AN APPLICATION OF DECISION THEORY IN SEISMIC ZONING[I]

by

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SYNOPSIS

Seismic zoning is considered as a problem of optimization of the distribution of the funds that a comunity devotes to the prevention of the seismic risk. It is shown that the optimum solution can be found through the repeated calculation of the optimum distribution over two sites. A special control of the model used for this calculation is carried out on the basis of decision theory, taking into account the uncertainties in the quantitative definition of the hypotheses.

INTRODUCTION

The problem considered in this paper is the optimization of the distribution over a seismic area of the funds that the community devotes to the prevention of the seismic risk. In the seismic codes, the formula giving the intensity of the lateral forces contains in general many coefficients to which the same intensity is proportional. Let C_i be the coefficient, at the point i, depending on local seismicity. As a first approximation we consider an area where only local seismicity differs from point to point, while all the remaining conditions (local soil conditions, shape of the response spectrum, social and economical conditions, characteristics of the buildings, etc.) are assumed as constant. Our problem, now, is the optimization of the distribution over the considered area of the coefficients C_i.

Let G be the total monetary cost per year of seismic risk prevention over the area (considering both the cost of seismic design and the cost of future damage due to earthquakes). The following principle is here accepted: the distribution of the coefficients C_i must minimize the expected number of victims over the area for a given value of G; i.e. starting from the optimum distribution of the coefficients C_i it must be impossible to reduce the expected number of victims maintaining constant the total cost G. If the area is divided into n small portions which can be considered as points, we face the pro-

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blem:

$$\min\left[\sum_{1}^{n}{}_{i}\ V_i\ (C_i)\right]$$

$$\text{with constr. } \sum_{1}^{n}{}_{i}\ D_i\ (C_i) = G \tag{1}$$

where V_i and D_i are respectively the expected number of victims and the total monetary cost of seismic risk prevention at the point i per year. The optimum distribution $\overline{C}_1 \ldots \overline{C}_n$ for a given G must then satisfy the following equations:

$$\begin{cases} V_i'\ (C_i) + \lambda D_i'\ (C_i) = 0 \\ \sum_i D_i\ (C_i) - G = 0 \end{cases} \qquad (i = 1 \ldots n) \tag{2}$$

where the prime is the symbol of derivative and λ is a Lagrange's multiplier.

Consider one particular point, for instance point 1, as reference for all other points. It is easy to prove that the distribution $\overline{C}_1 \ldots \overline{C}_n$ satisfies the eq. (2) if and only if each couple $\overline{C}_1$, $\overline{C}_k$ (k = 2 .... n) satisfies the equations:

$$\begin{cases} V_1'\ (C_1) + \lambda D_1'\ (C_1) = 0 \\ V_k'\ (C_k) + \lambda D_k'\ (C_k) = 0 \\ D_1\ (C_1) + D_k\ (C_k) = G - \sum_{2}^{n}{}_{j \neq k}\ D_j\ (\overline{C}_j) \end{cases} \tag{3}$$

In fact, if $\overline{C}_1 \ldots \overline{C}_n$ is the solution of eq. (2), the eq. (3) are identically satisfied by each couple $\overline{C}_1$, $\overline{C}_k$. Viceversa, if each couple $\overline{C}_1$, $\overline{C}_k$ satisfies eq. (3), the first set of n equations (2) is evidently satisfied; moreover, summing up the last equations (3) for k = 2 ....n, we verify that the last eq. (2) is satisfied.

It is also useful to point out that, if all the buildings at the sites 1 and k have the same characteristics, the solution $\overline{C}_1$, $\overline{C}_k$ of the problem (3) does not depend on the number of persons living at the two sites. In this case, in fact, if we multiply by two arbitrary numbers n_1 and n_k the population of the two sites, the optimum solution is:

$$\min\left[n_1\ V_1\ (C_1) + n_k\ V_k\ (C_k)\right]$$

$$\text{with constr. } n_1\ D_1\ (C_1) + n_k\ D_k(C_k) = n_1\ D_1(\overline{C}_1) + n_k\ D_k\ (\overline{C}_k); \tag{4}$$

and it is immediately verifiable that $\overline{C}_1$, $\overline{C}_k$ is the solution of the problem (4) too.

With reference to a standard building it is then possible to obtain the different optimum distributions $\overline{C}_1 \ldots \overline{C}_n$ corresponding to different values of G in the following way. Fix an arbitrary value for $\overline{C}_1$ at the reference site. For each k = 2...n,

eliminate λ from the first two eq. (3) and find $\overline{C}_k$. Calculate $\overline{G} = \sum D_i(\overline{C}_i)$. The set of values $\overline{C}_1$, $\overline{C}_k$ (k = 2 ....n) is the optimum distribution for the total cost $\overline{G}$.
In conclusion the optimum distribution of the coefficients C_i over n sites can be found through the repeated calculation of the optimum distribution over two sites. As the number of persons at each site is arbitrary, V_i and D_i can be expressed for instance in victims/year/person and dollars/year/person.

