PREDICTION OF MAXIMUM STRUCTURAL RESPONSE BY USING SIMPLIFIED ACCELEROGRAMS

by

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SYNOPSIS

Maximum displacements of single-degree-of-freedom systems in both elastic and inelastic range due to earthquake motion are calculated by an economical method. The efficiency of this method is achieved by constructing a four-rectangular pulse model accelerogram of short duration. This model accelerogram retains both the intensity and the frequency content of the actual ground motion. Examples of this method are given by constructing model accelerograms for two parent ground motions - El Centro 1940 NS*2, and Pacoima Dam 1971 S14W. Both elastic and inelastic displacement response spectra due to these model accelerograms are close to those due to the parent accelerograms. One application of this method is presented to construct a model accelerogram from an elastic design sepctrum. The inelastic response spectra due to this design model accelerogram are compared with those obtained by currently used methods.

INTRODUCTION

The maximum displacement of structures due to earthquake motion is of primary interest in structural analysis and design. To calculate the maximum displacement of a structure in both elastic and inelastic range would be uneconomical in the preliminary design stage by using the time history of ground motion. This study provides a simple rational method to construct a model accelerogram of short duration, therefore the computation of maximum response is economized.

The model accelerogram is constructed by retaining the energy stored in structure during an earthquake. The vibration energy, E(t), of an elastic oscillator is defined as the sum of the relative kinetic energy and the potential energy, PE(t). For practical purposes the maximum PE(t), which indicates the maximum displacement of structure, is considered equal to the maximum E(t). This maximum E(t) is no less than the energy estimated by using the Fourier Amplitude Spectrum, FAS, of the ground accelerogram when structure has no damping (1). When structure is damped, this maximum E(t) is reduced because of energy dissipation. The reduction depends on both the amount of damping and the characteristics of accelerogram. The reduction is small for structures subjected to accelerograms that generate maximum E(t) early in the time history. When the influence of damping is large, the maximum E(t) reduces below FAS and is best estimated by using the damped velocity response spectrum, S^e_v,β.

Damping has little influence, however, on the structural response caused by the model accelerogram because of its short duration. Thus, the maximum E(t) equals approximately the energy estimated from the FAS of the model

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accelerogram. Therefore, the energy stored in a structure can be retained by constructing a model accelerogram whose FAS equals either the FAS or the $S^{e}_{V,\beta}$ of the parent ground motion, depending on the influence of damping. It is more practical, in general, to construct the model accelerogram so that the energy delivered to a range of structures is retained. This is achieved by minimizing the energy difference in the range of structures of interest.

CONSTRUCTION OF THE MODEL ACCELEROGRAM

A model accelerogram (Figure 1) with 8 undetermined parameters (GA*P1, T1, GA*P2, T2, GA*P3, T3, GA*P4, T4) which represent 4 rectangular pulses, is used to retain the energy delivered to structures. The total number of independent parameters is reduced to 2 by relationships among parameters derived from physical constraints and parametric study. A zero end velocity of the model accelerogram is among the physical constraints. These two independent parameters are; GA, which relates to the acceleration level, and T1, which relates to the time duration of the model. Therefore, the FAS of this model accelerogram can be expressed in closed form and is denoted by FAS(w|GA, T1), where w represents the natural frequency of the structure and is the domain of FAS.

Either the FAS or the $S^{e}_{V,\beta}$ of the parent ground motion can be used as an objective function, $\phi(w)$ (Figure 2). The $\phi(w)$ is fitted to the FAS(w|GA, T1) with least square difference in the frequency range. The difference (σ_m) in fitting is defined as

$$\sigma_m = \left\{ \frac{1}{N} \sum_{i=1}^{N} \left[\phi(w_i) - \mathrm{FAS}(w_i | \mathrm{GA},\ \mathrm{T1}) \right]^2 \right\}^{\frac{1}{2}} \tag{1}$$

where N covers the range of w such that $\frac{2}{3}\pi < w_i < 4\pi$, which is the period range for common structures. The minimization process for σ_m leads to a set of two nonlinear equations in GA and T1, which are then solved by the steepest descent method. This results in an iterative process to obtain values for GA and T1. Formally the iteration process is a non-linear fixed point operator. Therefore a contraction of the operator implies the convergence of the process and the error bound can be estimated. The constructed model accelerogram is labeled and is referred to as an F-series model when $\phi(w)$= FAS, and an S-series model when $\phi(w)=S^{e}_{V,\beta}$. The F-series model is identified by F in model's label (e.g. EF1522), similarly the S-series model is identified by S.

