TRANSFER FUNCTIONS FOR SURFACE WAVES

by

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SYNOPSIS

An approximate technique is developed to calculate the surface wave transfer functions for propagation between two stations with different site conditions. The technique is applied to Love waves propagating in a two dimensional single-layered model of the soil containing a discontinuous change in the layer thickness. The displacement spectra produced by sudden dislocation near the free surface at two stations located on either side of the transitional zone are calculated and compared. The influence of the higher modes is shown to be highly significant, especially at higher frequencies.

INTRODUCTION

In most of the previous studies on the influence of local site conditions on earthquake ground motion it has been implicitly assumed that no significant seismic energy is propagated along the sedimentary layers and that the motion produced within the underlying bedrock can be adequately described by means of body waves with appropriate decay factors. It is difficult to justify the validity of these assumptions in shallow earthquakes where a significant amount of energy is trapped in the layers as surface waves. In order to improve the applicability of such studies the present authors are in the process of developing a linear system model which incorporates the transfer functions of both body and surface waves [1].

In an attempt to give a proper definition of the surface wave transfer functions and to understand the mechanics of energy transfer across transitional regions we consider a single layered model of the earth containing a sudden change in layer thickness caused by a step change in the elevation of the interface. We study the propagation of Love waves in the model, with special emphasis at high frequencies. The technique can be extended to multilayered media and Rayleigh waves.

In almost all of the previous studies on the propagation of surface waves in layers of varying thickness the influence of the higher modes have essentially been ignored. Since the higher modes become highly significant at high frequencies, the results of such studies cannot be directly used in earthquake engineering applications. We describe an alternate approximate technique which is more accurate at higher frequencies.

THEORY

Let H_1 and H_2 denote the layer thicknesses to the left and the right of the discontinuity. μ_1, β_1 are the shear modulus and the shear wave velocity of the layer while μ_2, β_2 are those for the basement complex. We introduce a

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cartesian system (x,y) at the surface on the left side of the step at a distance d from it. The x axis is horizontal, to the right and y-axis vertically downward.

At a given frequency ω let N_1, N_2 denote the number of Love modes that can exist in the layers of thickness H_1 and H_2 respectively. Then an incident Love wave with mode number ℓ, surface amplitude one and frequency ω will give rise to N_1 reflected modes and N_2 transmitted modes at the step. Let $A_{\ell m}$, $B_{\ell n}$ denote the surface amplitudes of the m^{th} reflected mode and the n^{th} transmitted mode. Then the spectral surface displacements to the left and the right of the discontinuity may be expressed in the form,

$$u_\ell(x,0,\omega) = e^{ik_\ell x} + \sum_{m=1}^{N_1} A_{\ell m} e^{ik_\ell d - ik_m(x-d)}, \quad 0 < x < d, \tag{1a}$$

$$= \sum_{n=1}^{N_2} B_{\ell n}\, e^{ik_\ell d + ik_n(x-d)}, \quad x > d, \tag{1b}$$

where k_ℓ, k_m, k_n denote the wave numbers of the incident, reflected and the transmitted modes and the phase shift of each wave has been taken into account. The second expression (1b) in the right hand side gives the amplitude and phase of the transmitted Love wave and will be called the modal transfer function for the ℓ^{th} mode. It is to be noted that although the incident wave amplitude is a constant (unity) the transmitted and reflected waves have amplitudes which vary with x.

The coefficients $A_{\ell m}$, $B_{\ell n}$ cannot be calculated exactly. It can be shown, by using the orthogonality properties of the Love modes and the continuity of the displacement across x = 0, that,

$$A_{\ell m} = \int_0^\infty \mu(y,H_1)\ U_m(y,H_1)\ \left\{V_\ell(y) - U_\ell(y,H_1)\right\} dy \Big/ R_m(H_1)$$

$$B_{\ell n} = \int_0^\infty \mu(y,H_2)\ U_n(y,H_2)\ V_\ell(y)\, dy \Big/ R_n(H_2) \tag{2}$$

where

$$R_m(H) = \int_0^\infty \mu(y,H)\,[U_m(y,H)]^2\ dy$$

$$\mu(y,H) = \mu_1, \quad 0 < y < H$$

$$= \mu_2, \quad y > H$$

$U_m(y,H)$ is the Love wave mode shape in the m^{th} mode defined in [2] and $V_\ell(y)$ is the (unknown) displacement along the line x = 0, y > 0. We calculated $V_\ell(y)$ approximately by utilizing a technique similar to that used in [3]. The details are omitted. We only note that the accuracy of the approximation increases with increasing frequency.

