THEORETICAL AND OBSERVATIONAL INVESTIGATION OF STRUCTURAL RESPONSE TO INSTANTANEOUS PEAK ACCELERATIONS DURING STRONG EARTHQUAKES

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S Y N O P S I S

A sinusoidal earthquake motion is considered with a sudden acceleration increment at its n-th period and the response of a one degree of freedom structure is theoretically investigated by successive application of the final conditions of one stage as initial conditions of the following stage. The results are compared with recorded strong motion accelerograms of this type and the corresponding acceleration and displacement response spectra. Some conclusions are derived concerning the validity of the theoretical results as well as the structural behaviour for transient forced vibration of this type.

1. Theoretical formulation

An ideal sinusoidal earthquake motion as indicated in Fig.1 is considered. Stages 1, lasting n periods, and 3 , have the same frequencies (ω) and amplitudes (α_1) while stage 2 occuring after stage 1 and consisting of only one period is showing the same frequency (ω) but an amplitude $\alpha_2 = \nu\alpha_1$ ($\nu>1$).

The equation of the motion of a single degree of freedom undamped system subjected to the above mentioned sinusoidal motion for the stage i, is :

$$\ddot{x}_i + \omega_0^2 x_i = \alpha_i \sin \omega t_i \qquad (1)$$

where ω_0 the natural frequency of the system.

The general solution of equation (1) is given by the formula :

$$x_i = \frac{A_i}{\cos \psi_i} \sin(\omega_0 t_i + \psi_i) + \frac{x_{st}}{1-\frac{\omega^2}{\omega_0^2}} \sin \omega t_i \qquad (2)$$

$$\dot{x}_i = \frac{A_i \omega_0}{\cos \psi_i} \cos(\omega_0 t_i + \psi_i) + \frac{x_{st}\omega}{1-\frac{\omega^2}{\omega_0^2}} \cos \omega t_i \qquad (3)$$

where $x_{st} = \frac{\alpha_1}{\omega_0^2}$ is the displacement of the system under a static load $\alpha_1 m$, and A_i, ψ_i constants depending on the initial conditions of each stage.

By the successive application of the final conditions of stage i as initial conditions to stage i+1, omiting in this description the intermediate operations, we obtain the following formulae for max x_i , occuring at time intervals where $\dot{x}_i = 0$.

$$\frac{\max x_1}{x_{st}} = \frac{1}{1-\frac{\omega^2}{\omega_0^2}} \left[-\frac{\omega}{\omega_0} \sin \frac{2k\pi}{1+\frac{\omega}{\omega_0}} + \sin \frac{\omega}{\omega_0} \frac{2k\pi}{1+\frac{\omega}{\omega_0}} \right] \qquad (4)$$

$$\frac{\max \chi_2}{\chi_{st}} = \frac{1}{1-\frac{\omega^2}{\omega_0^2}}\left[-\frac{\omega}{\omega_0}\nu tg\lambda_2 \sin(k\pi-\frac{\pi}{2}-\lambda_2) + \nu \sin\frac{\omega}{\omega_0}(k\pi-\frac{\pi}{2}-\lambda_2-\psi_2)\right] \quad (5)$$

where $tg\psi_2 = \dfrac{\sin \omega_0 nT}{\nu-1+\cos\omega_0 nT}$ (6)

and $tg\lambda_2 = \dfrac{\nu-1+\cos\omega_0 nT}{\nu \cos \psi_2}$ (7)

$$\frac{\max \chi_3}{\chi_{st}} = \frac{1}{1-\frac{\omega^2}{\omega_0^2}}\left[-\frac{\omega}{\omega_0} tg\lambda_3 \cos(k\pi-\lambda_3) - \cos\frac{\omega}{\omega_0}(k\pi-\lambda_3-\psi_3)\right] \quad (8)$$

where : $tg\psi_3 = \dfrac{\nu tg\lambda_2 \sin(\omega_0 T+\psi_2)}{\nu tg\lambda_2 \cos(\omega_0 T+\psi_2)+\nu+1}$ (9)

and $tg\lambda_3 = -\dfrac{1}{\cos\psi_3}\{\nu tg\lambda_2 \cos(\omega_0 T+\psi_2)+\nu+1\}$ (10)

