EARTHQUAKE RESPONSE CHARACTERISTICS OF DETERIORATING HYSTERETIC STRUCTURES

by

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SYNOPSIS

A Simple method to represent deteriorating hysteretic structures is developed. Equivalent stiffness and energy absorbing capacity of structures are assumed to degrade with decreasing residual strength derived from the theory of low-cycle fatigue. Calculated mean-square response of the model subjected to earthquake-type random excitation exhibits typical dynamic failure process of structures. The proposed method can cover wide ranges of deteriorating hysteretic structures suggesting that it is useful in practical earthquake response analysis.

1. INTRODUCTION

In most of earthquake response analyses of hysteretic structures, it has been assumed that dynamic properties of structures depend only on response amplitude. Recently, however, deterioration effects of reinforced concrete structures during strong earthquakes have been emphasized on the basis of recorded seismograms and loading tests of structural elements.[1)]

Fig.1 shows the first mode hysteretic response of the Millikan Library on the campus of California Institute of Technology during the San Fernando earthquake in 1971. It was obtained directly from strong motion seismograms.[2)] The amplitude and time dependent stiffness and energy absorbing capacity of the building were calculated from the slope and area of each hysteresis loop and they are plotted in Fig.2 in terms of equivalent natural frequency ω_{eq} and damping factor h_{eq}. It is clear from Fig.2 that dynamic properties of the building deteriorated during the earthquake. Deterioration of reinforced concrete structures has also been suggested from restoring force characteristics of structural elements subjected to cyclic loading.[3)] Examination of these data required to introduce general deterioraing model to explain structural response and earthquake damages.

In this study, a new simple method to represent deteriorating biliear hysteretic structures is proposed. As a basic measure of structural deterioration, cumulative damage and residual strength are defined as functions of number of cycles and amplitude of loading. Then equivalent linear parameters of the hysteretic structures are controlled to degrade with decreasing residual strength of structures. Effects of structural deterioration to earth-response are examined by comparing nonstationary mean-square response of linear, conventional bilinear and proposed deteriorating bilinear structures.

2. EQUIVALENT LINEARIZATION OF DETERIORATING HYSTERETIC STRUCTURES

2-1 Cumulative Damage and Residual Strength of Structures

As a basic measure of structural deterioration of stiffness and energy absorbing capacity with cyclic loading, cumulative damage fuction defined in the theory of low-cycle fatigue[4)] is adopted. Let us define the increment in cumulative damage ΔD_i due to one cycle loading with amplitude of μ_i in ductility factor (D.F.) as:

$$\Delta D_i = 4(\mu_i/\mu_f)^a \qquad \text{.......................................(1)}$$

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where μ_f= D.F. at failure under static loading; a= a parameter which determines the pattern of damage function (2.0 is used in this study). Then accumulated damage $D(n)$ after n_i cycles of loading with amplitude of μ_i (i=1,2,··) is written as

$$D(n)= \sum_i n_i \Delta D_i = 4/\mu_f^a \sum_i n_i \mu_i^a \quad \ldots\ldots (2)$$

Using R.W. Lardner's damage-rate function;[5] R. Minai[6] proposed the cumulative damage $D(t)$ over the time interval of $(0,t)$ in the form of

$$D(t)= \int_0^t F(\mu)|\dot{\mu}|dt = a/\mu_f \int_0^t |\mu|^{a-1}|\dot{\mu}|dt \quad \ldots\ldots (3)$$

The expected value of cumulative damage $E[D(t)]$ in nondeterministic random response will be estimated when joint probability density function $p(\mu,\dot{\mu},t)$ of D.F.μ and its rate $\dot{\mu}$ at time t' $(0\leq t'\leq t)$ is known.