EXAMPLES OF THE MODEL ACCELEROGRAM

Model accelerograms are constructed for two ground motions (El Centro 1940 NS magnified 2 times and Pacoima Dam 1971 S14W) by the method described above and shown in the flow chart of Figure 2. The resulting model accelerograms are shown in Table 1. In this table the value of TA*10 (TA being the total duration of the four-pulse model) appears in the third and fourth places of the label. Three sets of relative values of P1, P2, P3 and P4 chosen for the study are: (1) 1, -1, 1, -1; (2) 1, -2, 2, -1; (3) 1, -2, 1, -1. This is represented by the digit in the fifth place of the label. D_m represents the maximum ground displacement of these models. Elastic displacement spectra due to these model accelerograms are calculated. The mean square difference (σ_x) between the elastic displacement spectra due to these model accelerograms and their parent ground motions is shown in Figure 3. The correlation indicates that smaller the σ_m smaller the σ_x will be. Therefore, the models with minimum σ_m are used to represent their parent ground motions. These models are EF1522, ES1522 for El Centro ground motion, and EF1511, PS1511 for Pacoima Dam ground motion. The time histories of the F-series model accelerograms are shown in the upper right corner of Figure 4 for these two ground motions. The velocity and the displacement time histories can be obtained by simple integration and are not shown here.

Figure 4, also shows the comparison between the FAS of the model accelerograms (EF1522, PF1511) and the FAS of their parent ground motions. Because of large underestimation of FAS of PF1511 in the long period range, a second model is used to better represent the energy delivered. This model accelerogram is denoted by PF4011 which matches the long period content of Pacoima Dam ground motion more closely.

Both the elastic and the inelastic displacement spectra are calculated due to these model accelerograms, for both the F- and the S- series models. These spectra are shown in Figure 5 for structures with 2% of critical viscous damping and with elastic-perfectly plastic restoring force characteristics. The yield level (X_y) ranges from infinity down to 0.5 inch to cover a broad range of structures including those designed according to Uniform Building Code. The displacement spectra due to the model accelerograms are close to those due to the parent ground motions in both the elastic and the inelastic range.

APPLICATION OF THE MODEL ACCELEROGRAM

A design model accelerogram, DS1522, is constructed by matching its FAS with an elastic design spectrum (Figure 6) with 2% viscous damping as obtained from Reference (2). This elastic design spectrum is normalized to have the same spectrum intensity as that of El Centro*2 within the period range 0.5 to 3.0 seconds. The resulting time history of DS1522 and its FAS are shown in Figure 7. Both the elastic and the inelastic displacement spectra are calculated due to DS1522 and are shown in Figure 8 for various X_y values. The displacement spectra due to El Centro*2 are also shown for comparison. Inelastic design displacement spectra as constructed from the given elastic design spectrum according to a currently used procedure given in Reference (3) for the period range 0.5 to 3.0 seconds are also shown in Figure 8 for comparison with those due to DS1522. It is noticed that the displacement spectra due to DS1522 are closer to those due to the actual ground motion (El Centro*2) than those calculated from the procedure of Reference (3).

CONCLUSION

A procedure for constructing model accelerograms of short duration is presented. The model consists of four-rectangualr pulses which have 8 unknown parameters. These parameters are determined by a minimization process based on matching the Fourier Amplitude spectrum of the model in a certain period range with that of the Velocity Response spectrum of an actual ground motion. In this study only 2 parameters are obtained from the minimization process and the others are examined parametrically. The results indicate that generally the model with least minimization error also gives the closest displacement spectrum. Therefore, theoretically all the parameters can be generated from one minimization process and a yet better model can be obtained.

