysteretic energy to the dissipated energy due to equivalent linear damping. When slowly varying amplitude and phase angle are assumed for the response $\mu(t)$, the two methods are found to result in the same expression of $\beta_{eq}(\mu_\circ)$ and $\omega_{eq}(\mu_\circ)$ as function of response amplitude $\mu_\circ$.

2-2 Probability Distribution of Maximum Hysteretic Response

Probability distribution of maximum hysteretic response is discussed through probabilistic analysis of equivalent linear structures.
Let μ_{max} denote the maximum absolute value of the response $\mu(t)$ during earthquake motion, its probability distribution is represented by

$$\Phi(\mu_{max})=P[\max|\mu(t)|\leq\mu_{max};\ 0\leq t<\infty\ ]= a_\circ(\mu_{max})\exp\{-\int_0^\infty c_\circ(\mu_{max},t)dt\} \text{....(3)}$$

where

$$a_\circ(\mu_{max})= P[|\mu(0)|\leq \mu_{max}]$$
$$c_\circ(\mu_{max},t)dt= P[|\mu(t+dt)|>\mu_{max}\ |\ \max|\mu(t')|\leq\mu_{max};\ 0\leq t'\leq t\ ] \qquad \text{.....(4)}$$

in which P[A] is the probability of event A and P[A|B] is the conditional probability of event A on the hypothesis of event B. The significance of $c_\circ(\mu_{max},t)$ is the rate of the upward crossing of $|\mu(t)|=\mu_{max}$ under the condition that no such crossing took place in the past response. Because of difficulties in obtaining the exact solution for $c_\circ(\mu_{max},t)$, several approximate methods have been proposed.[5),6)]

2-3 Monte Carlo Simulation

Monte Carlo simulation is carried out on a digital computer to check the accuracy of predicted rms and maximum response of hysteretic structures.

Stationary non-white excitation is generated through following procedure. First, band limited white noise is generated by summation of a few hundreds of sinusoidal time function whose frequency and phaze angle are random variables with uniform probability densities. Then relative velocity response of a linear simple structures with parameters of ω_f and h_f is obtained; its stationary part is used as the excitation $f(t)$. Hence the power spectrum density of $f(t)$ which has the variance of unity is the form, we have

$$S_f(\omega)= 4h_f/(\pi\omega_f)\cdot(\omega/\omega_f)^2\cdot[\{1-(\omega/\omega_f)^2\}^2+4h_f^2(\omega/\omega_f)^2]^{-1} \qquad \text{......(5)}$$

Response of hysteretic structures is calculated by the linear acceleration method. The stationary rms response of ductility factor is estimated as the time average of stationary portion with duration 20 times the natural period of structures.

Earthquake-type nonstationary random excitation is generated as the product of deterministic nonstationary envelope function $\psi(t)$ shown in Fig. 6 (a) and the stationary random process $f(t)$. The nonstationary rms response is estimated as the ensemble average of 50 samples.

3. STATIONARY RESPONSE TO NON-WHITE RANDOM EXCITATION

3-1 Stationary RMS in Ductility Factor

Stationary rms response σ_μ of hysteretic structures is predicted by an iterative method[7] which approximately determines the equivalent linear parameters $\omega_{eq}(\sigma_\mu)$, $\beta_{eq}(\sigma_\mu)$ and corresponsding rms response $\sigma_\mu(\omega_{eq},\beta_{eq})$.

In this study, bilinear hysteresis loops shown in Fig.1 are adopted. A parameter n expresses the nonlinearity of the second stiffness. Predicted values of σ_μ, ω_{eq} and β_{eq} for medium-period bilinear structures with $\omega_\circ$ equal to ω_f ($\omega=\omega_f/\omega_\circ=1.0$) are shown in Fig.2. The abscissa is the linear rms response σ_l in terms of ductility factor. Hence it is proportional to the excitation level and inversely proportional to the yield level.

Compound effects of ω_{eq} and β_{eq} on the hysteretic rms response σ_μ will be clarified from the concept of transition of structural receptance shown in Fig.4. It is easily understood that the softening spring of the yielding and its hysteretic damping have different effects on its response depending on $\omega_\circ$ relative to ω_f.

It is clear from Fig.3 (a) that the response of short-period bilinear structures relative to linear response increases with the excitation level σ_μ and with increasing nonlinearity n because of the transition of the structural receptance closer to the peak of the excitation spectrum as shown in Fig.4. In long-period structures, this transition effects reduces the bilinear response relative to linear response as found on Fig.3 (b). This effect is also shown in Fig.4.