Applying to formulae (4), (5) and (8) for different values of $\frac{\omega}{\omega_0}$, ν and n the curves showed in Fig.2 are obtained, giving the corresponding values of $\dfrac{\max \chi_i}{\chi_{st}}$

2. Comparison with real strong earthquakes

Two characteristic earthquakes of similar type with the above examined ideal sinusoidal earthquake motion are investigated.

a) Parkfield (U.S.A.) earthquake occured in June 27, 1966

From the accelerogram showed in Fig.3 we obtain α_1= 0,108 g, α_2= 0,50 g, $\nu = \dfrac{0,50}{0,108} = 4,63$, T = 0,692 , n = 3 .
From the corresponding acceleration and displacement response spectra (Fig. 4 and 5), we obtain for a structure with natural period $T_0 = 1_{sec}$, ω_0= 6,28 sec^{-1} max χ = 17 cm.

For this structure χ_{st} = 2,686 cm. From the curves of Fig.2 and for $\frac{\omega}{\omega_0} = \frac{T_0}{T} = \frac{1,00}{0,692} = 1,445$ we obtain $\frac{\max \chi}{\chi_{st}} = 6,33$ and max χ = 6,33X2,686 = = 17 cm.

b) Leukas (Greece) earthquake occured in November 4, 1973

Compared with Parkfield earthquake, having an almost constant amplitude at stages 1 and 3, Leukas earthquake is showing an irregularity of the variation of its amplitude (Fig.6). In order to obtain a sinusoidal form convenient for the application of the above theoretical formulae, the use of a Fourier analysis gave four harmónics with periods in sec., 0,415 , 0,83 , 1,25 , 1,66 , which coincides with the values resulting from the displacement and acceleration response spectra (Fig. 7 , 8)

For a structure with natural period $T_0 = 1_{sec}$, max χ results according to the following table :

T	ω	$\frac{\omega}{\omega_0}$	ν	n	α_1	χ_{st}	$\frac{\max \chi}{\chi_{st}}$	$\max \chi$
0,415	15,73	2,59	6,8	9	0,078g	0,31	3	0,93
0,830	7,56	1,30	2,1	4,5	0,053g	0,84	12	10,08
1,25	5,04	0,86	1,0	2,25	0,031g	1,11	6	6,66
1,66	3,79	0,65	1,0	1,125	0,0124g	0,787	2,25	1,77
							$\max \chi =$	19,44

The displacement response spectrum gives a value of max χ = 20 cm.

3. Conclusions

a) The above theoretical formulae are giving satisfactory results in cases of accelerograms of the type of figure 1,with almost constant amplitude in stages 1 and 3.

b) In cases of varying amplitudes a Fourier spectral analysis is recommended and the total max.displacement is the sum of max χ of each harmonic component.

c) The influence of the instantaneous acceleration peak to the structure, is limited during only the time interval of the acceleration increment, without having any effect on stage 3.

d) The sequence of occurence of peak acceleration does not affect considerably the overcharge of the structure, except of a range near resonance, where this influence is considerable.

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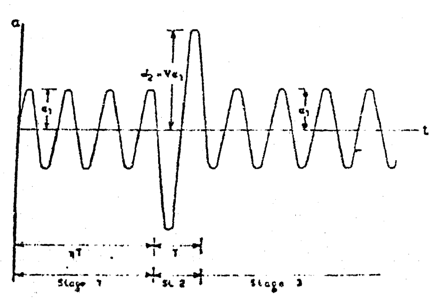
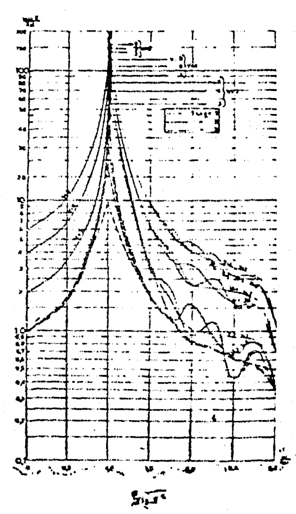
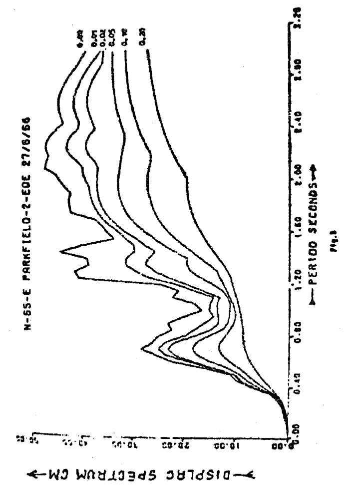


