STATISTICAL MODELING OF SEISMIC DAMAGES [I]

by

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SYNOPSIS

Cost-benefit analysis in earthquake engineering expresses the terms related to seismic damage as the product of the amount of direct and indirect damages times the probability of their occurrence. On the basis of few historical records the paper points out the importance of the indirect damages (which are usually neglected) due to disruption of one or more urban functions, such as production, transportation systems, etc.. The paper works out a model to describe logical connexions among the different functions of an urban system hit by the earthquake. The fault tree technique is used to derive the probability of indirect damages (disruption of urban functions) on the basis of a given probability of collapse of a single structural system or building.

INTRODUCTION

The choice of an appropriate seismic coefficient can be seen as the result of an optimization process where monetary costs connected with seismic occurrences and the expected number of victims are involved [1], [2], [3]. Consistently with this approach the problem is stated as follows :

$$\min_{V \leq K} (C_c + C_D P_D + D_I P_I) \qquad (1)$$

where C_c is the cost of the building as constant investment per unit time, V the expected number of victims and K its limiting value. The term C_D is usually seen as the cost of structural repairing or re-building while P_D is the probability of the building failure mode causing C_D taking place due to seismic events in the unit time. The terms C_I and P_I indicate similar quantities for indirect damages, i.e. loss of serviceability of one or more urban functions. Most of the literature on seismic risk analysis is concerned with the evaluation of P_D and C_D; usually the terms C_I and P_I are disregarded. Sometimes they are accounted for by means of an amplification factor on C_D that is felt to be appropriate, [1], [2].

Two questions arise: 1) to what extent indirect damages C_I are relevant with respect to C_D and 2) what kind or relationship holds between P_I and P_D. Statistical data to answer question 1) are scarse and scattered. As an example for Italian earthquakes since 1962 (Irpinia 1962, Belice 1968, Viterbo 1971, Ancona 1972) the Italian Ministry of Public Works reports a loss of 506.2 billion lire of which 423.1 billions for structural damages and 83.1 connected with "first aid" services (ratio 20%). During the S. Fernando earthquake, damage to Los Angeles city was reported [4] to

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be 170 mil. dollars for structural damage and 50 mil. dollars for personal property and inventory damage (ratio 22%). Summing up the two addenda deduced by the above examples (which, however, are only two of the many items contributing to indirect damages) a figure of C_I/C_D = 42% can be obtained. As far as the second question is concerned, a basic distinction holds. Probability P_D depends on the structural behaviour of a given building (or system of buildings) while P_I is connected to the function of the building (or system of buildings), hence it may also be determined by the structural behaviour of different systems of buildings. As a matter of fact, loss of serviceability in a building may be due to loss of communication, loss of public utilities, etc. even if the structure of the building survived the seismic shock. Therefore, to evaluate P_I we need to investiagate the structural reliability of an urban system. The fundamental urban functions (dewelling, transportation and communications, electrical and water supply systems, agriculture, services) must be taken into account and for each of them the path (or the paths) leading to their loss of serviceability, must be considered. In the following this is attempted by means of the "fault tree" technique [5], [6] which has been already used in system reliability. This procedure enables the probability of loss of urban serviceability to be expressed in terms of the seismic vulnerability of single urban components. Moreover it becomes possible to make explicit the brittleness of the overall urban system with respect fo its basic components and to perform a more rational evaluation of indirect damages connected with its disruption.

URBAN FUNCTIONS AND RELATED FAULT TREE

In the following the disruption of a given urban function will be denoted as a "top event". In turn, it depends on a number of events which can be grouped in two classes: derived events and basic events. The latter group consists in events which can be considered as the sources of the overall process leading to the top event; intermediate phases of this process are denoted as derived events. The explicit formulation of the logical connections between the top event and the basic events is called the fault tree for the top event. Logical operators involved are of two types: intersection (x) and union (+). Suppose that event A is connected with two basic events B and C. If the first operator is applied, event A occurs if B and C arise simulaneously, in the second case event A occurs if B or C take place. Basic and derived events show a mutual interaction so that the top event is the result of a complex logical chain which may produce synergical effects. This means that the disruption of a given urban function is not necessarily determined by the "collapse" of the weakest link. The choice of basic events is arbitrary in principle: however risk analysis for seismic actions implies basic events whose occurrence depends on the behaviour or single items (buildings, equipments, etc.) under earthquake loads.

