RESPONSE OF SIMPLE ELASTOPLASTIC SYSTEMS TO EARTHQUAKES OF VARIOUS MAGNITUDES AND DISTANCES

by

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SYNOPSIS

The responses of one degree-of-freedom elastoplastic structures to earthquake motions is examined by least-squares regression using magnitude and distance as independent variables. The distribution of peak elastoplastic displacement at a site subjected to a range of seismic motions from a nearby fault is well-approximated by the distribution of displacement of an equivalent elastic system, substantiating results reported by others based on comparison of displacements during individual motions. Total hysteretic energy absorbed by the elastoplastic models is used as a measure of damage; it is found that, contrary to an often-expressed notion, two earthquakes (one a small, close event, the other a large distant event) which produce the same expected elastic displacements are also expected to produce approximately the same amount of hysteretic energy in elastoplastic systems. For events with the same peak ground acceleration, the large, distant event generally produces more damage.

INTRODUCTION

The response of various types of nonlinear structural models to earthquake motions has been studied by many investigators (1, 4, 5, 9, 10, 11). Nonlinear models are appealing intuitively because they reflect the behavior of structures during intense earthquake motions in a more realistic fashion than linear models. Methods of investigation have included time integration using real and artificial motions, and random vibrations analyses. Relatively little attention has been given to the distribution of response of nonlinear systems as a function of earthquake size and distance.

In this study systems with elastoplastic force-deformation behavior are examined, since they are the simplest nonlinear structural model. Two measures of response of these systems to different earthquakes are studied: the peak displacement, and the total hysteretic energy absorbed during ground shaking, which is a simple indicator of cumulative damage.

DATA SET AND SYSTEMS EXAMINED

The strong motion records used in this study consisted of 68 horizontal components of motion recorded at 21 sites during 22 events which occurred in western North America. Magnitudes (M_L) ranged from 5.3 to 7.6, and epicentral distances were from 11 to 120 km. In general the smaller events were associated with closer source-to-site distances. The records used are reported in Reference 7.

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In all, fifteen elastoplastic systems were considered, with initial (small deformation) natural frequencies ranging from 0.25 to 5 hz. Table 1 indicates the properties of five systems for which results are presented here; substantially similar results were obtained for ten other elastoplastic systems identical to those of Table I except for different yield displacements. For all systems, 2 percent viscous damping was assumed.

MAXIMUM DISPLACEMENT OF SYSTEMS

To determine the maximum displacements of the systems of Table I, the responses to each of the 68 horizontal components of motion was computed using a constant-velocity time-integration procedure (2). The five systems experienced inelastic response during 42 to 56 of the 68 components of motion, depending on the frequency of the system. To make comparisons to elastic systems, the peak responses of elastic systems with the same natural frequencies were obtained for the 68 components of motion (these are published in Reference 3).

The peak responses of each system were regressed on earthquake magnitude and distance, using an equation of the form

$$\log\ (\text{displacement}) = C_1 + C_2 M + C_3 \log\ (R + 25) \qquad (1)$$

where M is magnitude and R is hypocentral distance in kilometers. The distance term R+25 was selected to conform to attenuation functions used by others. For the elastoplastic systems, only records causing inelastic response were used in the regression; for elastic systems, all 68 components of motion were used. Results of all regression analyses are reported in Reference 7. Residuals of log displacement about the predicted values were found to conform to a normal distribution.

It has been observed by several investigators (8, 10) that the peak displacement of an elastoplastic system can be estimated by the peak displacement of an elastic system with the same initial natural frequency and viscous damping, for low and medium frequency systems. This approximation is substantiated by examining and comparing the responses of systems to single components of motion. To determine if the distribution of peak displacements in elastoplastic and elastic systems are similar, in the context of seismic hazard, a site located 40 km from a fault capable of producing magnitudes in the range 5.0 to 7.6 was examined. The distribution of magnitudes was taken to be exponential (truncated at 7.6) with a "Richter b" value of 0.88 (typical of California), and events within 100 km of the closest point to the site were considered.

Figure 1 indicates the upper fractiles of displacement for both elastoplastic and elastic systems at the site described, for a single event of random magnitude and epicentral location on the fault. Also shown on Figure 1 is the mean displacement of the elastic systems, which is also the (chosen) yield displacement of the elastoplastic systems. Thus these chosen yield displacements represent a decision in the design context of requiring the structure to remain elastic during the average level of motion from the next event on the fault, but relying on plastic strength for larger-than-average motions. The comparison of displacements for the 0.9

and 0.99 fractiles indicates close agreement except for the high frequency system, where agreement is not expected as mentioned above. Thus it can be concluded that the distribution of displacements of elastic systems indicates this distribution for elastoplastic systems, for low and medium frequencies. Because the seismic threat at a site can be represented for design purposes in terms of a single event (7), it is expected that this conclusion holds for other sites as well.