Simulated bilinear rms response is plotted in Figs.2 (a) and 3 (a),(b) in comparison with predicted values. When nonlinearity is not strong (n= 0.25), both results show satisfactory agreement. Strong nonlinearity (n= 0.75) makes the simulated bilinear response larger than predicted values presumably due to growth of plastic deformation.

3-2 Probability Distribution of the Maximum Ductility Factor

Probability distribution of maximum bilinear response in stationary state is predicted by assuming $c_\circ(\mu_{max})$ is approximately equal to the unconditional crossing rate $N_\mu(\mu_{max})$ of ductility factor response $\mu(t)$ at the level of μ_{max}. In calculating $N_\mu(\mu_{max})$, the Gaussian distribution of $\mu(t)$ and $\dot{\mu}(t)$ is assumed. The duration of stationary response is taken as 20 times the natural period of linear structures.

Fig.5 shows the predicted and simulated results for medium and short-period bilinear structures. Agreement is satisfactory for both of linear structures with relatively large damping factor ($h_\circ$=0.1). In these cases, assumtion of Poisson process arrival of ductility factor response to the level of μ_{max} can be used successully. Hysteretic effects decrease the maximum response of medium-period structures, whereas it increases that of short period-structures. Difference between the simulated and theoretical values increases with the nonlinearity parameter. These properties are almost consistent with those of rms response discussed in the previous section.

4. NONSTATIONARY RESPONSE TO EARTHQUAKE-TYPE EXCITATION

4-1 Nonstationary Response and Equivalent Linear Structures

Nonstationary response of hysteretic structures is predicted by the step-by-step method[7] which consits of two procedures. One is nonstationary random response analysis of equivalent linear structures and the other is determination of equivalent linear parameters β_{eq} and ω_{eq} as function of response level.

Fig.6 shows time depending mean-square $\sigma_\mu^2(t)$ of ductility factor response and corresponding equivalent linear parameters ω_{eq} and h_{eq} for short-period bilinear structures. The large value of $\sigma_\mu^2(t)$ and the time lag between the peaks of response and the nonstationary envelope function are found with increasing nonlinearity inspite of larger hysteretic damping h_{eq}. This is due to reduction of ω_{eq} which makes the receptance function closer to the peak frequency of the excitation.

Predicted $\sigma_\mu^2(t)$ is compared with simulated result in Fig.7. Although simulated result shows fluctuations presumably due to insufficient sample size and a little larger value than predicted response which may be attributed to growth of plastic deformation, agreement between the two is fairly good.

4-2 Probability Distribution of Maximum Ductility Factor

For prediction of the probability distribution of the maximum response in nonstationary state, $c_0(\mu_{max}, t)$ is approximated by the unconditional crossing rate $N_W(\mu_{max}, t)$ of response envelope $W(t)$ at the level of μ_{max}. Is is reported that the assumption of Poisson Process arrival of response envelope to a level of μ_{max} can be used more successfully than that of ductility response $\mu(t)$ itself[6].

Predicted and simulated probability distribution of maximum response of short-period bilinearstructures are shown in Fig.8 and their mean values and coefficients of variation are compared in Table 1. Both the predicted and simulated results show larger maximum response for stronger nonlinearity. It is seen in Table 1 that the nonlinearity of short period bilinear structures makes not only mean values but also coefficients of variation larger. This result suggests that the effect of hysteretic damping should not be expected in earthquake resistant design of relatively short-period structures.

REFERENCES

1) Caughey, T.K.;"Random Vibration of a System with Bilinear Hysteresis," Jour. of Appl. Mech., ASME, Vol.27, 1960, pp.649-652.
2) Hudson, D.E.; "Equivalent Viscous Friction for Hysteretic Systems with Earthquake-like Excitations," Proc. of the Third WCEE, Vol.II, 1965, pp.II 185-206.
3) Rice, S.O.; "Selectd Paper on Noise and Stochastic Processes," edited by N. Wax, Dover Pub., 1945, pp.184-246.
4) Kobori, T. and Minai, R.; "Linearization Techniques for Evaluating the Elasto-Plastic Response of a Structural System to Nonstationary Random Excitation," Ann. Rep. of EPRI, Kyoto Univ., No.10A, 1969, pp.235-260.
5) Lin, Y.K.; "Probabilistic Theory of Structural Dynamics," McGraw-Hill, 1972.
6) Kameda, H.; "On Estimation of the Maixmum Structural Response to Random Earth-quake Motion from Response Envelope," Mem. Fac. Eng., Kyoto Univ., Vol.XXXVI, Part 4, 1975, pp.458-472.
7) Goto, H. and Iemura, H.; "Linearization Techniques for Earthquake Response of Simple Hysteretic Structures," Proc. of JSCE, No.212, 1973, pp.109-119.

