#### CRITICAL SEISMIC EXCITATION AND RESPONSE OF STRUCTURES

by
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#### SYNOPSIS

The practicality of the critical excitation method for the design of structures is investigated. In the terminology of this paper, an excitation is critical for a structure if it generates a larger response peak in one of the design variables than any other possible excitation in some given class. The basic idea of the method is to draw up the design in such a way that the structure has sufficient reserve strength to sustain its own critical excitations up to a certain maximum intensity. This paper investigates more specifically a modification of the idea. It is shown, by an analysis of several existing and planned structures, that the modification leads to fairly realistic, if somewhat conservative designs. The results encourage the conclusion that the modified mehtod, or a similar one, may become a useful design tool of structures whose impostance justifies conservative design.

#### INTRODUCTION

The design of structures against seismic excitation is a process of decision-making under uncertainty. In most seismically active sites in the world, few recorded accelerograms and little reliable geological information are available. Someone entrusted with the design of an important structure must nevertheless decide what kind of ground acceleration it is to withstand. Under the circumstances, he may study the records obtained elsewhere, at localities with similar geological features, and base his design on one of these records. He would do so in the hope that this accelerogram or response spectrum represents an excitation likely to happen at that site.

A somewhat more rational procedure was proposed sometime ago (1). A designer who follows it would select not a single accelerogram but a certain class of excitations which he considers to be realistic for the locality in question, and would then determine those in that class which generate the largest response peaks in each structural variable, such as a joint displacement or a member force. These excitations have been called "critical", and so have the responses of the structure. The idea is that the designer would draw up the design in such a way that it would have sufficient reserve strength to sustain its own critical excitations.

This design procedure can be cast in many forms. The paper begins by describing two. The first is one that is intuitively and conceptually appealing but unfortunately often leads to overly conservative designs. The second is a modification of the first (2), intended to avoid excessive conservativeness without introducing excessive computation complexity.

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The main purpose of the paper is to present results that were obtained using the modified procedure. In order to verify its practicality, a number of realistic structures were analyzed. The response peaks were calculated in each case for the modified critical excitation, along with those generated by several recorded ground motions of the same intensity. The ratios of the peaks, clled the "critical design factors" in this paper, are indicators of the degree of conservativeness of the procedure. These factors are shown to fall into the range of 1.1 to 2.9. There are two reasons for believing that factors in that order represent a reasonable degree of conservativeness. For one, the modified critical excitations appear to be fairly realistic samples of possible ground motions that cannot be disregarded out of hand. Secondly, as a fairly broad sample of strength calculations shows, designs by experienced engineering firms frequently have sufficient reserve strength to sustain these excitations.

The results suggest the conclusion that the modification of the critical excitation method, or some similar procedure, may become a practical engineering design tool for structures whose importance justifies some conservativeness.

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### CRITICAL EXCITATION

In the case of forced vibration, the most damaging excitation on a structure can be expected to have a frequency content that closely matches that of the structure. This is actually so, under certain assumptions. As has been demonstrated before  $\binom{1}{l}$ , the critical excitation  $\mathbf{x}_{\mathbf{c}}^*(t)$  for a linear system is the time-reversed impulse response, multiplied by an intensity modification factor,

$$x_c^*(t) = \frac{E}{N} h(-t), \qquad (1)$$

if E, the reference intensity of the ground acceleration is defined as  $t_{\rm g}$ 

$$E^{2} = \int_{0}^{\tau_{e}} \tilde{x}_{g}^{2}(t) dt$$
 (2)

and N is the square integral of the unit impulse response

$$N^2 = \int_0^t e^{h^2(t)dt}, \qquad (3)$$

In both Eqs. (2) and (3), the "effective duration"  $t_{\rm e}$  of a ground excitation is the period over which the excitation contributes significantly to the maximum response of the structure.  $t_{\rm e}$  depends on the modes as well as the damping of the structure. The response peak under

the critical excitation always occurs at the end of that period. It is

$$y_c^*(t_e) = EN$$
 (4)

The frequency content of  $x_c^*(t)$  is the same as that of the structure, in the sense that its Fourier amplitude spectrum differs from that of the structure only by the factor (E/N). A plot of the typical critical excitation with El Centro intensity is shown in Fig. 1.

# MODIFICATION OF CRITICAL EXCITATION

The response peak (4) is often found to be unrealistically high. The intuitive reason for this is quite plain. The frequency content of  $\mathbf{x}_{\mathbf{c}}^{\mathbf{t}}$  often differs greatly from that of any realistic ground acceleration. It is therefore necessary to exclude from the class of excitations that are being considered, all those with frequency contents that are unlike those of realistic ground motions. This can be done in many ways. One that seems particularly simple is the following.

Among the ground motions that are considered realistic for a particular site, one should presumably include a number n of recorded ones,  $\mathbf{x_i}$  (i = 1, 2,..n), preferably those that have occurred at locations with similar geological features. In addition, all linear superpositions of the  $\mathbf{x}$  might be considered to be realistic as well, provided only that their combined intensity does not exceed a prescribed maximum. This, at any rate, was done in the study reported here. Moreover, in order to avoid computational complication, it was not the critical excitation among these superpositions that was determined. Rather, an excitation  $\mathbf{x}$  was calculated which differed least (in the least-squares sense) from  $\mathbf{x}$ . In symbols

$$X_{\ell}^{*} = \sum_{i=1}^{\infty} a_{i}^{X} i$$
 (5)

so that

$$\int_{0}^{t_{e}} (x_{\ell}^{*} - x_{c}^{*})^{2} dt = Minimum$$
 (6)

and

$$\int_0^t e^{x^2} (t) dt = E^2$$
 (7)

A plot of a typical modified critical excitation x with El Centro intensity is shown in Fig. 2. It is, by all appearances, a sample of a perfectly realistic ground motion during an earthquake. One cannot, in other words, ignore it in the process of a design on grounds of its being "unrealistic" or "unlikely".