Two actual ground motions (El Centro 1940 NS and Pacoima Dam 1971 S14W) are used for examples to construct the model accelerograms. Generally the duration of the model accelerogram is about 1.5 seconds for structures with period in the range from 0.5 to 3.0 seconds. When large inelastic behavior of structures is expected, the long period content of ground motion becomes important and should be matched. This results in a model accelerogram with duration extending to about 4.0 seconds as is in the case of Pacoima Dam ground motion. Generally both the elastic and the inelastic displacement spectra due to the model accelerograms are quite close to those due to the actual ground motions.

The scope of this study was limited to single-degree-of-freedom systems and to a few actual accelerograms. Nonetheless, it is shown that a short duration model accelerogram can be obtained by a rational procedure which retains certain important characteristics of an expected ground motion. Such models can be conveniently used in the preliminary design stage of structures to predict maximum displacements in elastic and inelastic range and also to include effects such as overturning due to gravity forces.

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TABLE 1 Model Accelerograms of El Centro * 2 and Pacoima Dam

S-series Models of Real Accelerograms

quake	model	GA	T1	T2	T3	T4	σ_m	D_m
El Centro	ES1512	42.39	0.220	0.530	0.530	0.220	17.28	2.05
	ES1522	40.41	0.594	0.460	0.257	0.189	9.96	10.71
	ES1532	53.81	0.414	0.666	0.224	0.196	11.66	9.23
	ES2012	39.52	0.458	0.792	0.542	0.208	19.86	8.30
	ES2022	29.89	0.779	0.610	0.350	0.260	13.96	13.61
	ES2031	33.25	0.689	0.543	0.582	0.186	19.39	11.85
	ES2512	26.47	0.516	1.027	0.734	0.223	20.19	7.04
	ES2522	25.12	1.050	0.795	0.398	0.257	16.89	20.79
	ES2532	27.13	0.698	1.198	0.368	0.236	15.39	13.23
	ES3022	20.82	0.270	0.451	0.971	1.308	18.89	26.74
Pacoima	PS1511	118.30	0.038	0.414	0.712	0.336	12.51	16.61
	PS1532	110.50	0.381	0.784	0.246	0.089	12.41	16.04
	PS2011	100.80	0.242	0.684	0.758	0.316	21.42	13.82
	PS2022	64.32	0.660	0.676	0.452	0.212	13.86	21.01
	PS2032	72.80	0.209	0.821	0.527	0.443	15.44	17.28
	PS2511	80.10	0.247	0.804	1.003	0.446	33.73	19.91
	PS2522	54.13	0.777	0.800	0.582	0.340	27.38	24.53
	PS2532	59.05	0.351	1.106	0.600	0.444	25.94	17.98
	PS4011	49.68	0.343	1.240	1.657	0.760	51.23	34.16

F-series Models of Real Accelerograms

quake	model	GA	T1	T2	T3	T4	σ_m	D_m
El Centro	EF1512	73.45	0.394	0.573	0.356	0.177	19.51	11.39
	EF1522	49.98	0.591	0.427	0.248	0.234	14.93	13.09
	EF1531	55.09	0.543	0.391	0.403	0.164	16.06	12.17
	EF2012	52.56	0.519	0.769	0.481	0.231	26.01	14.14
	EF2022	39.72	0.778	0.555	0.333	0.334	19.16	19.19
	EF2031	42.45	0.740	0.525	0.522	0.212	24.72	17.45
	EF2512	39.20	0.717	1.052	0.533	0.198	27.14	20.13
	EF2522	30.26	1.043	0.739	0.384	0.334	23.14	24.68
	EF2532	35.45	0.718	1.089	0.355	0.339	22.46	18.26
Pacoima	PF1511	126.90	0.066	0.418	0.684	0.332	22.98	15.09
	PF1522	71.73	0.572	0.597	0.318	0.014	23.07	17.60
	PF1532	118.40	0.386	0.762	0.243	0.109	24.59	17.61
	PF2011	109.40	0.296	0.704	0.704	0.296	30.70	9.61
	PF2022	67.25	0.628	0.645	0.463	0.264	24.49	19.89
	PF2031	79.57	0.472	0.516	0.786	0.226	25.75	13.31
	PF2511	78.51	0.341	0.909	0.909	0.341	45.22	16.18
	PF2522	58.38	0.772	0.768	0.575	0.385	36.23	26.12
	PF2531	61.71	0.441	0.582	1.100	0.377	34.81	15.14
	PF4011	45.53	0.323	1.302	1.677	0.698	62.94	38.92