When the value of cumulative damage $D(n)$ or $E[D(t)]$ reaches to 1.0, it is regarded as complete failure of the structure. Therefore residual strength $R(n)$ or $E[R(t)]$ of structures is written as

$$R(n)= 1- D(n), \quad E[R(t)]= 1- E[D(t)] \quad \ldots\ldots (4)$$

2-2 Equivalent Linear Parameters of Conventional Bilinear Structures

Equation of motion of single-degree-of-freedom structures with conventional bilinear restoring force $q(n_p,\mu,\dot{\mu},t)$ is written as

$$\ddot{\mu}+ \beta_\circ\dot{\mu} + \omega_\circ^2 q(n_p,\mu,\dot{\mu},t)= - a_f\psi(t)f(\omega_f,h_f,t) \quad \ldots\ldots (5)$$

where $\beta_\circ$ and $\omega_\circ$= damping coefficient and natural frequency, respectively, in infinitesimal vibration; a_f= intensity parameter; $\psi(t)$ = nonstationary envelope function; $f(\omega_f,h_f,t)$= stationary random process with unit variance; ω_f= predominant frequency of excitation; h_f= shape parameter of excitation spectrum. Hysteretic effects in Eq.(5) can be replaced by equivalent damping coefficient β_{eq} and natural frequency ω_{eq} as[7]

$$\ddot{\mu} + \beta_{eq}\dot{\mu} + \omega_{eq}^2\mu= - a_f\psi(t)f(\omega_f,h_f,t) \quad \ldots\ldots (6)$$

in which β_{eq} and ω_{eq} are determined from either the least mean-square error method or the energy balance method as functions of amplitude μ_i in D.F..

$$\begin{aligned}\omega_{eq}^2(\mu_i)&=\omega_\circ^2\{2n_p(2-\mu_i)\sqrt{\mu_i-1}/(\pi\mu_i^2)+ n_p\cos^{-1}(1-2/\mu_i)/\pi+(1-n_p)\}\\ \beta_{eq}(\mu_i)&= \beta_\circ+ 4n_p(\mu_i-1)/(\pi\omega_{eq}\mu_i^2)\end{aligned} \quad \ldots\ldots (7)$$

Equivalent linear parameters for nondeterministic random response are approximated by the expected values of β_{eq} and ω_{eq}^2 corresponding to each peak of random amplitude. They are calculated from probability density function $p(\mu_i,\sigma_\mu,\rho_{\mu\dot{\mu}},\sigma_{\dot{\mu}})$ of peak μ_i as

$$\begin{aligned}\omega_{eq}^2(\sigma_\mu,\rho_{\mu\dot{\mu}},\sigma_{\dot{\mu}})&= \int_0^\infty \omega_{eq}^2(\mu_i)p(\mu_i,\sigma_\mu,\rho_{\mu\dot{\mu}},\sigma_{\dot{\mu}})d\mu_i\\ \beta_{eq}(\sigma_\mu,\rho_{\mu\dot{\mu}},\sigma_{\dot{\mu}})&= \int_0^\infty \beta_{eq}(\mu_i)p(\mu_i,\sigma_\mu,\rho_{\mu\dot{\mu}},\sigma_{\dot{\mu}})d\mu_i\end{aligned} \quad \ldots\ldots (8)$$

where σ_μ and $\sigma_{\dot{\mu}}$ =root mean square (rms) of μ and $\dot{\mu}$, respectively; $\rho_{\mu\dot{\mu}}$= crrelation coefficient between μ and $\dot{\mu}$.

2-3 Equivalent Linear Parameters of Deteriorating Bilinear Structures

Although there would be many ways to describe deterioration effects of structures, it will be a simple and practical approach to measure the degraded capacity of structural strength in terms of the residual strength discussed above. In this study, it is assumed that the stiffness (ω_{eq}^2) degrades proportional to the residual strength and that the energy absorbing capacity (β_{eq}) degrades more rapidly in proportion to the square of the residual strength as shown in Fig.4. This is written as

$$\omega_{eq}^2(\mu_i, R(n)) \doteq \omega_{eq}^2(\mu_i) R(n) \ , \quad \beta_{eq}(\mu_i, R(n)) = \beta_{eq}(\mu_i) R^2(n) \quad \ldots\ldots(9)$$

This assumtion is made according to experimental results of reinforced concrete shear walls performed by T.Shiga et al.[3] In this study, deterioration of the equivalent rigidity and the equivalent viscous damping under cyclic loading with constant amplitude is plotted against the number of load cycles, which suggest that the present approach is appropriate in investigating the effects of structural deterioration.