We now introduce a sudden strike slip dislocation of magnitude D on a small fault segment $\delta\ell$, oriented at angle θ with the horizontal located at (0,h) in the layer. Then the Love wave spectral displacement produced at a site (x,0) is given by

$$u(x,0,\omega) = \sum_{\ell=1}^{N_1} A_\ell u_\ell (x,0,\omega) \qquad (3)$$

where u_ℓ is given in equation (1) and A_ℓ can be found in [2].

NUMERICAL RESULTS

The computations are carried out for the case, β_1 = 1 km/sec., β_2 = 1.8 km/sec., μ_2/μ_1 = 3.3, H_1 = 1 km., H_2 = 1.3 km or, H_1 = 1.3 km, H_2 = 1 km.

In Fig. 1 the amplitudes of the modal transfer functions given by (1b) are plotted for the case x = d. Each curve in Figs. (1a) and (1b) represents the amplification of the ℓ^{th} mode Love wave just after it crosses the discontinuity. The amplitude of a given mode approaches unity at very high frequencies. It is to be noted however that at any given frequency at least one mode is strongly affected by the step. The waves propagating from the thick end of the layer are more severely affected by the transition than those propagating from the thin side.

The spectral amplitudes of the Love waves produced at two sites located on either side of the step at a distance of 1 km from it are plotted in Figs. 2 and 3. The source parameters used in these computations are, d = 8 km, h = .25 km, and θ = 45°. Only the incident waves are plotted for the site located to the left of the discontinuity since the reflection coefficients $A_{\ell m}$ have a relatively insignificant effect on the motion. It can be seen that in either case the spectral amplitudes at the two sites differ considerably especially at high frequencies. These difference cannot be related to the contrast in the site properties in a simple manner. They are caused mainly by the transmission of and subsequent interference between the higher modes.

REFERENCES

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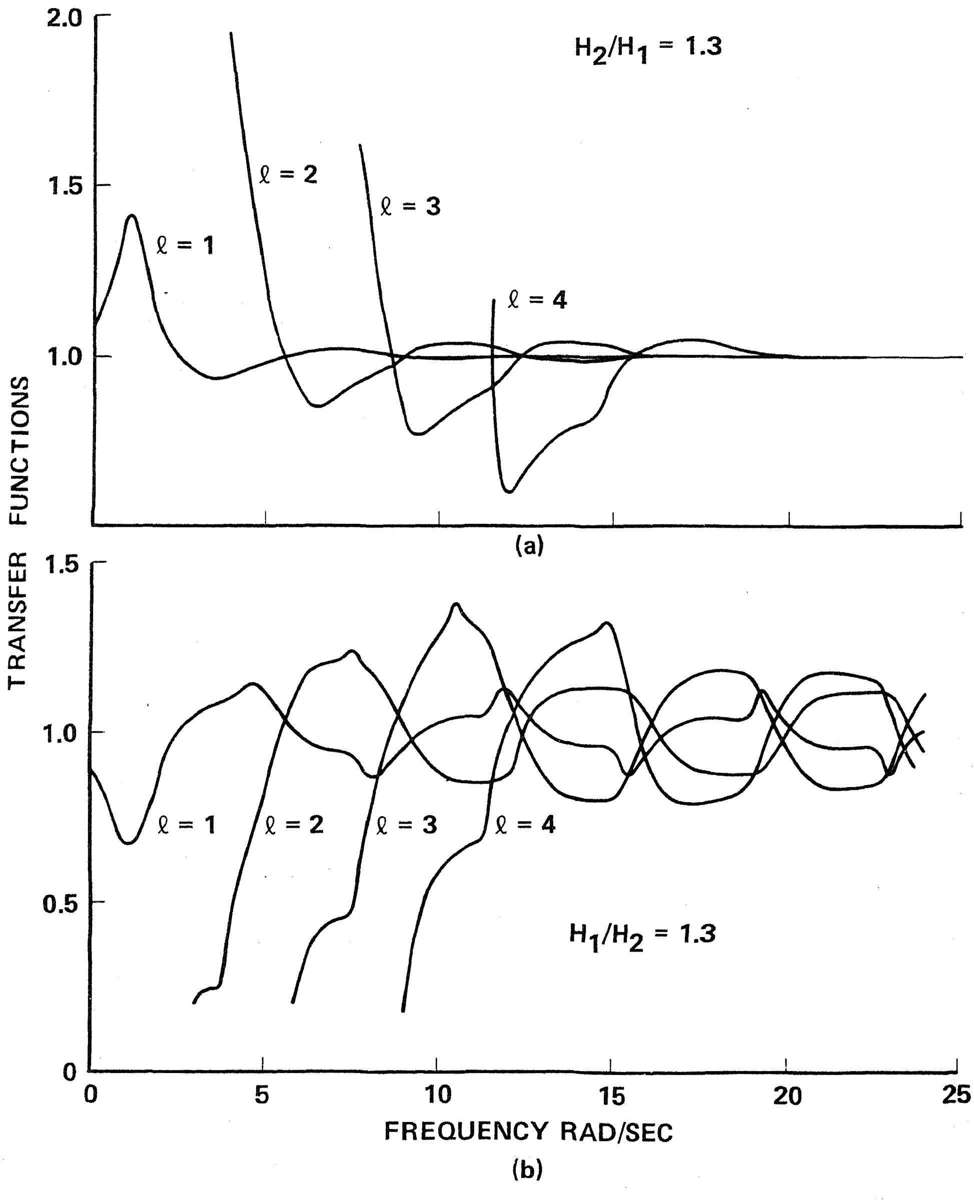