Note : the units are inch and second

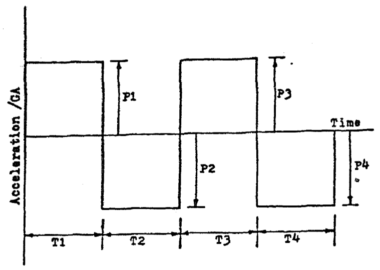


Figure 1. Four-pulse Model Accelerogram

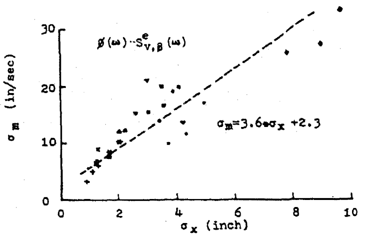


Figure 3. Correlation between σ_m and σ_x

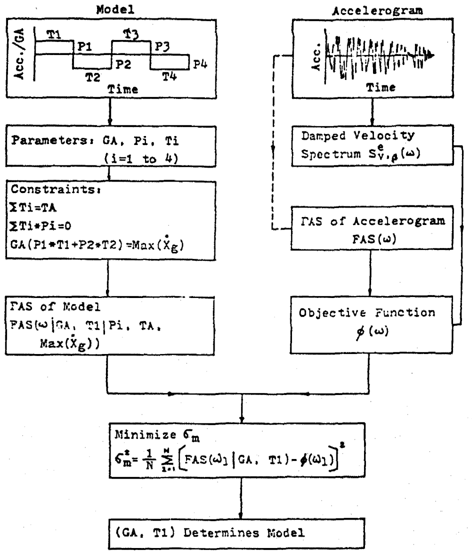
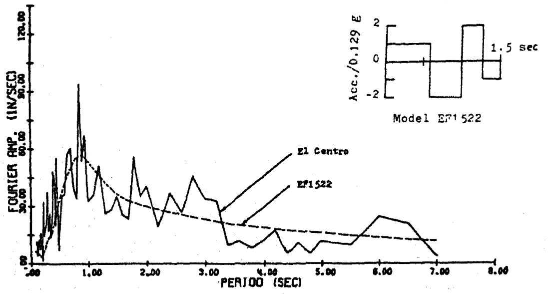
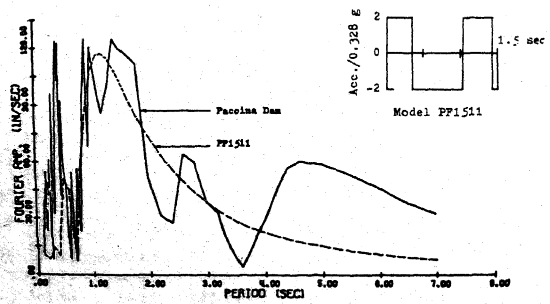