Using Eqs.(7) and (9), residual strength and deteriorated structural parameters of a model for sinusoidal cyclic loading are calculated and shown in Fig.5. Deterioration effects are shown for 1,5,10 cycles of loading. It is found that five cycles of loading with D.F. μ_i =5.5 lead to complete loss of stiffness and energy absorbing capacity; i.e., ω_{eq}^2 and β_{eq} vanish. In this figure, no deterioration means a conventional bilinear model.

When loading is random but deterministic like structural response subjected to recorded earthquake motions, structural damage is calculated at every half cycle of vibration from Eq.(2). According as increasing damage, deteriorated structural parameters are estimated from Eq.(9) and they are adjusted also at every half cycle of vibration. Fig.6 shows the calculated response of a proposed deteriorating model subjected to the San Fernando earthquake record used in Fig.1. The total force ($\beta_{eq}\dot{x} + \omega_{eq}^2 x$) is plotted against the relative displacement response x to reproduce deteriorating hysteretic loops. Although some discrepancy is found between Figs.1 and 6, general trends of deterioration of slopes and area of hysteresis loops agree well to suggest usefullness of the proposed model.

3. NONSTATIONARY RESPONSE TO EARTHQUAKE-TYPE EXCITATION

3-1 Step-by-Step Estimation of Response and Damage

Nonstationary mean-square response of the proposed deteriorating hysteretic model subjected to earthquake-type random excitation is predicted by the step-by-step linearization technique which consists of two procedures.[7] The first step is determination of equivalent linear parameters of deteriorating hysteretic model in nondeterministic random response. They are estimated from the covariances of response σ_μ , $\rho_{\mu\dot{\mu}}$, $\sigma_{\dot{\mu}}$ and the expected residual strength E[$R(t)$] following almost the same idea discussed in Eq(9). This procedure is expressed as

$$\omega_{eq}^2(\sigma_\mu, \rho_{\mu\dot{\mu}}, \sigma_{\dot{\mu}}, E[R(t)]) = \omega_{eq}^2(\sigma_\mu, \rho_{\mu\dot{\mu}}, \sigma_{\dot{\mu}}) E[R(t)] \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$
$$\beta_{eq}(\sigma_\mu, \rho_{\mu\dot{\mu}}, \sigma_{\dot{\mu}}, E[R(t)]) = \beta_{eq}(\sigma_\mu, \rho_{\mu\dot{\mu}}, \sigma_{\dot{\mu}}) E^2[R(t)]$$

The second step is estimation of covariances σ_μ , $\rho_{\mu\dot{\mu}}$, $\sigma_{\dot{\mu}}$ of nonstationary response of equivalent linear structures subjected to stationary excitation under specified initial conditions. These two procedures are applied to evry time-segment which is made small enough so as to let equivalent linear parameters and level of excitation constant within it.

3-2 Nonstationary Mean-Square Response

Deterioration effects of structural stiffness and energy absorbing capacity during earthquake response are examined by comparing nonstationary mean-square response of linear, conventional bilinear and proposed deteriorating bilinear structures. Calculated results for three models subjected to moderate and strong excitations are shown in Figs. 8 and 9. The nonstationary excitation is represented by the product of nonstationary envelope function $\psi(t)$ shown in Fig.7 and stationary random process $f(\omega_f, h_f, t)$.