In fig. 1 the fault tree for productive activity is represented. This fault tree may be considered as typical for the analysis of the vulnerability of an urban system; in fact by neglecting some parts of it, other urban functions can be recovered. For example, if "telecommunications" (basic event E_{11}) are disregarded the fault tree for medical service or hospitals is obtained; again if the branch headed by "labour" is neglected it depicts the process leading to the loss of serviceability of the residential function. In the case of the fault tree represented in fig. 1 it turns out that the top event can be expressed in the following way :

$$TOP = E_1+E_2+E_5+E_6+E_8+E_{10}+E_{17}+E_{18}+E_{19}+E_{20}+ \\ + E_{12}xE_{14}+E_{13}xE_{14}+E_{16}xE_9+E_{16}xE_{14}+E_{15}xE_9 \quad (2)$$

The above expression is obtained by means of set theory algebra whose basic relationships are listed below :

$$E_i x E_i = E_i + E_i = E_i + E_i x E_k = E_i$$

By inspection of fig. 1 and eq. (2) it can be observed: a) basic events E_3, E_4, E_7, E_{11} do not affect the top event; b) each one of the basic events (E_1), (E_2), (E_5), (E_6), (E_{10}), (E_{17}), (E_8),(E_{18}) (E_{19}),(E_{20}) may produce the loss of serviceability of the urban function considered,by acting alone; c) the same effect can be obtained by any one of the combinations $(E_{12}xE_{14})$, $(E_{13}xE_{15})$,$(E_{16}x E_9)$, $(E_{16}xE_{14})$, $(E_{15}xE_9)$. The events in brackets are called cut-sets for the fault tree.

The probability $P^{TOP}(I)$ that the top event occurs due to an earthquake of intensity$\geqslant$I is now sought. Eq. (2) allows to express P^{TOP} as a sum of probabilities of the above cut-sets. As far as the probability of single basic events is concerned, it must be noted that the analysis of damage from past earthquakes shows an uneven distribution of seismic effects on buildings and infrastructures. Moreover, a little correlation holds between damages to a particular building and to other buildings of the same type. This leads to the assumption of considering basic events as statistically independent events. As a consequence the following equations hold :

$$P(E_i x E_j) = P(E_i) \cdot P(E_j) \\ P(E_i + E_j) = P(E_i) + P(E_i) - P(E_i) \cdot P(E_j) \quad (3)$$

SOME NUMERICAL RESULTS

The probabilities of the basic events are derived from a cumulative distribution function (C.D.F.) of the random variable I(earthquake intensity) and therefore the same is true for the top event.

The object of this section is to show some relationship between the C.D.F. of the input (basic events) and the output (top event), with reference to the fault tree of fig.1. Let us subdivide the basic events into three groups each corresponding to a class of vulnerability: 1) events (1,2,5,8,9,12,13,14,15) having the same probability $P_1(I)$; 2) events (16,19,20) having a lower $P_2(I)$; 3) extreme events (6,10,17,18) with probability $P_3(I)$. Fig. 2 shows the above mentioned C.D.F.'s when they are normal and their expected values m_i have the relationship :

$$m_3 = 2\ m_2 = 4\ m_1 \quad (4)$$

The coefficient of variation is assumed v = 0,5 for all. Remark that m_1 represents the average intensity at which the building is expected to be out of service (most likely the expected yielding point of the structure in earthquake intensity units). The C.D.F. of the top event is indicated as $P^{TOP}_{0,5}$ in fig. 2 (*). It is evident that

(*) The derivation of top event probability has been performed by the computer code discussed in [7]. Thanks are due to Dr.Reina for cooperation.