Figure 2. Flow Chart to Construct Model Accelerogram

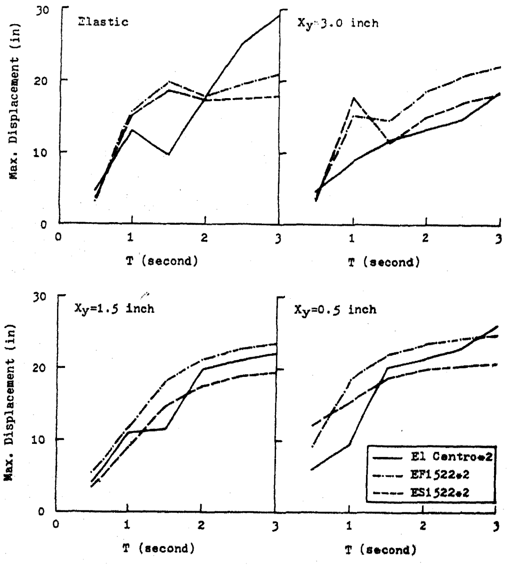


(1) El Centro Accelerogram



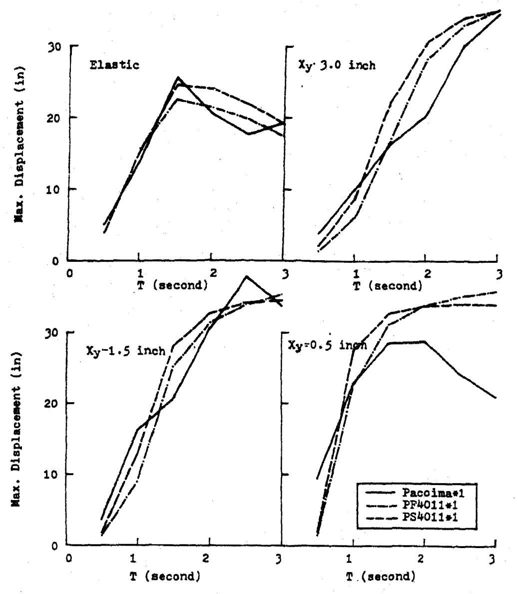
(2) Pacoima Dam Accelerogram

Figure 4 Fourier Amplitude Spectra of Parent and Model Accelerograms



(1) El Centro * 2

Figure 5. (TO BE CONTINUED)



(2) Pacoima Dam * 2

Figure 5. Displacement Spectra due to Real and Model Accelerograms.

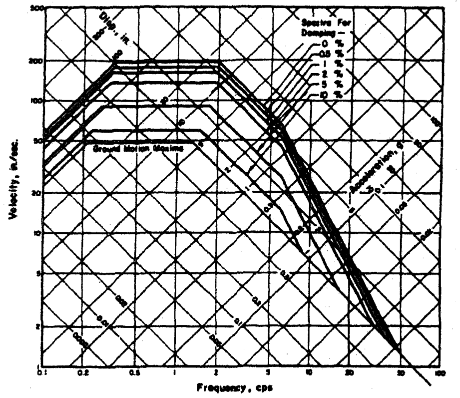


Figure 6. Basic Design Spectra (after ATC-2).

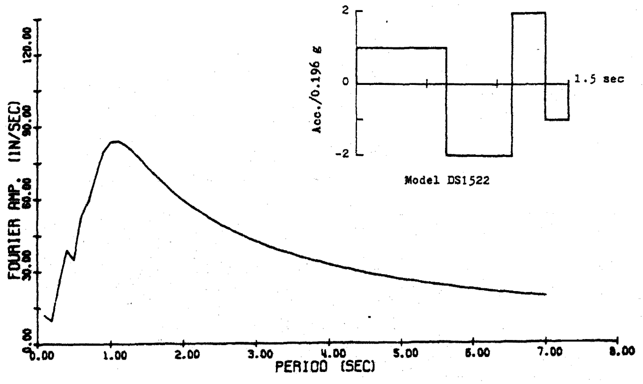


Figure 7. FAS of Model Accelerogram of Design Spectrum (DS1522).

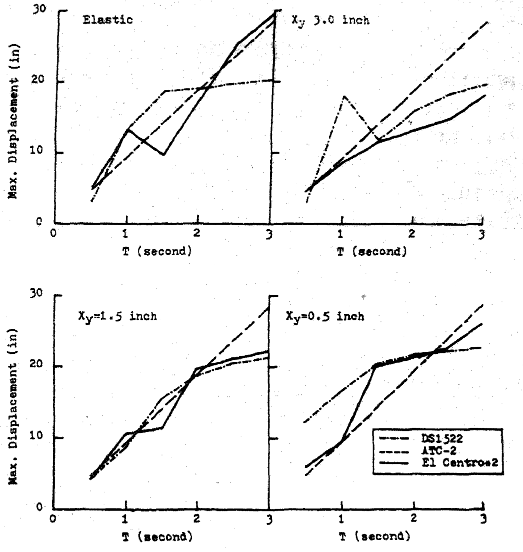


Figure 8. Displacement Spectra due to Design Model Accelerogram.