Fig.1

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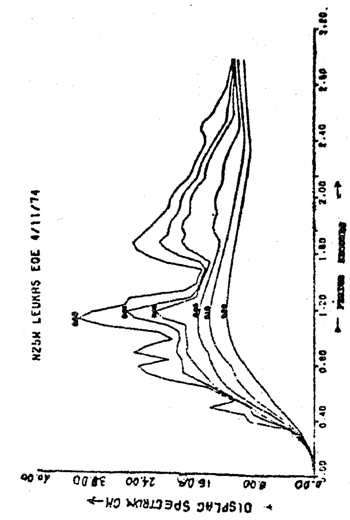
DISPLAC SPECTRUM CM
PERIOD SECONDS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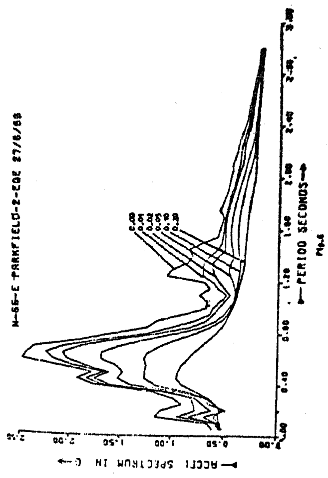
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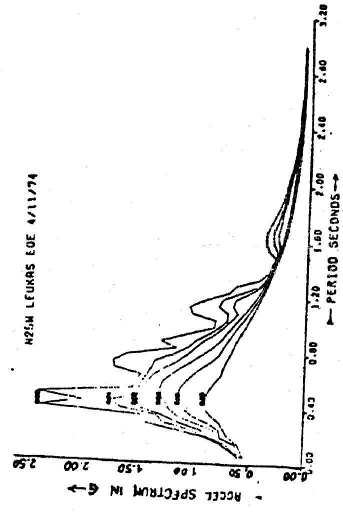
ACCEL SPECTRUM IN G
PERIOD SECONDS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ACCEL SPECTRUM IN G
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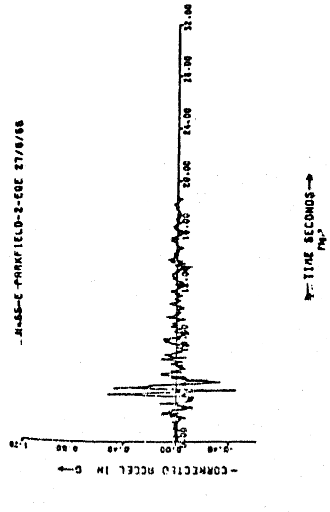
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CORRECTED ACCEL IN G
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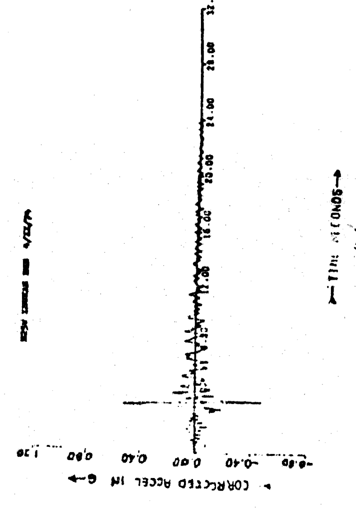
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