

STOCHASTIC SEISMIC RESPONSE AND RELIABILITY OF HYSTERETIC STRUCTURES

by

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SYNOPSIS

The nonstationary, nonlinear response of hysteretic structural systems to stochastic earthquake-like excitations is studied in relation to the assessment of the structural reliability. An analytical procedure to be available for the stochastic response analysis of hysteretic systems with strong nonlinearity is developed without recourse to the equivalent linearization technique. The ultimate anti-seismic safety of structures subjected to destructive earthquakes is discussed mainly from a random low-cycle fatigue viewpoint. Numerical example is given for the typical bilinear hysteretic system, and the present results are compared with those of the Monte Carlo simulation.

INTRODUCTION

In relation to the ultimate anti-seismic design procedure which guarantees the safety of structures to intense earthquakes in their ultimate state, the probabilistic determination of the seismic response and reliability of severely nonlinear hysteretic system are essentially important. In the present study, an approximate method for the stochastic response analysis of hysteretic systems with strong nonlinearity is presented without recourse to the equivalent linearization technique. The piecewise hysteretic characteristics considered here can be expressed in the forms of appropriate first-order quasi-linear differential equations which make the Fokker-Planck formulation possible. The time-dependent statistics of responses, including cumulative plastic deformation and low-cycle fatigue damage, are obtained as solutions to a set of nonlinear first-order differential equations.

Under the anti-seismic design philosophy which permits inelastic response during an intense earthquake, there is the possibility of some repeated plastic deformations. Hence, it is important to evaluate the safety of structures from the point of view considering the so-called low-cycle fatigue phenomenon (1,2). The problem of random low-cycle fatigue is complex and a universally available theory for fatigue failure does not seem to exist to date. In this study, the statistics of fatigue damage and the reliability function (the probability of survival) for such failure measure are examined according to a linear cumulative damage theory. The analytical method is illustrated by an example for bilinear hysteretic systems to nonstationary Gaussian white noise, and the analytical results are compared and checked by the results of the Monte Carlo simulation.

HYSTERETIC SYSTEM

The restoring force in a structural system may be modeled by hysteretic characteristics which depend on the past time history of response. The hysteretic system considered here is mainly the well-known bilinear hyster-

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etic type as shown in Fig. 1. The bilinear hysteretic characteristic $\Phi(x, \dot{x})$, nondimensionalized to have unit rigidity for the first branch and a rigidity Γ for the second branch, is expressed as

$$\Phi(x, \dot{x}) = x - (1 - \Gamma) \varphi(x, \dot{x}), \quad \dot{\cdot} = \frac{d}{dt} \quad (1)$$

where x is the nondimensional displacement with reference to the elastic limit deformation, and $\varphi(x, \dot{x})$ represents the backlash characteristic as shown in Fig. 2. Introducing the new state variable z defined on the range $[-1, 1]$, the backlash characteristic φ and its time derivative $\dot{\varphi}$ are given by the following forms (3,4):

$$\varphi = x - z \quad \text{where } |z| \leq 1, \quad \dot{\varphi} = g(\dot{x}, z) = \begin{cases} \dot{x} U(\dot{x}) & ; z = 1 \\ 0 & ; |z| < 1 \\ \dot{x} U(-\dot{x}) & ; z = -1 \end{cases} \quad (2)$$

where $U(\cdot)$ is the Heaviside unit step function.

Furthermore, the above expressions can be easily extended to the polylinear hysteretic characteristics as follows:

$$\Phi_j(x, \dot{x}) = x - \sum_{j=1}^J (r_j - r_{j+1}) \varphi_j(x, \dot{x}), \quad \varphi_j = x - z_j; |z_j| \leq \delta_j; \delta_1 = r_1 = 1 \quad (3)$$

where φ_j is the backlash characteristic with a clearance $2\delta_j$ of dead zone and unit slope, and r_j and δ_j are the rigidity ratio of the j -th branch and the deformation at the end of j -the branch in the virgin curve, respectively.

