UNIQUENESS PROBLEMS IN STRUCTURAL IDENTIFICATION

FROM STRONG MOTION RECORDS

BY

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ABSTRACT

Problems in the identification of the structural parameters of a building system from strong motion records have been investigated. Concepts of local and global nonuniqueness have been introduced. Through an analysis of a linear N-degree of freedom system used to model an N-story structure, conditions under which global uniqueness follow have been found. The studies indicate that global nonuniqueness may be obtained by proper instrument location and point out that the roof and basement records do not have sufficient information to uniquely determine the estimates of stiffness and damping. It has been shown that nonuniqueness may occur locally even when the initial guess is close to the true parameter values.

INTRODUCTION

The prediction of structural response to strong ground shaking requires the use of suitable dynamic models for structural systems. Typically, for structures vibrating in the linear range, this would necessitate a knowledge of the mass, stiffness and damping distributions in the structure. This paper studies some of the problems underlying the determination of these vibratory characteristics from ground motion records.

Many structures have been typically instrumented with two strong motion accelerographs, one of which is often placed in the basement of the structure while the other is placed at the roof level. The investigation carried out here deals with such situations and points out that the determination of structural parameters from such "input-output" data is an ill-posed inverse problem leading to nonunique solutions, thus requiring great care in the interpretation of the dynamic models so obtained.

THE IDENTIFICATION PROBLEM RELATED TO STRONG MOTION RECORDS

The procedure for establishing a parametric structural model from strong motion "input-output" records is shown in Figure 1. The parametric identificaiton problem starts with an assumed model of the structural system in which a set of unknown parameters are required to be "identified" through the use of the records. Typically, since the mass distribution in most structures is fairly well known, it is the damping and stiffness distributions designated by $\underset{\sim}{c}$ and $\underset{\sim}{k}$ which need identification.

The response of the actual structural system to the ground input $v^*(t)$, at some location $\underset{\sim}{a}$ in the structure, is measured by strong-motion accelerographs suitably located in the structure. Both the input and the system response records so obtained are contaminated by measurement noise $m(t)$ and

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n(t) respectively. Beginning with an initial guess of the parameters to be estimated, the model parameters are then so adjusted that when the structural model is subjected to the recorded input v(t), the model response $w_m(a,t)$ at the location a is "as close as possible" to the measured response $w^{obs}(a,t)$. These adjusted parameters, which yield the "best" match between w_m and w^{obs}, are then said to be the "best" estimates of the stiffness and damping parameters.

Through the matching of model and measured response at a given location in a structure is often used [1] as the criterion for the establishment of the model parameters, the question that has received little attention in the past, is whether or not there exists a unique set of parameters which yield such a "best" match. This may become a problem of considerable engineering importance, because if several different structural models could all provide the same degree of history matching, then the structural analyst would not be able to deduce the exact structural model from such input-output data.

In this regard, two types of nonuniqueness problems can be isolated a) global nonuniqueness and b) local nonuniqueness. Global nonuniqueness concerns itself with the question of whether it is at all possible to obtain from an infinite ensemble of input-output records, a definitive knowledge of the parameters sought. It attempts to find out how many different structural models would all provide the same input-output time histories. In cases where several such structural models exist, the analyst may still be able to find the correct model, if he has some prior information about the range in which the parameters to be estimated lie. This leads to the problem of local nonuniqueness, which studies the determination of the structural properties from a few recorded input-output histories when some prior information is available. Such information can be used to restrict the parameter search space leading to a 'localized' identification problem.

GLOBAL NONUNIQUENESS PROBLEMS IN BUILDING IDENTIFICATION

Consider the lumped mass model of an N-story structure represented by the masses m_i, $i = 1,2,\ldots N$ and the stiffnesses k_i $i = 1,2,\ldots N$ (Figure 2b). Using the notation in the figure, for the system starting from rest, we have $M\ddot{w} + Aw = f$ where $M \equiv \text{diag}[m_1, m_2 \ldots m_N]$, $w = [w_1, w_2, \ldots w_n]^T$, $f = [0, 0, \ldots o, k_N v(t)]^T$ and A is the symmetric, tridiagonal positive definite stiffness matrix. In the following discussions soil structure interaction will be neglected and the data will be assumed noise free. Using Laplace Transforms we have

$$\frac{W_n(\lambda)}{V(\lambda)} = \frac{k_n}{m_N} \cdots \frac{P_{n-1}(\lambda)}{P_N(\lambda)} \qquad (1)$$

where $W_n(\lambda)$ is the transform of $w_n(t)$ and $P_i(\lambda)$ is the upper ixi submatrix of $[K-\lambda I]$ with $K = M^{-1/2}AM$.