In Fig.8 (a), a linear structure shows large value of mean-square response in D.F. with large time-lag between the peaks of response and excitation. This is due to small value of damping factor $h_o(=\beta_o/(2\omega_o)=0.02)$. The maximum mean-square response of the conventional bilinear structure is found less than 50% of linear structures and there is almost no time-lag between the peaks of response and excitation because of the energy absorbing effects of hysteresis loops. The deteriorating bilinear structure shows, except at the beggining of response, larger response than that of conventional bilinear structures due to structural damage. The cumulative damage shows rapid growth when the response attains its maximum value ($t/T_o \simeq 4.0$) and then gradually increases up to 50% of the complete failure value ($E[D(t)]=1.0$). Equivalent Linear parameters of conventional bilinear structures depending only on response amplitude recover their initial values at the end of vibration. On the contrary, those of proposed deteriorating bilinear model calculated from Eq. (9) loses their capacity according as the incresing damage and does not recover their initial values. Deterioration of ω_{eq} in this figure is found very similar to that in Fig.2.

In Fig.9, square of an intensity parameter a_f of the excitation is increased from 0.75 used in Fig.8 to 1.0 to represent strong earthquake motions. Other parameters are same as those used in Fig.8. It is a natural result that linear response in Fig.9 is 133% of that in Fig.8 from the theory of linearity between excitation and response. Bilinear response in Fig.9 shows the rate of increase as almost same as the linear structure because of a little change between equivalent linear parameters in Fig.8 and 9. Incontrast, response of the deteriorating bilinear model decreases only slightly after its peak value at $t/T_o=5$ inspite of rapid decrease in excitation level. After $t/T_o=13$, the response becomes larger than that of a linear structure and finally shows very rapid growth at $t/T_o=24$. This is the effects of deterioration of structural stiffness and energy absorbing capacity with increasing cumulative damage. Extreme loss of structural capacity results in rapid growth of response to cause the collapse;i.e.,$E[D(t)]=1.0$.

For the purpose of measuring deterioration of structural capacities and investigating their effects on earthquake response, the proposed method is much simpler than conventional methods of controlling hysteresis loops with deterioration effects. The proposed method can also cover wide range of deteriorating structures by choosing a suitable value for parameter a in Eq.(1) and by defining appropriate relation between cumulative damage and equivalent linear parameters. Hence the method in this study seems promissing for practical use in earthquake response analysis.

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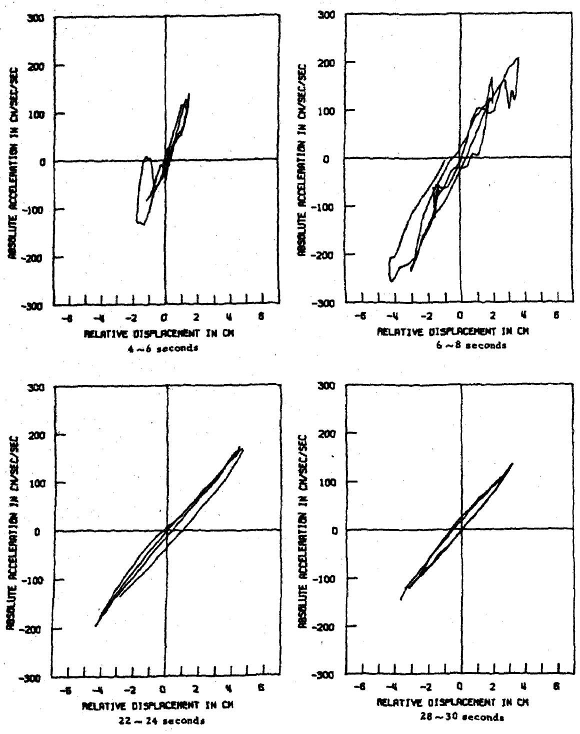

Fig.1 First Mode Hysteretic Response of the Millikan Library Determined from the Recorded Motions (by Iemura & Jennings)