CALCULATION OF THE OPTIMUM DISTRIBUTION FOR TWO SITES

The seismicity of a site is here represented by the following correlation between the maximum ground acceleration a_m (measured in terms of g) and the return period T (measured in years):

$$\ln T\,(a_m) = \frac{1}{\nu} \ln a_m - \frac{\mu}{\nu} \quad , \tag{5}$$

where μ, ν depend on the site.
As a numerical example, two sites A and B will be considered with the following coefficients. Site A : $\mu = 3.270$, $\nu = 0.4$; site B : $\mu = 2.365$, $\nu = 0.4$.
A standard building is considered at both sites, defined by the following characteristics. The additional construction cost due to seismic design, d_1, expressed as per cent of the construction cost of the building, is: for $C \leq 0.01$, $d_1 = 0$; for $C \geq 0.05$, d_1 is a linear function of C defined by its values in C = 0.05 and in C = 0.1; for $0.01 \leq C \leq 0.05$, d_1 is a cubic function which connects the preceeding branches with regularity conditions in C = 0.05.
The peak ground acceleration a_m^c corresponding to the collapse of the building is given by the following "failure correlation":

$$a_m^c = \alpha + \beta C \tag{6}$$

The cost of direct damage (per cent of construction cost) caused by an earthquake, d (a_m, C), will begin to be appreciable when a_m rises above $a_1 = \gamma + C$, and will grow in proportion with a_m until reaching 100 per cent when $a_m = a_m^c$. Then the annual cost of damage, d_2, in per cent of the construction cost of the building, is:

$$d_2(C) = \int_0^\infty -d(a_m, C) \frac{d\,\frac{1}{T}}{d\,a_m}\, d\,a_m = \frac{100\,\nu\, e^{\mu/\nu}}{(1-\nu)\,(a_m^c - a_1)} \left(a_1^{\frac{\nu-1}{\nu}} - a_m^{c\,\frac{\nu-1}{\nu}}\right) \tag{7}$$

The indirect costs d_3 are taken into account as a percent ξ of direct cost:

$$d_3 = \xi\, d_2 \;.$$

As far as the expected number of victims is concerned, the assumption is made that V will be proportional to the number of failures F:

$$V = q \cdot F \;,$$

where the coefficient q is arbitrary, provided it is the same at both sites. In fact the value of q has no influence on the solution of eq. (3).
From the described model we obtain the expressions of V and of the total annual cost d^* in per cent of the construction cost:

$$V = q\, e^{\mu/\nu} (a_m^c)^{-1/\nu} \tag{9}$$

$$d^* = i\, d_1 + d_2 + d_3 \tag{10}$$

where $i \cdot d_1$ is the annual cost of d_1.
In the numerical example, assume the values of the coefficients listed in the table I. Translate then costs and damage into dollars/year/person based on the following data: accomodation of 25 m^2 per person; Italian market prices for 1975; 1 dollar = 850 liras; capital investment at 10 per cent p.a.; nominal life of the building > 50 years. Using the conditions (1) for the sites A and B, for each value of G it is possible to derive a couple of values C_A and C_B; i.e. the ratio C_A/C_B as a function of C_A (fig. 1).
The numerical values assumed for the coefficients in Table I are rather uncertain. In a previous research[IV] it has been shown that the uncertainty about the coefficients α and β, defining the failure correlation, is the most important one as regards the reliability of the result of fig. 1.
A first control of the influence of this uncertainty on the final result can be carried out as follows. Consider a "pessimistic" failure correlation, marked 1 in fig. 2, and an "optimistic" one, marked 2. The "average" correlation represented with a dotted line corresponds to the values of α and β of Table I. The extreme correlations 1 and 2 correspond to the values:

$$\alpha_1 = 0.25 \qquad \beta_1 = 4$$
$$\alpha_2 = 0.5 \qquad \beta_2 = 8 \tag{11}$$

If the calculation of C_A/C_B is repeated on the basis of correlations 1 and 2, the results of fig. 3 are obtained (the dotted line is the same line of fig. 1, obtained from the average correlation). The variations in the ratio C_A/C_B appear rather small, but the control is not exhaustive. Consider for instance the correlation defined by $\alpha = \alpha_1 = 0.25$ and $\beta = \beta_2 = 8$, which is obviously included in the band between correlation 1 and 2. This new correlation could be proposed by the person who is pessimist as regards the strength of the building without

IV G. Grandori and V. Petrini: Comparative analysis of the seismic risk in sites of different seismicity, to be published in Int. Journal Earthq. Eng. and Struct. Dynamics.

seismic design and is optimist as regards the increase of strength due to seismic design. The results obtained from this correlation are marked 3 on fig. 3 and show not negligible variations of the ratio C_A/C_B. For a more complete control it is then necessary to consider all possible positions of the failure correlation in the band of fig. 2, through a probabilistic approach.