STOCHASTIC RESPONSE OF HYSTERETIC SYSTEM

The dimensionless equation of motion of a single-degree-of-freedom bilinear hysteretic system is expressed as

$$\ddot{x} + 2h\dot{x} + \Phi(x, \dot{x}) = -f(t), \quad f(t) = \alpha(t)\xi(t), \quad \max|\alpha(t)| = 1 \quad (4)$$

where h is the critical damping ratio, $\alpha(t)$ represents a deterministic envelope function, and $\xi(t)$ is the white noise with zero mean and the spectral density S_0 . The joint probability density function $p(x, y, z; t)$ of x , $y (= \dot{x})$ and z is governed by the Fokker-Planck equation

$$\frac{\partial p}{\partial t} = -y \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} [2hy + rx + (1-r)z] p + \frac{\partial}{\partial z} [g(y, z) - y] p + \frac{S(t)}{2} \frac{\partial^2 p}{\partial y^2}, \quad S(t) = 2\pi S_0 \alpha^2(t) \quad (5)$$

No exact solution to Eq. 5 is obtained up to date. However, an infinite set of the first-order differential equations for moments $M(\ell, m, n; t)$ of the responses can be derived from Eq. 5, namely

$$\dot{M}(\ell, m, n) = \ell M(\ell-1, m+1, n) - m \{ zh M(\ell, m, n) + r M(\ell+1, m-1, n) + (1-r) M(\ell, m-1, n+1) \} + \frac{S(t)}{2} m(m-1) M(\ell, m-2, n) + n \{ M(\ell, m+1, n-1) - N(\ell, m, n) \}, \quad \ell, m, n = 0, 1, 2, \dots \quad (6)$$

where

$$M(\ell, m, n; t) = \iiint_{-\infty}^{\infty} x^\ell y^m z^n p(x, y, z; t) dx dy dz \quad (7)$$

$$N(\ell, m, n; t) = \iiint_{-\infty}^{\infty} x^\ell y^m z^{n-1} g(y, z) p(x, y, z; t) dx dy dz$$

Here, the unknown probability density function $p(x, y, z; t)$ is expressed in series of multi-dimensional Hermite polynomials. In usual cases, attention is focused on the covariance between responses which are of primary interest. Therefore, an approximate analysis may be effectively performed by introducing a simplified probability density function derived from the normal probability density function. By taking account of the non-Gaussian distribution of z which is evident from its definition, the probability density function may be assumed to be approximately given by the truncated form

$$p(x, y, z; t) = [U(z+1) - U(z-1)] w(x, y, z; t) + \delta(z-1) \int_{-\infty}^{\infty} w(x, y, z'; t) dz' + \delta(z+1) \int_{-\infty}^{\infty} w(x, y, z'; t) dz' \quad (8)$$

where $w(x, y, z; t)$ is the normal density function with the covariance matrix h .

By making use of Eqs. 6 to 8, the covariance matrix \mathbb{K} of responses can be numerically obtained (4). Note that in general $\mathbb{K} \neq \mathbb{I}_k$.

The above mentioned method can be applied to the evaluation of cumulative plastic deformation which has attracted interest in relation to the anti-seismic safety of hysteretic structures (5). Although the plastic deformation process is naturally discrete process in time, the cumulative plastic deformation can be expressed as the continuous integral with respect to time

$$|\eta_p(t)| = (1-r) \int_0^t |\dot{\phi}(t')| dt', \quad \eta_p^+(t) = (1-r) \int_0^t \dot{\phi}(t') U(\dot{\phi}) dt, \quad \eta_p^-(t) = (1-r) \int_0^t \dot{\phi}(t') U(-\dot{\phi}) dt \quad (9)$$

where $|\eta_p(t)|$ is the absolute accumulation of plastic deformation, and $\eta_p^{\pm}(t)$ represents the accumulation of plastic deformation with positive (negative) velocity. The mean value of cumulative plastic deformation can be obtained by using Eqs. 2, 8 and 9.