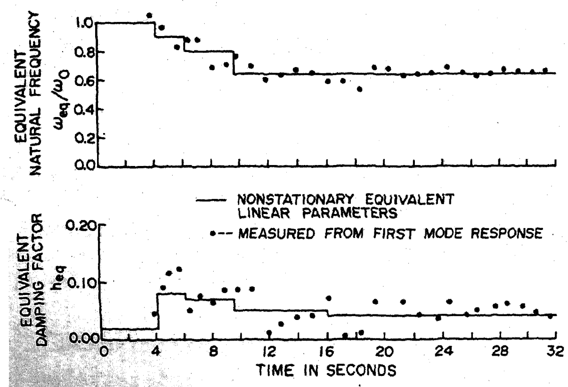

Fig.2 Equivalent Linear Parameters of the Library during the Earthquake

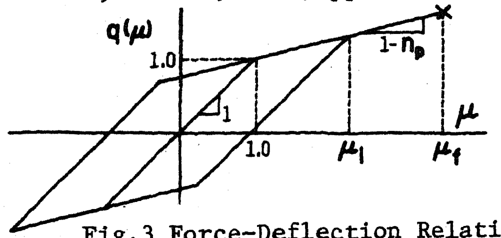

Fig.3 Force-Deflection Relation of the Deteriorating Model

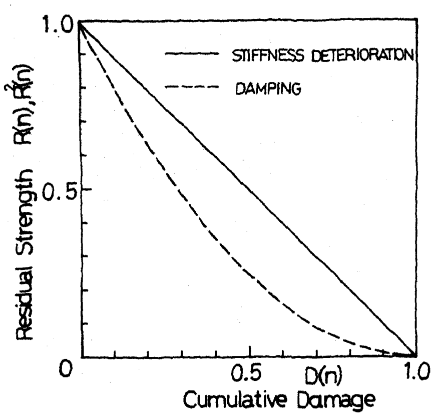

Fig.4 Deterioration of Structural Stiffness and Damping

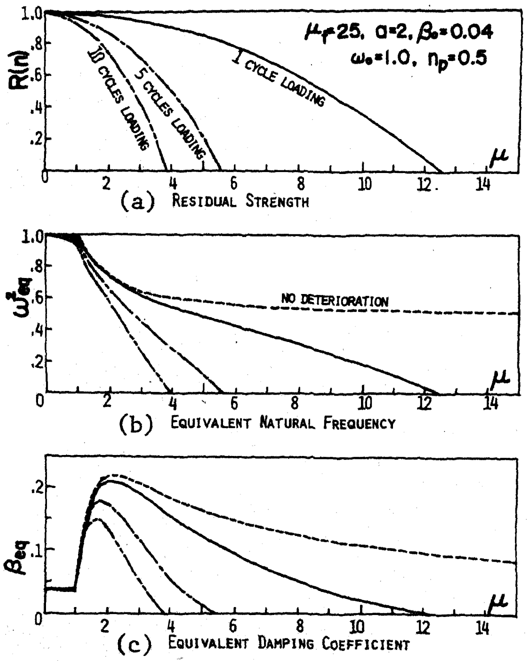

Fig.5 Resiual Strength and Equivalent Linear Parameters

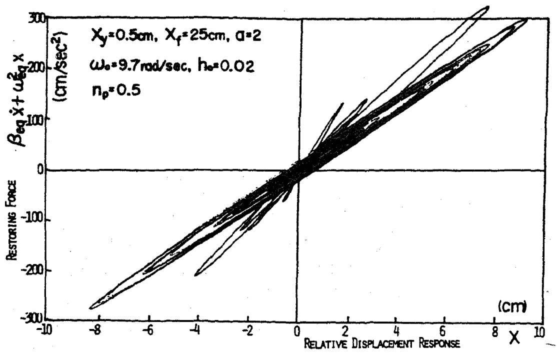

Fig.6 Hysteretic Response of a Deteriorating Bilinear Model Subjected to the Earthquake Record