CALCULATION OF C_A/C_B BASED ON DECISION THEORY

Consider the coefficients α and β as independent random variables defined in the set Δ limited by (11) with the probability densities $p(\alpha)$, $p(\beta)$. Then the expected number of victims and the expected monetary cost are:

$$E(V) = \iint_\Delta V(C,\alpha,\beta)\ p(\alpha)\ p(\beta)\ d\alpha\ d\beta$$

$$E(D) = \iint_\Delta D(C,\alpha,\beta)\ p(\alpha)\ p(\beta)\ d\alpha\ d\beta$$

In these conditions the decision criterion which is generally accepted is:

$$\begin{aligned} &\min \quad E(V_A + V_B) \\ &\text{with constr.} \quad E(D_A + D_B) = \text{constant} \end{aligned} \qquad (12)$$

If there are no elements suggesting particular distributions of $p(\alpha)$, $p(\beta)$, it is spontaneous to assume that they are uniformly distributed in their intervals. In this case the conditions (12) lead to the values of C_A/C_B represented with the solid line in fig. 4, compared with the "deterministic" result of fig. 1 (dotted line in fig. 4) obtained from the average failure correlation. The differences are very small.
But even if we want take into account different distributions $p(\alpha)$, $p(\beta)$ the variations in respect of the "deterministic" result of fig. 1 remain rather small. Consider for instance the distributions $p(\alpha)$, $p(\beta)$ of fig. 5. The case I is the one already considered, leading to the results of fig. 4. The case II corresponds to the position, described before, of the person who has elements in order to be pessimist about α and optimist about β, and so on. All the distributions of fig. 5 lead to results which are contained in the band of fig. 6. The maximum variation in respect of the "deterministic" result based on the average failure correlation does not exceed 10%.

TABLE I

$d_1(0.05) = 2.45$	$d_1(0.1) = 6.95$	$\gamma = 0.05$	$\xi = 0.5$

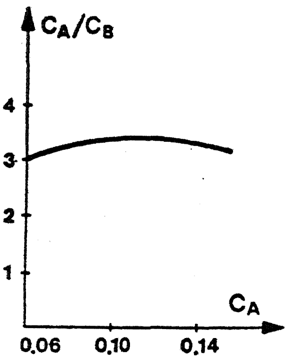

Fig. 1

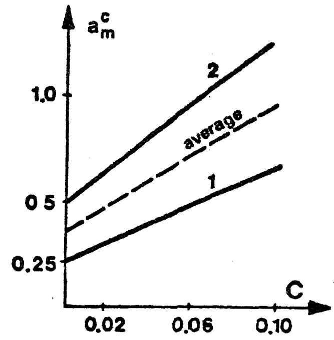

Fig. 2

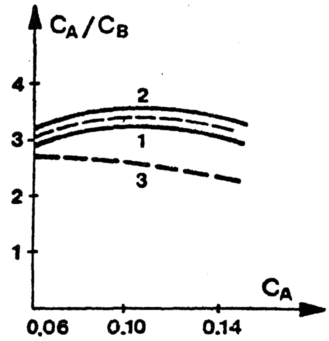

Fig. 3

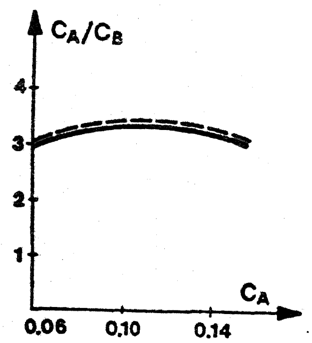

Fig. 4

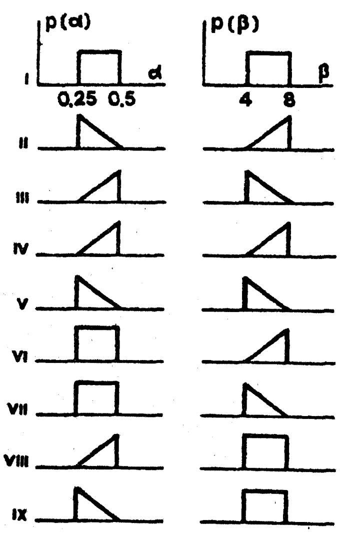

Fig. 5

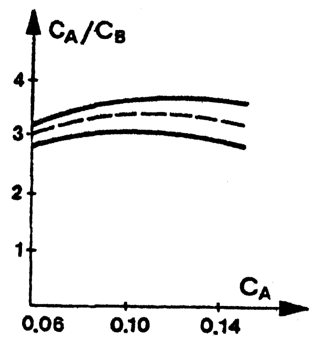

Fig. 6

DISCUSSION

Elizabeth Pate (U.S.A.)

In your minimization over the total loss, do you assume fixed the total investment to be allocated for seismic effects mitigation (Therefore fixing the upper limit of the gain* from such an investment), (this in itself implies an "acceptable" or "accepted" risk). How do you think this total investment can be rationally fixed in the global frame of a general public policy ?

Loss - Cost

Author's Closure

With regard to the question of Miss. Pate Elisabeth, we wish to state that at present we deal with the problem of the optimum distribution over a country of the resources devoted to seismic risk prevention. The choice of the total amount of these resources (i.e. the choice of the acceptable seismic risk) is a second step for which a wider approach becomes necessary. For the moment we cannot offer any rational suggestion for this choice.