$P_{0,5}^{TOP} > P_1$ for any I, and it turns out to be basically of normal type with an expected value :

$$m_{TOP}^{0,5} = 0,4\ m_1 \qquad (5)$$

If an other value of v = 0,2 is assumed, $P_{0,2}^{TOP}$ turns out to be closer to the structural risk exposure P_1 with :

$$m_{TOP}^{0,2} = 0,6\ m_1 \qquad (6)$$

A sensitivity analysis of the result m_{TOP} with respect to variations of the mean values of the statistical distribution of the basic events can be carried out. In the particular case under consideration one obtains as a maximum: $\partial m_{TOP}/\partial m_1 = 15,6\%$. Considering the latter results and comparing eq. (5) to (6) we may deduce that the ratio m_{TOP}/m_1 is rather stable and depends more on the logical structure of fault tree then on the numerical input data.

CONCLUDING DERIVATIONS

The mean value m_1 can be considered as a design parameter related to the design seismic coefficient. Therefore each term in eq. (1) can be expressed as a function of m_1:

$$\min_{m_1} \left(C(m_1) + C_D \cdot P_D(m_1) + C_I P_I(m_1) \right) \qquad (7)$$

$$V(m_1) \leqslant K.$$

Calling $P_S(I)$ the probability that the earthquake intensity will exceed I in the unit time one obtains:

$$P_D(m_1) = \int_0^{\infty} P_S(I)\ f_1(I)\ dI \cong P_S(m_1) \qquad (8)$$

where f_1 is the density function of P_1. Using the same approximation,

$$P_I(m_1) = \int_0^{\infty} P_S(I)\ f_{TOP}(I)\ dI \cong P_S(m_{TOP}) = P_S(B.m_1) \qquad (9)$$

The above numerical results give for B a value around 0.5 (see eqs.(6), (5)). In other words: in the realm of this case study the term accounting for indirect damage can be expressed by the cost of indirect damages times the probability of occurrence of an earthquake having intensity one half of that for which the building is designed.

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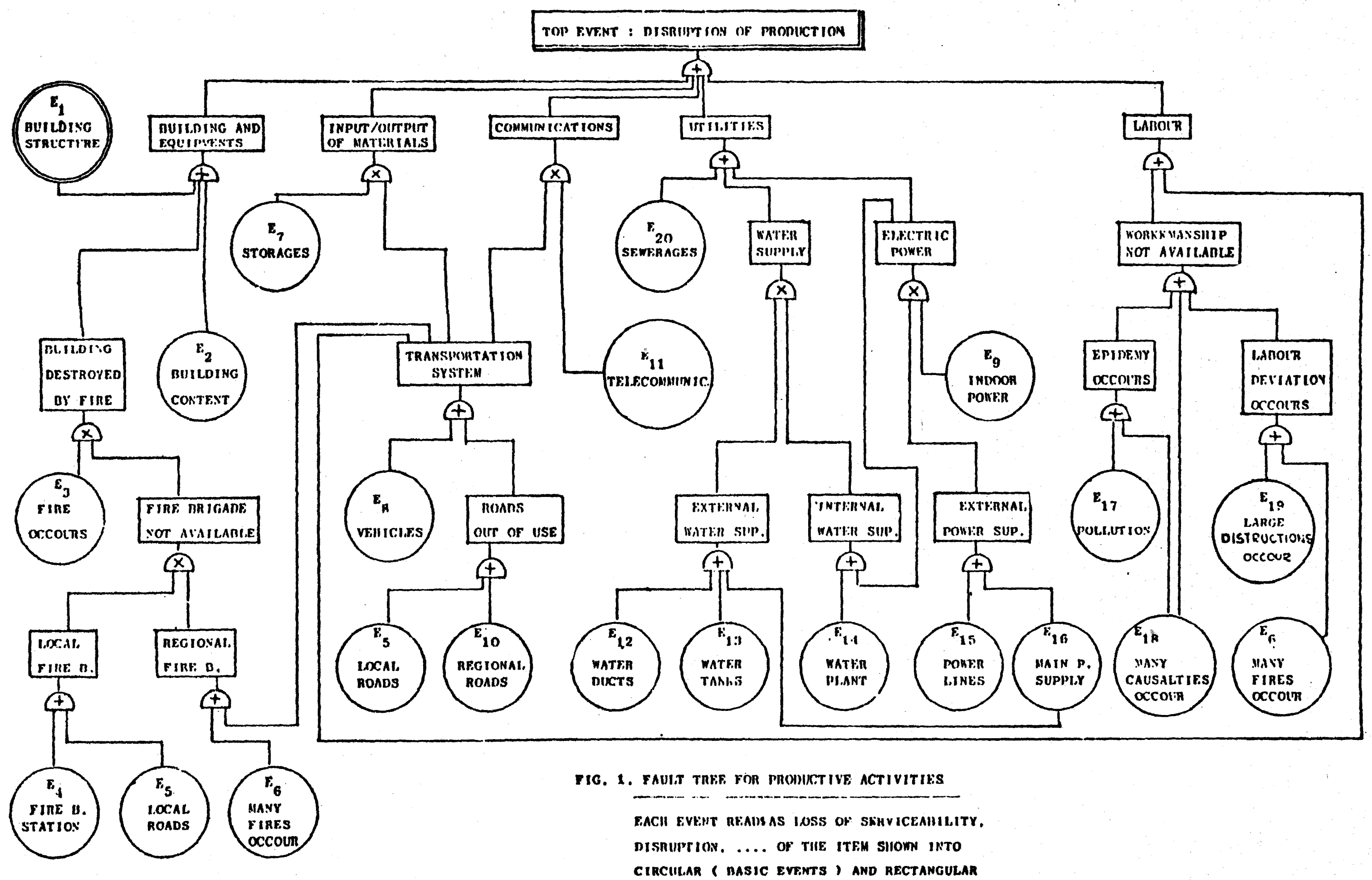

