STOCHASTIC PREDICTION OF SEISMIC RESPONSE
OF INELASTIC MULTIDEGREE-OF-FREEDOM STRUCTURES

by

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SYNOPSIS

This paper presents an approximate probabilistic dynamic analysis of the interstory displacement response to earthquake-like excitation of n-story, elastoplastic, shear structures. The starting point is the (elastic) response of an associated linear system having the initial properties of the elastoplastic system. The yielding activities of the stories are grouped into n 'states,' each characterized by the story which yields first. Hysteretic dissipation of energy and change of vibrational frequencies due to yielding are estimated in order to compute statistics of the response during each yielding state. The transition from the elastic to the elastoplastic response is determined by a set of first-crossing probabilities. Finally, combination of the n-states, conditioned by their probabilities of occurrence, leads to a statistical description of the maximum plastic displacement for each story. Results of the method for 2- and 4-dof structures are compared with the results of simulation studies (2,3).

INTRODUCTION

The starting point for the developed theory is the work by Knarnopp and Scharton (4), extended by Vanmarcke (5) and Vanmarcke and Veneziano (6), on the probabilistic response of one-degree-of-freedom inelastic oscillators. In order to extend their analysis to m-dof structures, extensive simulation studies (2,3) were conducted. The results of these studies served as a guide in making a number of necessary simplifying assumptions, and were used to compare with the results of the method. In this paper only the stochastic theory is presented. The input is a power spectral density function, $G(\omega)$, and a strong motion duration, S; and the output the probability distribution of the story ductility factors, μ.

RESPONSE DURING ELASTIC INTERVALS

At times when no plastic action occurs in any of the stories of a structure, the elastoplastic system behaves like an elastic m-dof oscillator (associated linear system). The response of this system can be described by the variances of the interstory displacements, σ^2, and the (elastic) apparent frequencies of vibration, Ω. From elastic random vibration theory (8)

$$\sigma_i^2 \simeq \sum_{k=1}^{n} \sigma_k^2 \phi_{ki}^2 \, \Gamma_k^2 \qquad i = 1, 2, \ldots, n \tag{1}$$

where the modal variances σ_k^2 are given by

$$\sigma_k^2 \simeq \frac{\pi G(\omega_k)}{4\zeta_k \omega_k^3} + \frac{\int_0^{\omega_k} G(\omega)\,d\omega}{\omega_k^4} \tag{2}$$

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and ϕ_{ki}, Γ_k are the modal shape and participation factor of the k^{th} mode, respectively, ζ_k is the damping ratio, and ω_k is the k^{th} eigenvalue. The frequency, Ω_i, is computed (2,3) by a weighted superposition of all the modes

$$\Omega_i \simeq \left\{\sum_{k=1}^{n} p_{ki}\, \Omega_k^2\right\}^{1/2} \qquad i = 1, 2, \ldots, n \qquad (3)$$

where the weighting factors, p_{ki}, represent the fraction of total response of the story of interest contributed by the k^{th} mode; the apparent modal frequency, Ω_k is itself a weighted combination of ω_k and the central frequency of the input motion (2).

Equivalent stationary intervals are estimated, and it is assumed that the (elastic) response is Gaussian. The first-crossing probability theory (1,5) is then used to give the probability of any story yielding before the others. For the i^{th} story this probability is

$$f_i \simeq \frac{\alpha_i}{\sum_{i=1}^{n} \alpha_i} \qquad i = 1, 2, \ldots, n \qquad (4)$$

where the decay-rate of the first-crossing time is approximated (5,6) by

$$\alpha_i \simeq \frac{\Omega}{\pi} \exp\,(-r_i^2/2) \cdot \frac{1-\exp\,\{-r_i\sqrt{2\zeta}\}}{1-\exp(-r_i^2/2)} \qquad (5)$$

The dimensionless r_i ratio is related to the yield displacement Y_i, by the relation: $r_i = Y_i/\sigma_i$.