LOW-CYCLE FATIGUE DAMAGE AND RELIABILITY

Before going into the main argument, consider the following two failure criteria: 1) The failure occurring when the instantaneous displacement response exceeds its limiting value, 2) The failure occurring when cumulative hysteretic energy exceeds its capacity. The reliability functions, for the two failure criteria, obtained through the Monte Carlo simulation to a bilinear hysteretic system subjected to the stationary white noise excitation are plotted in Fig. 8. Here, η_F is the ductility ratio of static fracture deformation. By comparing the two reliability functions for the same value of η_F , quite large difference is observed. It is suggested from results of experimental studies of structural members and frames that the failure of structures with relatively stable hysteresis may occur according to the low-cycle fatigue criterion rather than the above two extreme criteria, because the instantaneous deformation criterion is too conservative, while the cumulative energy criterion is too unconservative (1,2).

To apply the low-cycle fatigue damage criterion to the continuous random response process (6), the cumulative damage function $\psi(t)$ is defined by the following integral of the continuous damage rate function $\dot{\psi}(t)$ (7):

$$\psi(t) = \int_0^t \dot{\psi}(t') dt', \quad \dot{\psi}(t) = \frac{a}{\eta_F^a} |x|^{a-1} |\dot{x}| \quad (10)$$

where a is the parameter representing the type of low-cycle fatigue. By considering the cumulative damage as one of the state variables, the Fokker-Planck equation for $p(x, y, z, \psi; t)$ is given by

$$\frac{\partial p}{\partial t} = -y \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} [2hy + rx + (1-r)z] p + \frac{\partial}{\partial z} [g(y, z) - y] p + \frac{a}{\eta_F^a} |x|^{a-1} |y| \frac{\partial p}{\partial \psi} + \frac{s(t)}{2} \frac{\partial^2 p}{\partial y^2} \quad (11)$$

Similarly as in the preceding section, from moment equations and a simplified probability density function by taking account of the range where $\psi \geq 0$, the statistics of cumulative damage can be obtained. Under the assumption that the limiting value of cumulative damage is unity, the reliability function is given by

$$R(t) = \text{Prob} [\psi(t) < 1] = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \int_0^1 d\psi p(x, y, z, \psi; t) \quad (12)$$

NUMERICAL RESULTS

To verify the results obtained by the present approximate method, a numerical simulation based on the Monte Carlo technique has been carried out. Analytical and simulated results are presented for both stationary and non-stationary Gaussian white noise excitations.

Under stationary excitation, i.e., $\alpha(t) = U(t)$, the variance K_{xx} of displacement, the variance K_{yy} of velocity and the average cumulative plastic deformation $E[|\eta_p|]$ are shown in Figs. 3(a), (b) and 6(a). For a nonstationary excitation with the envelope function shown in Fig. 4, the results of K_{xx} and $E[|\eta_p|]$ are shown in Figs. 5 and 6(b). The abscissas of these figures show the nondimensional time with reference to the natural period $t_0 (=2\pi)$ of the associated linear system. In these figures, the simulation results indicated by dots are also plotted. The agreement between the present analytical results and the simulation results is satisfactorily good except for the case of nonpositive rigidity ratio. It is shown that the response characteristics of bilinear hysteretic systems are quite sensitive to the rigidity ratio, and there exists the rigidity ratio minimized the displacement response. For the case where $r < 0$, both the displacement response and the cumulative plastic deformation increase rapidly within the short time-duration.

The mean damage $E\Psi(t)$ and the reliability function $R(t)$ obtained by the analytical and simulation methods based on Eq. 10 are shown in Fig. 8. Choosing η_F and a as parameters, $E\Psi(t)$ and $R(t)$ under stationary and nonstationary excitations are shown in Figs. 9 and 10, respectively. From these figures, it is shown that the reliability function tends to decrease abruptly after $E\Psi + 3\sigma_\Psi$ reaches the critical value of cumulative damage, where σ_Ψ is the standard deviation of Ψ . It is also found that the reliability function is significantly influenced by the values of η_F and a , which depend on the structural materials, member compositions and geometry, as well as by the properties of earthquakes, such as the nonstationarity and the duration-time.

CONCLUDING REMARKS

An analytical procedure to determine the stochastic response and the reliability according to low-cycle fatigue failure criterion of hysteretic structures to earthquake-like excitations has been developed. The method is basically the Fokker-Planck approach which is based on the differential representations of polylinear hysteretic characteristics and cumulative fatigue damage. It is emphasized that the method is applicable to the nonstationary stochastic response analysis of hysteretic systems even if the response process is broad band. The cumulative plastic deformation can be also evaluated by using the differential representation of hysteretic characteristics.