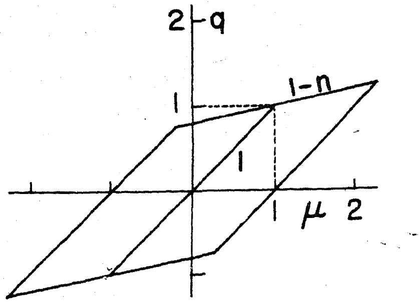
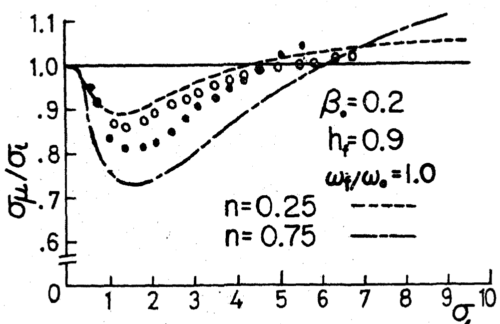
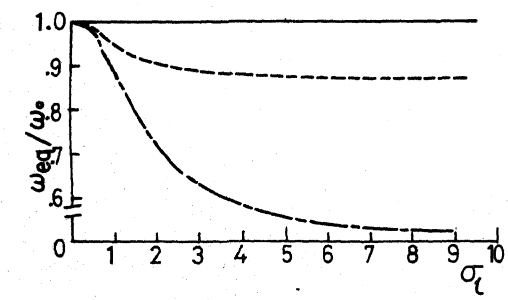


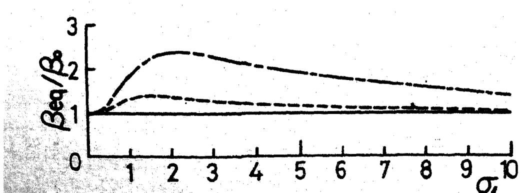
Fig.1 Bilinear Hysteretic Restoring Force



(a) Normalized RMS Response

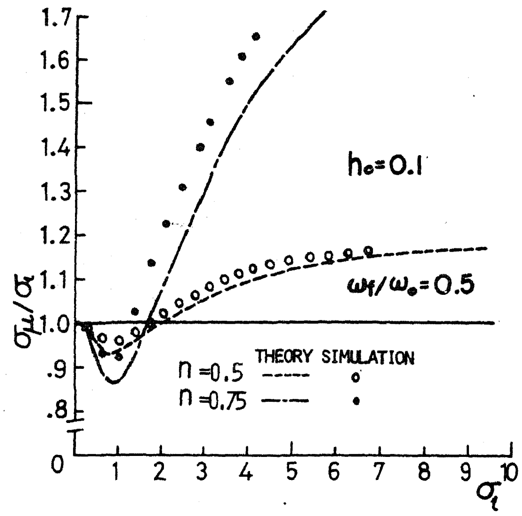


(b) Equivalent Natural Frequency

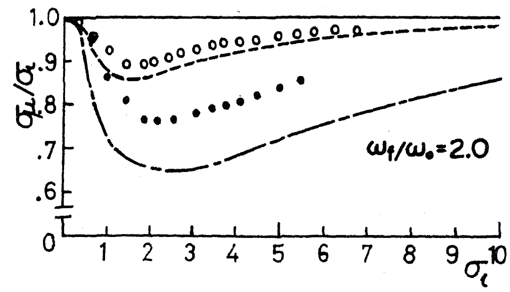


(c) Equivalent Damping Factor

Fig.2 Stationary RMS Response of Medium-Period Bilinear Structures and Equivalent Linear Parameters



(a) Short-Period Bilinear Structures



(b) Long-Period Bilinear Structures

Fig.3 Predicted and Simulated RMS Response of Bilinear Structures

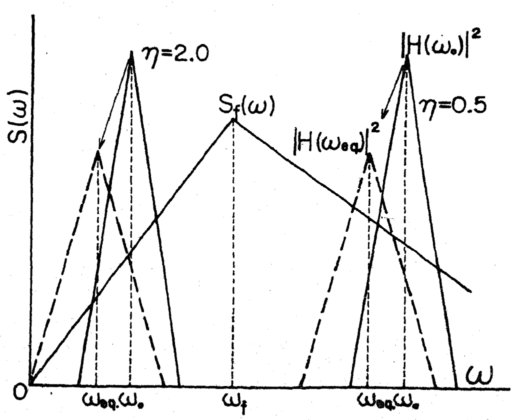
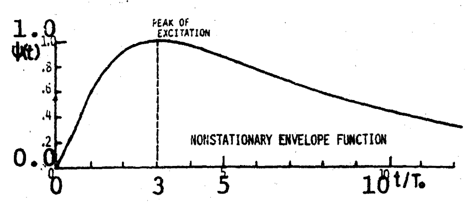
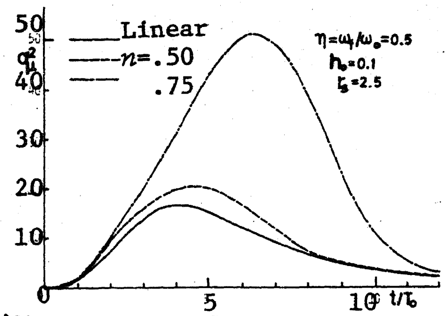