FIG. 1. FAULT TREE FOR PRODUCTIVE ACTIVITIES

EACH EVENT READS AS LOSS OF SERVICEABILITY, DISRUPTION, ... OF THE ITEM SHOWN INTO CIRCULAR (BASIC EVENTS) AND RECTANGULAR (DERIVED EVENTS) FIGURES. EXCEPTION ARE SPECIFIED.

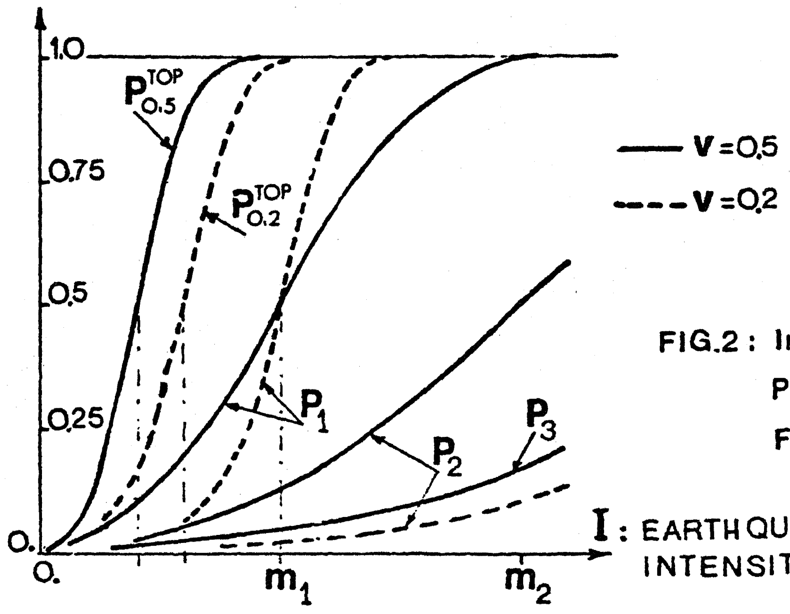

FIG.2 : Input/Output Probability Functions

DISCUSSION

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What are the constraints of the basic minimisation problem ?

Author's Closure

Not received.