RESPONSE DURING A YIELDING STATE

Once the relative displacement of a story crosses the yield level, chances are that this event will recur during a number of consecutive cycles of motion. The response of every story can then be viewed as a two-state process with purely elastic or elastic-plastic time intervals. When the elastic intervals of all the stories overlap, the response is elastic and can be described as was presented above. The question now is to describe the behavior of the structure when at least one story is in its yielding state.

Assume that the i^{th} story yields first following an elastic interval. During its yielding state, other stories may or may not yield. The plastic excursions which occur before the other stories yield are described in the same way as for a 1-dof system (4,6). By equating kinetic energy before yielding with hysteretic energy dissipated during yielding, the expected plastic displacement due to a single plastic excursion δ_i is estimated to be: $\delta_i \simeq \sigma_i/2r_i$. The expected number of consecutive plastic excursions (mean clump size) is approximated by the formula (5,7)

$$\eta_i \equiv E[N_i] \simeq \left\{1 - \exp\,[-\sqrt{2\zeta}\, r_i]\right\}^{-1} \qquad (6)$$

An initial estimate of the average duration associated with the i^{th} state, τ_i, is obtained by dividing η_i' by (Ω^*/π), where Ω^* is obtained from the elastic frequency Ω multiplied by an empirical reduction factor which accounts for the effect of yielding on vibration frequency.

At the same time, the other stories receive less power, and those that are above the yielding story vibrate faster than before yielding, as simulated responses clearly indicate (2). The first effect is translated into a reduction of the elastic r.m.s. value for each story m by a factor ξ_i, such that: (new)$\sigma_{mi} = \xi_i \sigma_m$. On the basis of rather mild assumptions, ξ_i is found to be

$$\xi_i = \left\{ 1 - \frac{\Omega_i K_i \sigma_i^2}{\sum_{k=1}^{n} \Omega_k K_k \sigma_k^2} \right\}^{1/2} \tag{7}$$

where K_i is the elastic stiffness of the i^{th} story. The increased frequencies are estimated by considering the stories above the yielding one as a separate substructure. The smaller number of stories implies a stiffer structure and therefore higher frequencies of vibration.

The question of whether or not other stories will yield during the yielding state of the i^{th} story is answered by again applying first-crossing probability theory to each story, using the reduced r.m.s. value σ_{mi}. The probability that the m^{th} story will yield is approximated by

$$p_{mi} \simeq 1- (1-\exp[-r_{mi}^2/2])\cdot\exp(-\alpha_{mi}\tau_i) \tag{8}$$

where r_{mi} and α_{mi} are computed on the basis of the reduced r.m.s. value (σ_{mi}).

Whenever another story yields, additional energy is hysteretically dissipated and new changes in the vibrational frequencies occur much in the same way as was previously described. These effects are accounted for, and the first-crossing theory is again used to find the probability of other stories yielding, and so on. While for a 2- or even a 3-dof structure the combination of the yielding activities (during one yielding state) is rather straightforward, it becomes very complicated for multistory structures; an approximate method has been developed to this end.

The above-mentioned procedure can be repeated in case another story yields first following an elastic interval. It is convenient to refer to "state i," where i is the first yielding story. The response of the structure can be viewed as a random sequence of n states (n = the number of stories) separated by elastic intervals. The statistics of the response of all the stories during each state are known, and the question is how to combine them in order to obtain statistics for the overall response.

COMBINATION OF STATES AND MAXIMUM RESPONSES

The expected total number of yielding states, N, is approximated as a weighted combination of the expected total number of occurrences of each state occurring individually. The latter is equal to S_o/ℓ_i, where S_o is the "equivalent stationary" response duration and ℓ_i is the expected duration of an i^{th} yielding state and the following elastic interval. The weighting factors are the probabilities f_i of occurrence of each state, i.