Numerical examples show the results of the present analytical method are compared satisfactorily with the results of the Monte Carlo simulation. The general remark found from the numerical results is that because the fatigue failure criterion prevails over the instantaneous deformation criterion, the consideration of the maximum displacement response alone is not sufficient for the evaluation of the anti-seismic safety of hysteretic structures, while, for the hysteretic structures with nonpositive rigidity ratio, careful attention must be paid to the unstable increase of plastic deformation due to one-sided drift of the center of hysteresis.

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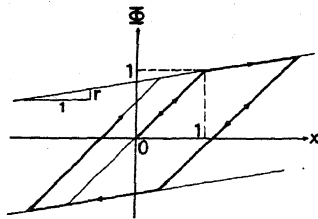


Fig. 1 Bilinear hysteretic characteristic

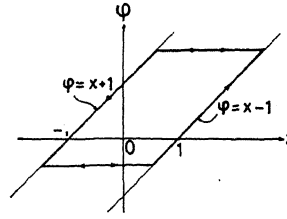
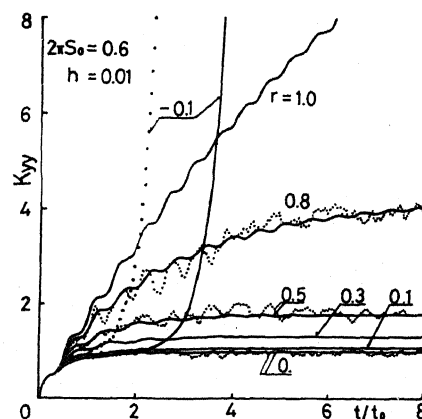
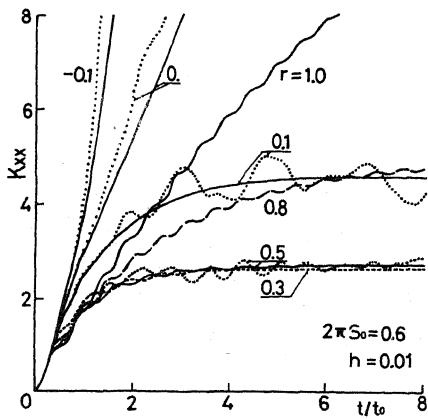


Fig. 2 Backlash characteristic



(a) Variance of displacement response (b) Variance of velocity response
Fig. 3 Effect of the rigidity ratio r on the response to stationary excitation

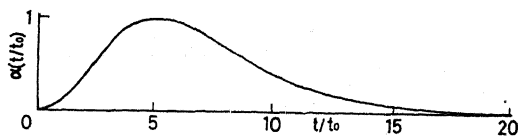


Fig. 4 Envelope function of nonstationary excitation
 $\alpha(t) = \beta t^\gamma \exp(-\nu t)$
 $\beta = 0.176, \gamma = 2.854, \nu = 0.571$

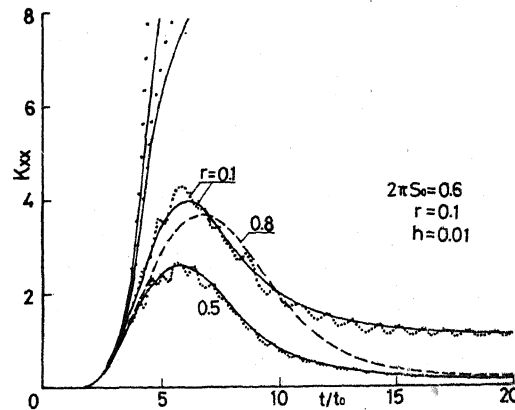
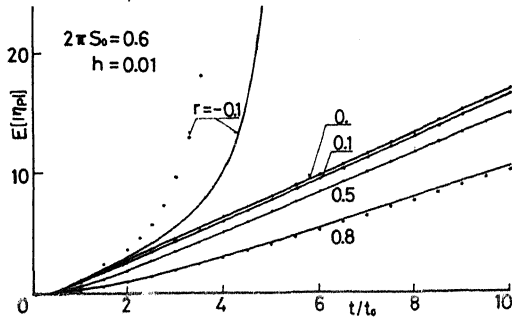
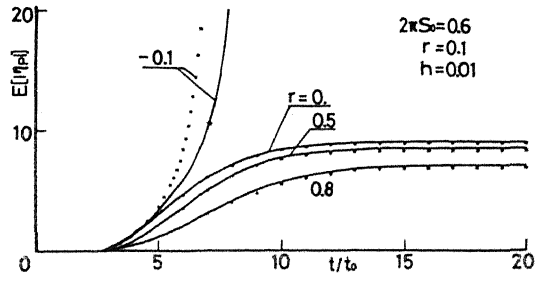