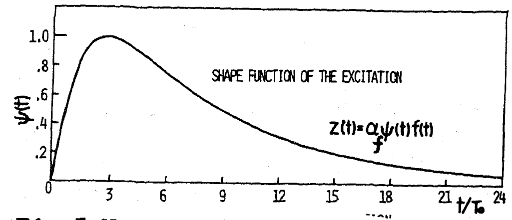

Fig.7 Nonstationary Envelope Function

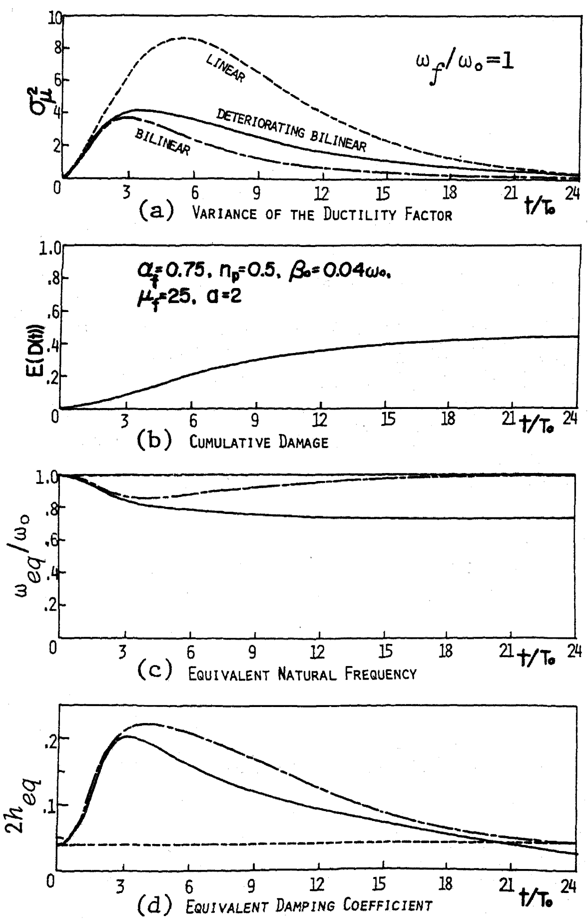

Fig.8 Nonstationary Response of Linear, Bilinear and Deteriorating Bilinear Structures Subjected to Moderate Excitation

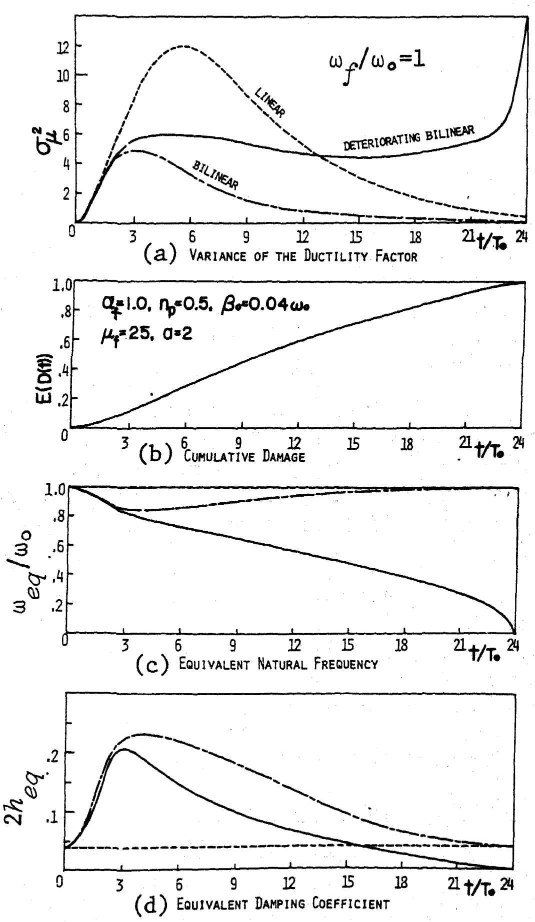

Fig.9 Nonstationary Response of Linear, Bilinear and Deteriorating Bilinear Structures Subjected to Strong Excitation