The global identification problem deals with attempting to find the stiffness k_i, $i = 1,\ldots N$ from a knowledge of v(t) and $w_n(t)$. Assuming that another system defined by $\tilde{K}$ having the same "structure" has the same (N-n+1)th story response (Fig. 2c) to the ground input v(t), we must have, for the two systems

$$\frac{k_N}{m_N} \cdots \frac{k_N}{m_N} \frac{P_{n-1}(\lambda)}{P_N(\lambda)} = \frac{\tilde{k}_N}{m_N} \cdots \frac{\tilde{k}_n}{m_n} \frac{\tilde{P}_{n-1}(\lambda)}{\tilde{P}_N(\lambda)} \qquad (2)$$

a) If $n=N$, the response is measured at the first floor level and it can be shown [4] that relation (2) leads to a unique determination of the k_i's. b) If $n=1$, the response is measured at the roof level, and it can be shown [4] that there may be as many as N! different systems which would all yield the same input-roof response time histories. Thus no matter how many inputs are looked at, the building stiffnesses would remain unidentifiable. c) Also, if history matching at some intermediate floor, n, $2<n<N$, is done, a maximum of $N-n!(n-1)!(n-1)$ different systems [4] may exist which yield the same input-nth story response. Table 1 summarizes the nonuniqueness situation for $N \leq 6$ stories.

NUMERICAL EXAMPLE: a) Undamped System: Consider a three degree of freedom system with $m_1=m_2=1$, $m_3=2$ and $k_1=k_2=1, k_3=2$ (Fig. 2b, N=3). This system will yield the same top story response [4] as the system $m_1=m_2=1$ and $k_1=1/2$, $k_2=1$, $k_3=4$ for any base input $v(t)$. Thus from only a knowledge of the base input and roof response it would be impossible to distinguish between the two models However, if the criterion for establishing a structural model is matching of the first story responses (that is the response of mass m_3) then the identification problem is unique.

b) Damped System: For a two degree of freedom damped system where k_1, k_2 are the story stiffness, c_1, c_2 the interstory damping values, and m_1 and m_2 the floor masses, it has been shown [5] that there exists another system which yields identical input-roof response pairs with parameters $\tilde{k}_1=k_2m_\gamma$, $\tilde{k}_2=k_1/m_\gamma$, $\tilde{c}_1=c_2m_\gamma$ and $\tilde{c}_2=c_1/m_\gamma$ where $m_\gamma=m_1/(m_1+m_2)$. These two systems are both physically reasonable so that without prior knowledge about the structural stiffness, it would be impossible to choose the right one only on the basis of input-roof response studies. Calculating the difference in the first story base shear between the two models, we have for a frequency ω,

$$\frac{k_2w_2(\omega) - \tilde{k}_2\tilde{w}_2(\omega)}{k_2w_2(\omega)} = \frac{m_2\omega^2[(m_1+m_2)k_1 - m_1k_2] - i\omega[(m_1+m_2)(k_1c_2-c_1k_2)]}{k_1(m_1+m_2)\ (m_2\omega^2 - ic_2\omega - k_2)} .$$

Clearly large differences in the base shear forces would occur for $\omega^2 \approx k_2/m_2$ thus showing that the nonuniqueness problem may become a critical one from the structural analysis viewpoint.

LOCAL NONUNIQUENESS PROBLEMS IN BUILDING IDENTIFICATION

In cases where global nonuniqueness occurs, it would be important to investigate if available information about the range of the structural stiffness in a structure can be utilized to converge to the true stiffness values. If, in other words, one starts 'close' to the true parameter values, can an iterative adjustment of the estimates be made from a few available input-output records so as to arrive at the true stiffness distribution? A commonly used procedure [2] for arriving at a good match between the model and the measured response is to determine the "sensitivity-coefficients" which give the rate of change of the model response at the measurement point a, with respect to the parameters estimated. These coefficients determine the manner in which the parameter estimates need to be changed to successively improve the history match between the measured and the calculated responses. However a determination of these coefficients involves the integration of the system equations (n+1) times (where n is the number of parameters to be estimated) at each iteration making the computation extremely inefficient. For a 50 story structure (n=50) considering that one may need 50 to 100 iterations to converge at a set of estimates, the computation times involved may become prohibitive. The algorithm provided in

[3] utilizes an optimal control formulation for the problem thus reducing the computation time by a factor of (n+1)/2.

If m(x) and k(x) represent the mass and stiffness distribution of a structure modelled as a shear beam of length L, starting from rest, we have (Figure 2a)

$$m(x)\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial x}[k(x)\frac{\partial w}{\partial x}] \; ; \; w(o,t) = v(t); \; \frac{\partial w}{\partial x}(L,t) = 0$$