HYSTERETIC WORK

While total elastoplastic displacements can be estimated from those of elastic systems, these displacements are not necessarily good indicators of low-cycle fatigue damage (6). One measure of damage (perhaps the simplest) is the total hysteretic work done by the system; for elastoplastic models this is calculated as the yield force times the sum of inelastic displacements. The hysteretic work cannot be easily related to maximum displacement; this is evident in Figure 2, where hysteretic work is compared to ductility (maximum displacement divided by yield displacement) for one second, 2 percent damped systems with various yield levels, as calculated from several horizontal components of motion. The comparison is particularly interesting for the two components of record E072, which show opposite trends.

The hysteretic work done by five elastoplastic models described previously, when subjected to the 68 components of motion, was regressed on earthquake magnitude and distance, using the form of Equation (1). (Again, only components which induced inelastic response in a given model were used in the regression analysis for that model.) Thus predictive equations were obtained which, for a given earthquake magnitude and distance, would indicate the expected hysteretic work in a structural model with chosen characteristics.

To indicate the effect of magnitude and distance on plastic energy absorbed by the systems, the total work predicted by different events is shown in Figure 3. The base for comparison was chosen to be a Magnitude 6 event at 16 km; predicted hysteretic work for the five systems during this earthquake is shown by x's in Figure 3. To make comparisons with other earthquakes, the elastic displacement of each system for this Magnitude 6 event at 16 km were calculated from the regression equations described in the previous section (Equation 1). A larger, more distant earthquake was then found for each system which had the same expected elastic displacement. (In general, a distance of 100 km was used, and the magnitude of the event was calculated. When this indicated a magnitude greater than 7.6, i.e., outside the range of data, a magnitude of 7.6 was used and the distance was calculated.) These events (one for each of the five models) are shown in Figure 3, along with the expected hysteretic work which they induce in their respective elastoplastic models (indicated by squares).

The striking observation about this comparison is that for all but high frequency systems, the larger, farther events, which have the same expected elastic response as the smaller, closer event, are expected to produce less damage (as measured by hysteretic energy absorbed). This is contrary to the commonly expressed idea that large, distant events produce more damage than small, close events, because of factors such as longer duration.

The notion that large, distant events are more damaging is only valid if earthquakes are compared on the basis of peak ground acceleration. To show this, a predictive equation similar in form to Equation (1) was developed for peak ground acceleration using the same 68 components of motion. An earthquake with M = 7.6 and R = 65 was found to have the same expected acceleration as the base earthquake (M = 6.0, R = 16). The predicted values of hysteretic energy absorbed by the five systems during this event are shown as open circles in Figure 3; they are above the values for the base earthquake. Thus, for different earthquakes with the same expected acceleration, the intuitive notion is substantiated that the larger, farther event is associated with more damage.

The variation in hysteretic work for a given event is large (7), with a typical coefficient of variation of 2 or more. This is evident from Figure 2. Hence some of the differences shown in Figure 3 (e.g. between the X's and the open squares for frequencies of 0.25,1, and 2 hz) are not statistically meaningful. The extremely low value of the 5 hz system for the M=6, R=16 km event is believed to result in part from statistically insignificant deviations of the regression coefficients for this system. However, the general trends suggested by the analysis are important; in particular, the results obtained for frequencies of 2 hz and lower do not show that large distant earthquakes are generally more damaging than small close shocks, when the events are compared on the basis of elastic response at the frequency of interest.

CONCLUSIONS

It is concluded from this study that the peak displacement of a low or medium frequency elastoplastic system is closely approximated by the displacement of an elastic system with the same initial frequency. This applies in calculations of seismic hazard, where integration is done over a range of event magnitudes and distances, as well as to single events which is the usual comparison reported.

Also, for two earthquakes with the same expected elastic displacement in a (low or medium frequency) one degree-of-freedom model, the smaller, closer event is expected to produce about the same amount of damage, as measured by hysteretic energy absorbed, as the large, distance event, in an elastoplastic model with the same initial frequency. These results are tentative and deserve further investigation with other inelastic models and other measures of low-cycle fatigue damage. However, if substantiated, they imply that the expected inelastic damage during an earthquake can be limited simply by specifying the proper pseudo-velocity response spectrum (which is proportional, at each frequency, to the peak displacement of an oscillator with that frequency). The same effect cannot be achieved by using a design acceleration; a large, distant event will generally produce more damage, as measured by hysteretic energy absorbed in low and medium frequency elastoplastic systems, than a small, close earthquake with the same expected acceleration.