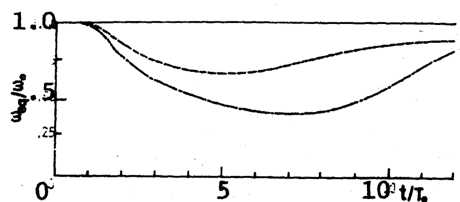
Fig.4 Transition of Receptance Function due to Nonlinearity and Hysteresis



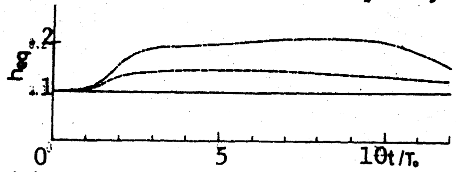
(a) Nonstationary Envelope Function



(b) Mean-Square of Ductility Factor



(c) Equivalent Natural Frequency



(d) Equivalent Damping Factor

Fig.6 Nonstationary Mean-Square Response of Ductility Factor and Transition of h_{eq} and ω_{eq}

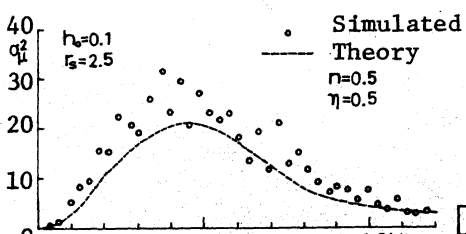
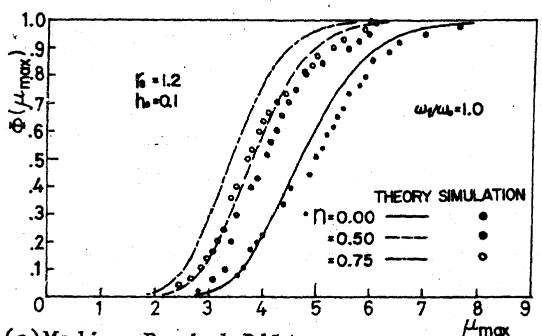
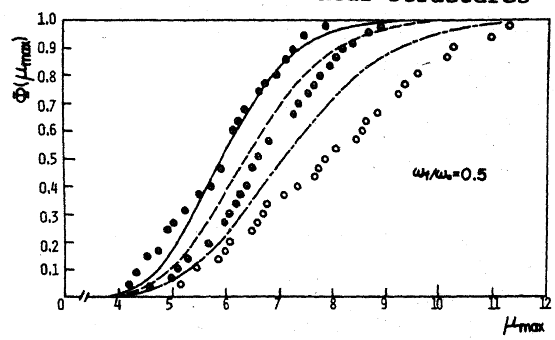


Fig.7 Predicted and Simulated Nonstationary response of Ductility Factor



(a) Medium-Period Bilinear Structures



(b) Short-Period Bilinear Strucutres

Fig.5 Probability Distribution of Maximum Ductility Factor in Stationary Response

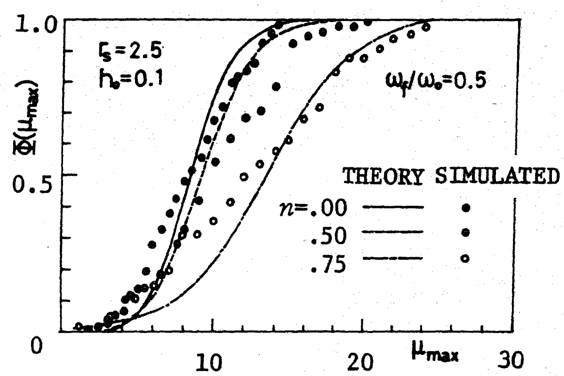


Fig.8 Probability Distribution of Maximum Ductility Factor in Nonstationary Response

Table 1

	Predicted		Simulated	
	Mean Max. Response	Coef. of Variation	Mean Max. Response	Coef. of Variation
Linear	8.31	0.28	7.79	0.39
n=0.50	9.00	0.30	9.60	0.43
n=0.75	12.91	0.33	12.09	0.52