Fig. 5 Variance of displacement response to nonstationary excitation



(a) Under stationary excitation



(b) Under nonstationary excitation

Fig. 6 Average cumulative plastic deformation

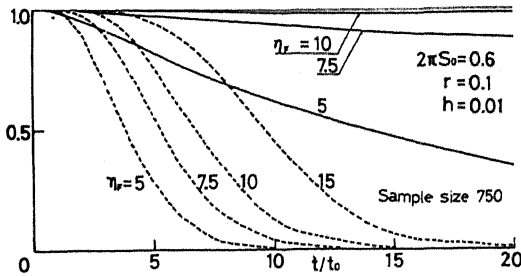


Fig. 7 Reliability function under stationary excitation

— Instantaneous failure criterion
 ---- cumulative energy failure criterion

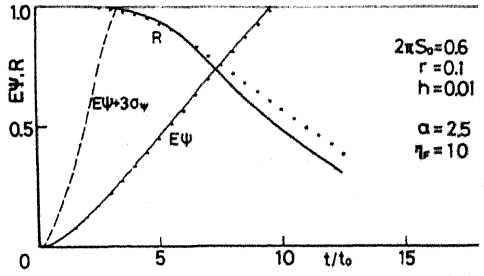
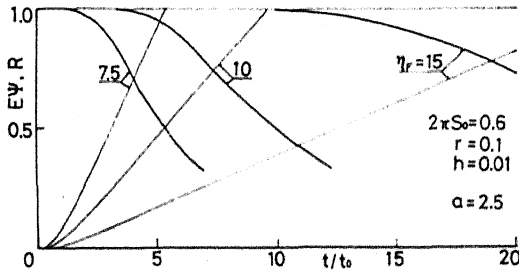
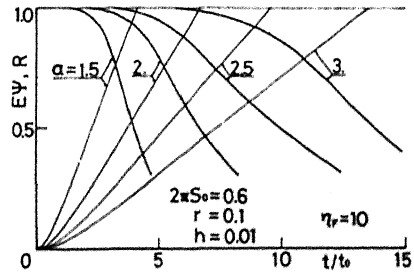


Fig. 8 Mean damage and reliability function

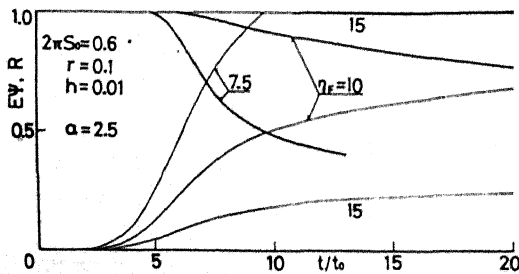


(a) Effect of η_r on the reliability

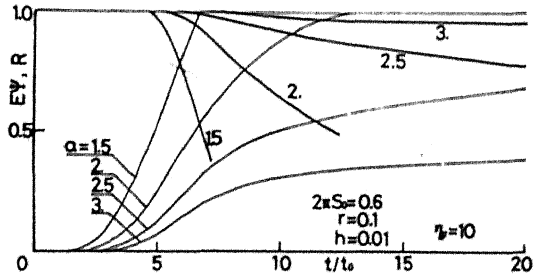


(b) Effect of α on the reliability

Fig. 9 Mean damage and reliability under stationary excitation



(a) Effect of η_r on the reliability



(b) Effect of α on the reliability

Fig. 10 Mean damage and reliability under nonstationary excitation