Using these equations we attempt to find k(x), from a knowledge of w(L,t) and v(t), such that the error functional

$$J = \frac{1}{2}[\int_o^T [w^{obs}(L,t)-w_m(L,t)]^2 dt + \frac{a}{2}\int_o^L (\frac{\partial k}{\partial x})^2 dx + \frac{b}{2}\int_o^L (\frac{\partial^2 k}{\partial x^2})^2 dx$$

is minimized. The positive weighting factors a and b correspond to first and second derivative penalties on k(x) and are adjusted from prior information about the stiffness distribution. Taking variations we have [3]

$$\delta J=[a\frac{\partial k}{\partial x} - b\frac{\partial^3 k}{\partial x^3} \quad k(x)\,|_o^L + b\frac{\partial^2 k}{\partial x^2}\frac{\partial \delta k}{\partial x}|_o^L - \int_o^L [a\frac{\partial^2 k}{\partial x^2} - b\frac{\partial^4 k}{\partial x^4} - \int_o^L \frac{\partial \Psi}{\partial x}\frac{\partial w}{\partial x}dt]\delta k(x)dx \qquad (3)$$

where $\Psi(x,t)$ satisfies

$$m(x)\frac{\partial^2 \Psi}{\partial t^2} = \frac{\partial}{\partial x}[k(x)\frac{\partial \Psi}{\partial x}] + [w^{obs}(L,t) - w_M(L,t)]\,\delta(x-L)$$

$$\Psi(x,T) = 0, \; \frac{\partial \Psi}{\partial t}(x,T) = 0, \; \Psi(o,t) = 0, \; \frac{\partial \Psi}{\partial x}(L,t) = 0$$

Expression (3) can be used to calculate $\delta J|\delta k(x)$ so that starting from a 'close' initial guess, adjustments can be made iteratively to the stiffness distribution to minimize J.

NUMERICAL EXAMPLE: The method outlined above was used to identify the linearly varying stiffness of a 16-story structure having a constant, known mass distribution (Fig. 3a). Assuming that a good initial guess of the stiffness distribution is available from prior information about the structure, an attempt at obtaining the true distribution by matching the model and measured response for the base input shown (Fig. 3b) is carried out. It is observed that though the model and measured roof responses are identical the estimated stiffness distribution is not correct. This example illustrates the nonuniqueness that may arise in inferring the stiffness distribution in structures, even if the initial guess is in close proximity to the true estimate by the use of one, or a few records of ground shaking. It points out that model and system responses may differ for inputs different from those used in the identification process.

CONCLUSIONS AND DISCUSSIONS

1) We have illustrated through the use of a simple shear model for structural systems that the matching of system and model responses at the roof level leads to nonunique structural identification. 2) Global nonuniqueness has been defined in this context as related to the lack of identifiability of a system from input-output records no matter how many records are used. Furthermore, it is shown that even when some prior knowledge of the structural system is available, estimates obtained from the use of a few input records may not lead to reliable parameter values. Thus nonuniqueness occurs even locally when the initial guess is in close proximity to the true parameter values. The later problem [3] is related to the nature of the input and the sensor location in the structure. 3) Globally unique solutions are guaranteed for the shear beam model if input-first floor responses are considered for identification. The analysis here

utilizes noise free data. In practice lower story responses would yield higher noise/signal ratios. Despite this, some preliminary work [6] appears to indicate that the first story response matching yields the best stiffness estimates.

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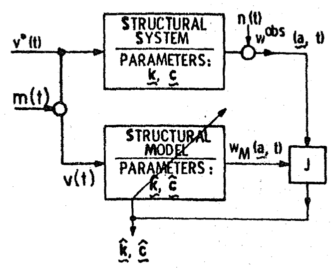
N \ n	6	5	4	3	2	1
6	6!	96	36	24	4	1
5		5!	18	8	4	1
4			4!	4	2	1
3				3!	1	1
2					2!	1

TABLE 1

Maximum number of solutions of the identification problem for an N-story structure given the response at the nth floor.



THE MODEL ADJUSTMENT TECHNIQUE FOR STRUCTURAL PARAMETER ESTIMATION PROBLEMS WITH SENSOR LOCATED AT a.

FIGURE 1

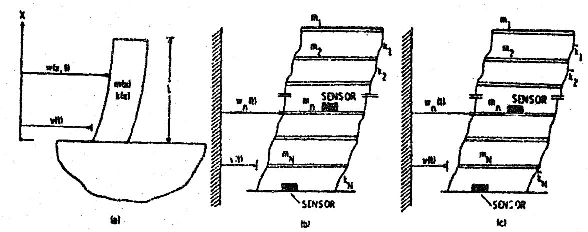


FIGURE 2

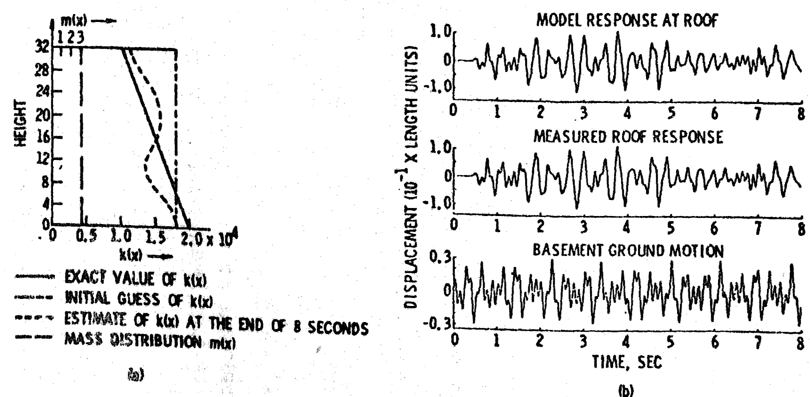


FIGURE 3