### APPLICATION TO REALISTIC STRUCTURES

Several realistic structures were analyzed and some of the structural members were investigated by the critical excitation and response approach. The essential results are summarized in Tables 1 and 2.

Table 1 shows the "critical design factors" of some of the design

variables of seven structures. The "critical design factors" are based on the ratio of the response of the second class critical excitation with that of the reference excitation of same intensity. The reference excitations are 1971 Pacoima dam S14W, 1940 El Centro S00E, and 1954 Eureka N79E. These factors range from 1.14 to 2.88.

Table 2 shows strength requirement for a modified critical excitation of El Centro intensity. The approximate ductility requirements for some of the members as they were designed are shown in the last column.

CONCLUSIONS

- 1. A modification of the critical excitation method is applied to several realistic structures. From the "critical design factos" calculated for each, and from strength checking on already designed ones, it appears that the method leads to results which are on the safe side but not overly conservative. This conclusion is further supported by plots of many of the modified critical excitations which are, by all indications, quite realistic samples of possible ground motions during earthquakes.
- 2. The modified mehtod, or some similar procedure, seems to have promise as a practical and useful tool for the design of structures in cases in which conservative design is desirable. This is likely to apply to structures of major importance, the destruction of which would cause severe human or economic losses.
- 3. Its attraction in such cases may lie in its ability to spot weak points in a design, and the fact that it eliminates much of the arbitrariness from the choice of the excitation on which designs now often are based.

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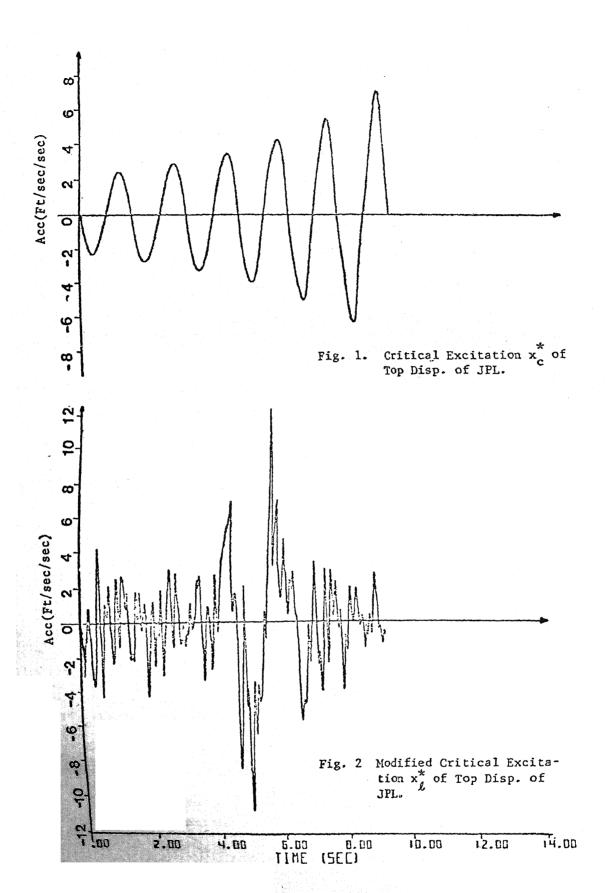


Table 1. CRITICAL DESIGN FACTORS

Structure	Fund. Period (sec)	Comparison Grd. Motion	Design Struct. Variable	Critical Design Factor	Source
R.C. Flat Plate Bldg. 16 stories	4.95.	Pacoima Dam El Centro	Base M of Ext. Col.	1.59 1.24 1.50	J. Vellozz: Amman & Whitney
Tapered R.C. Chimmey 1000 ft.	3.40	Pacoima Dam El Centro Eureka	Base V	1.48 1.33 1.32	J. Vellozzi Amman & Whitney
Reactor Shell	0.416	Pacoima Dam Eureka	Centroidal Defl.	1.39 1.14	Miller & (3
Steel Struc. on Conc. Dike	0.214	Pacoima Dam Eureka	Top Defl.	2.29 2.50	Stone & Webster
Bank of Cal. (steel frame) 24 stories	3.664	Pacoima Dam El Centro Eureka	Base M of Col.	2.33 2.86 2.13	Degenkolb & Assoc.
JPL Bldg. No. 180	1.488	Pacoima Dam JPL Basement	Base M of Col.	1.97 2.88	Wood(4)
Int'l Bldg. (steel frame) 24 stories	1.456	Pacoima Dam El Centro Eureka	Top Defl.	1.23 2.61 2.69	Degenkolb & Assoc.

Table 2. STRENGTH REQUIREMENT FOR THE CRITICAL EXCITATION OF EL CENTRO INTENSITY

Structure	10		
Structure	Structural	Requirements or	Approximate
	Element	Secs. Provided	Ductility Reg'd
R.C. Flat	Bottom Story	20"x20"col.f'=3ksi	
Plate Bldg.	Ext. Col.	fy=60ksi,	,
	1	12-#14	1
	land the state of	1	
	<del> </del>	#6 Ties @8"	-
R.C. Chimney	Bottom Sec.	fc-3ksi, fv=50ski,	
		#9 Vert. Reinf.@65"	4
		both faces	•
Bank of	Ground Floor	14WF456	
California	Ext. Col.	A441 Steel	3.75
JPL Bldg. 180	2-4 0		3./3
ort prag. 100	2nd Story Col.	14WF158	
		A36 Steel	S
Int'l Building	Bottom Ext.	14WF320+2P1.24x35	
ŭ	Column	A7 Steel	
	L	K, Steel	1.4