The problem then is to find the expected number of occurrences of each state if the total number of occurrences is N. This requires the joint probability distribution of the "number of successes" of n states, each having a probability f_i of happening, in an "experiment" with N trials. The answer is a multinomial distribution with parameters the probabilities f_i. From this the expected number of occurrences, x_i, of each state is computed.

$$E[x_i] = Nf_i \, , \qquad i = 1, 2, \ldots, n \tag{9}$$

So, the response process is viewed as a sequence of n groups, each consisting of $Nf_i(i^{th})$ states with known response statistics and separated by elastic intervals.

Simulation studies by other researchers (7) as well as the study in connection with the research reported here indicate that the probability distribution of the maximum plastic displacement, d, of each story during a plastic state has an exponential form. The distribution that was found to yield best results is

$$F(d) = 1 - \exp\,[-d/(\eta\delta)] \tag{10}$$

where η is the expected clump size of the story of interest in the plastic state (Eq. 6). One can then argue that, since a next plastic state starts adding plastic displacement from the final plastic displacement of the previous one either upwards or downwards (with equal probability), the average total plastic displacement is independent of the final plastic set of each state. Therefore, with regard to the expected value, the maximum plastic displacement is the maximum of each state maxima and therefore its probability distribution is

$$\phi(d) \simeq \prod_{i=1}^{n} \Phi_i(d) = \prod_{i=1}^{n} \left(1 - \exp[-d/(\eta_i\delta_i)]\right)^{Nf_i} \tag{11}$$

from which the expected maximum plastic displacement $\bar{d}$, is obtained numerically. The mean ductility factor is then simply $\bar{\mu} = \bar{d}/Y+1$.

For the variance of the maximum plastic displacement a generalized random walk model seems reasonable, because it is felt that what basically contributes to the variance of the response is the superposition of permanent plastic sets. Hence one can write for the variance of the plastic set of the n groups of plastic states

$$\sigma_D^2 = \sum_{i=1}^{n}\left\{E[x_i]\cdot\sigma_{D_i}^2 + \sigma_{x_i}^2\left(E[D_i]\right)^2\right\} \simeq \sum_{i=1}^{n} N\delta_i^2\left\{f_i(2-f_i) + (1+\alpha_i\tau_i)^{-2}\right\} \tag{12}$$

The standard deviation of the ductility factor is then simply $\sigma_\mu = \sigma_D/Y$.

The mean and standard deviation of μ are then used to compute the parameters of an Extreme Value Type I distribution which was found to be a good approximation of the probability distribution of the ductility factor (3).

COMPARISON WITH RESULTS OF TIME-HISTORY ANALYSES

Results of the theory are compared in the next page with statistics from the time-integration analyses. Fig. 1 compares the results for a 2-dof structure and two different motion intensities and Fig. 3 the results for a flexible 4-dof structure and two motion durations. The agreement is quite satisfactory and it seems that the model is able to predict even small changes in the response when various parameters are varied.

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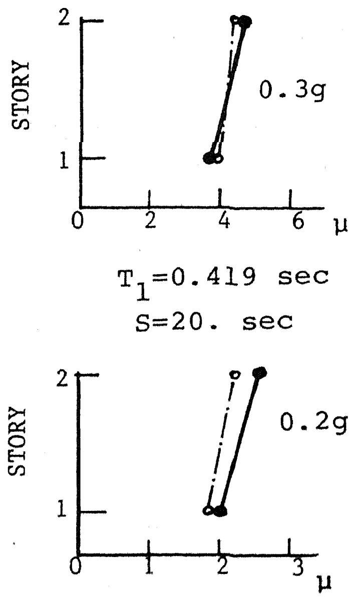

Fig.1.Mean Ductilities For a 2-dof Stru. and 2 Motion Intensities.