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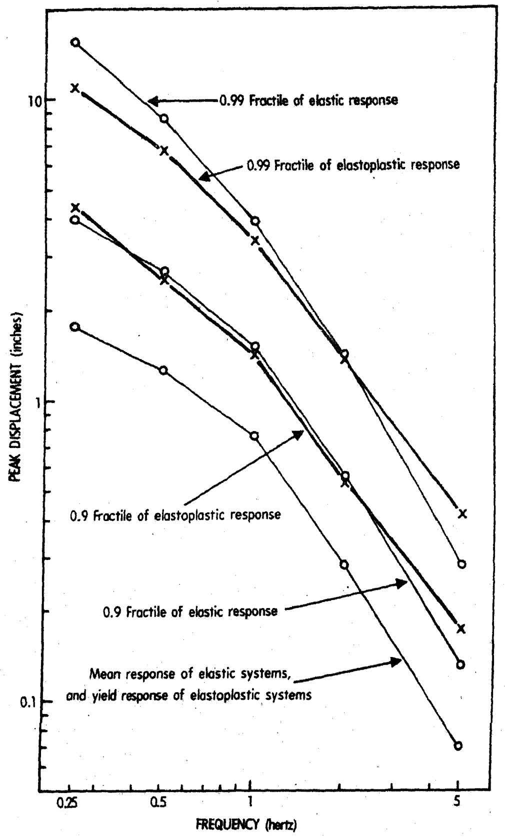

Fig. 1

MEAN AND UPPER FRACTILES OF RESPONSE FOR ELASTIC AND ELASTOPLASTIC MODELS

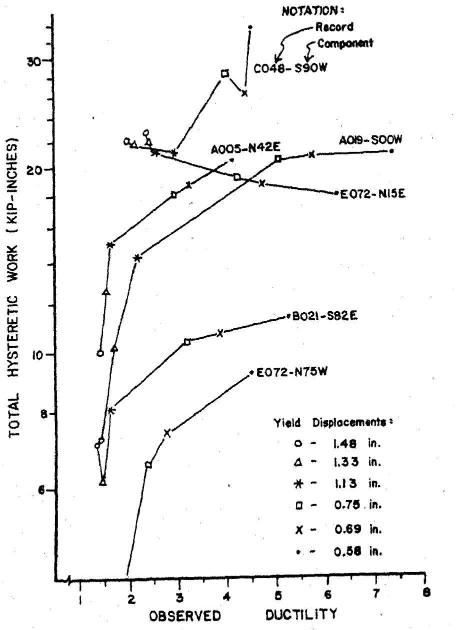

Fig. 2

TOTAL HYSTERETIC WORK VERSUS DUCTILITY FOR ELASTOPLASTIC MODELS WITH ONE SECOND PERIOD AND VARIOUS YIELD DISPLACEMENTS

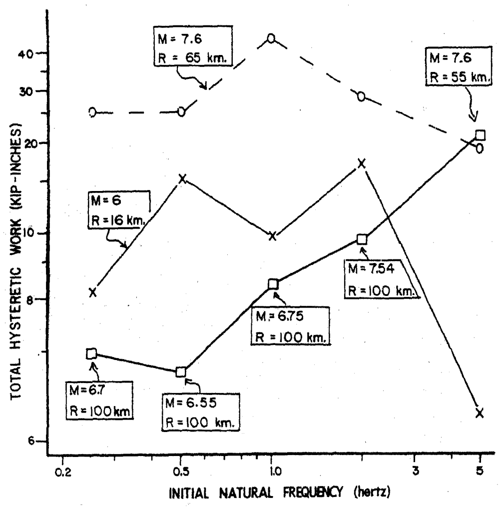

Fig. 3

TOTAL HYSTERETIC WORK IN ELASTOPLASTIC MODELS FOR A SMALL CLOSE EARTHQUAKE AND FOR SEVERAL LARGE DISTANT EARTHQUAKES

TABLE 1

PROPERTIES OF SINGLE DEGREE OF FREEDOM ELASTOPLASTIC SYSTEMS

(2% viscous damping assumed)

Mass 0.1 (kip-sec²)/in. for all systems

System No.	Spring Stiffness, Kips/Inch	Initial Natural Frequency, hertz	Yield Displacement, Inches
1	0.247	0.25	1.80
2	0.986	0.5	1.24
3	3.94	1.0	0.750
4	15.8	2.0	0.285
5	98.6	5.0	0.0835