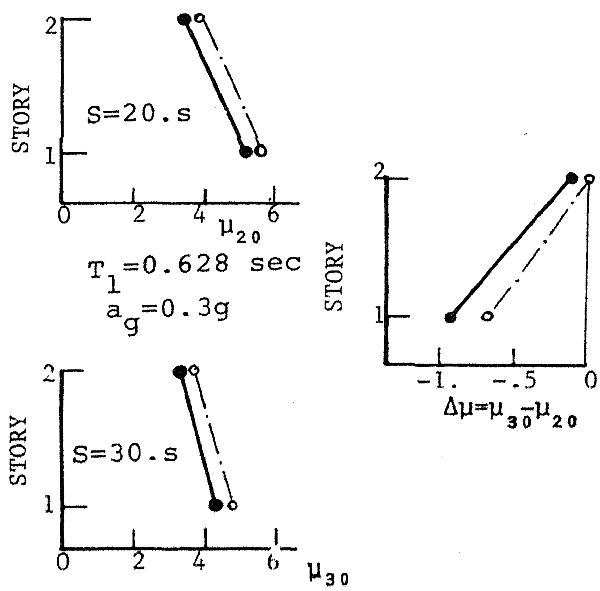

Fig.2.Mean Ductilities for a 2-dof Stru.and 2 Motion Durations.

Average from the 15 simulated response time-histories.

Mean predicted by Random Vibration Theory.

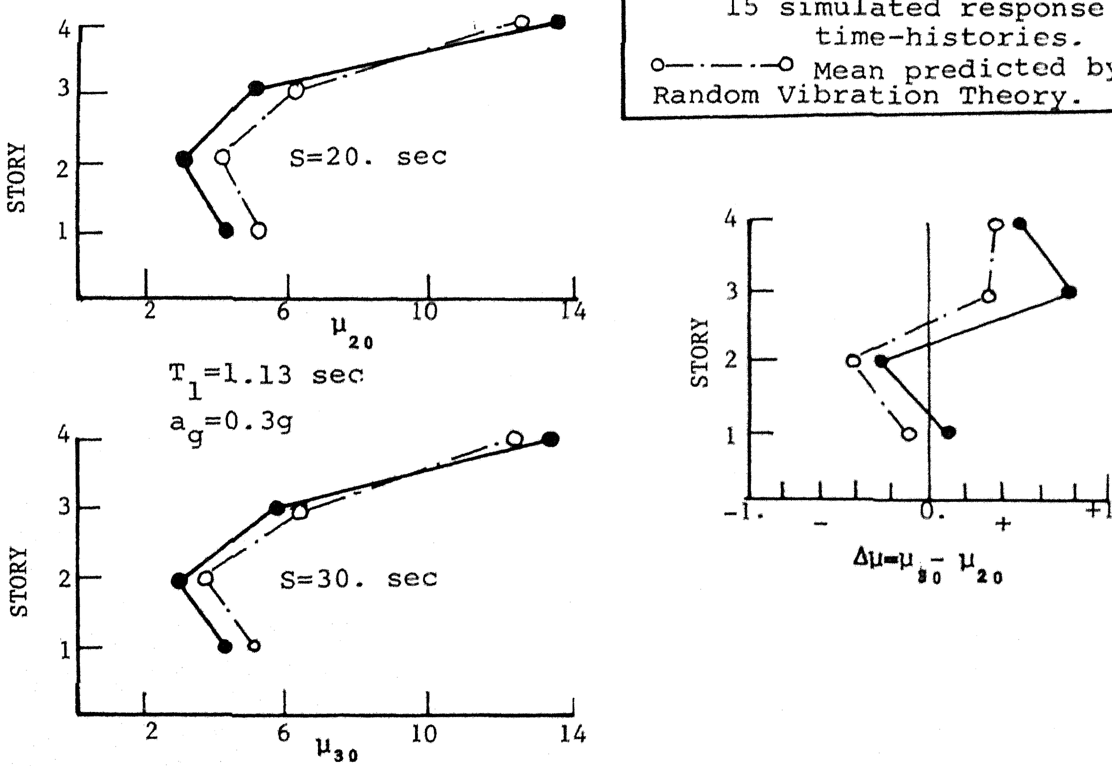

Fig.3.Mean Ductilities for a 4-dof Structure and 2 Different Motion Durations (S=20sec and S=30sec).