Figure 1. Modal Transfer Functions for Love Waves. ℓ is the Incident Mode Number.

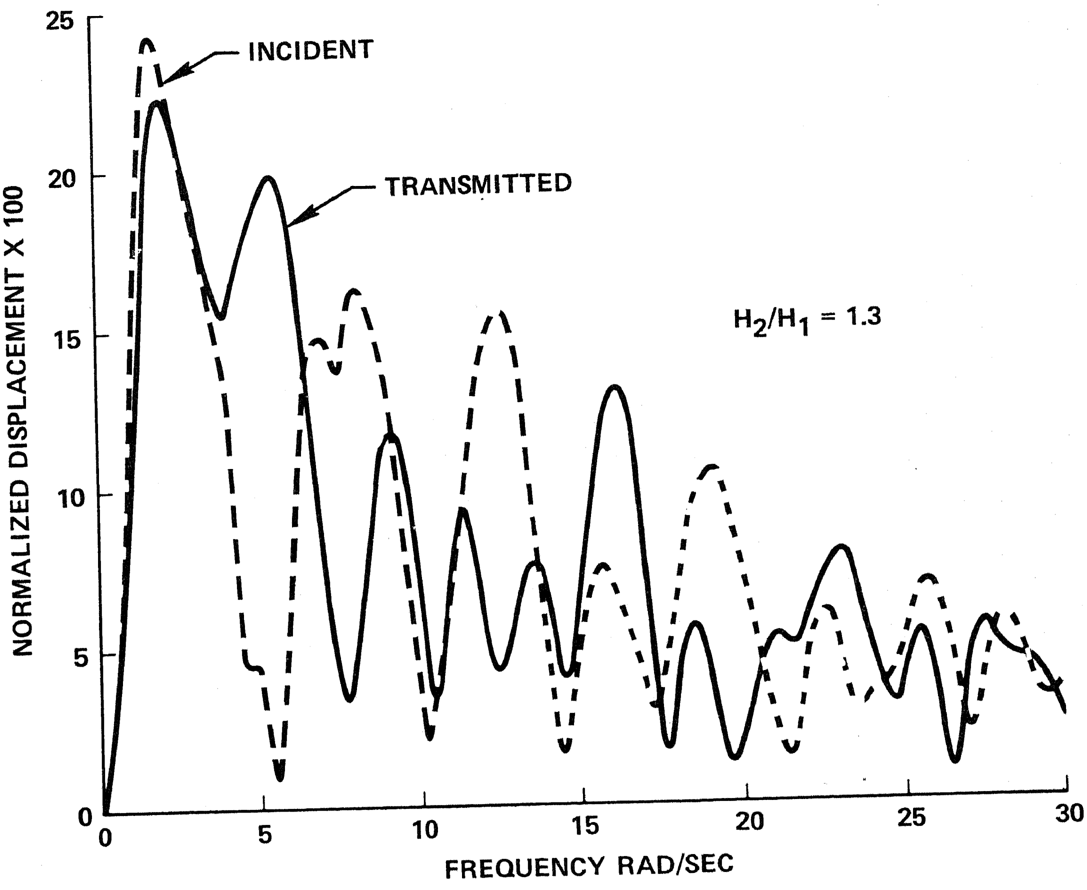

Figure 2. Normalized Love Wave Spectra $|\beta_1 u/D\delta\ell|$ Due to a Near-Surface Point Dislocation at Sites on Either Side of the Discontinuity.

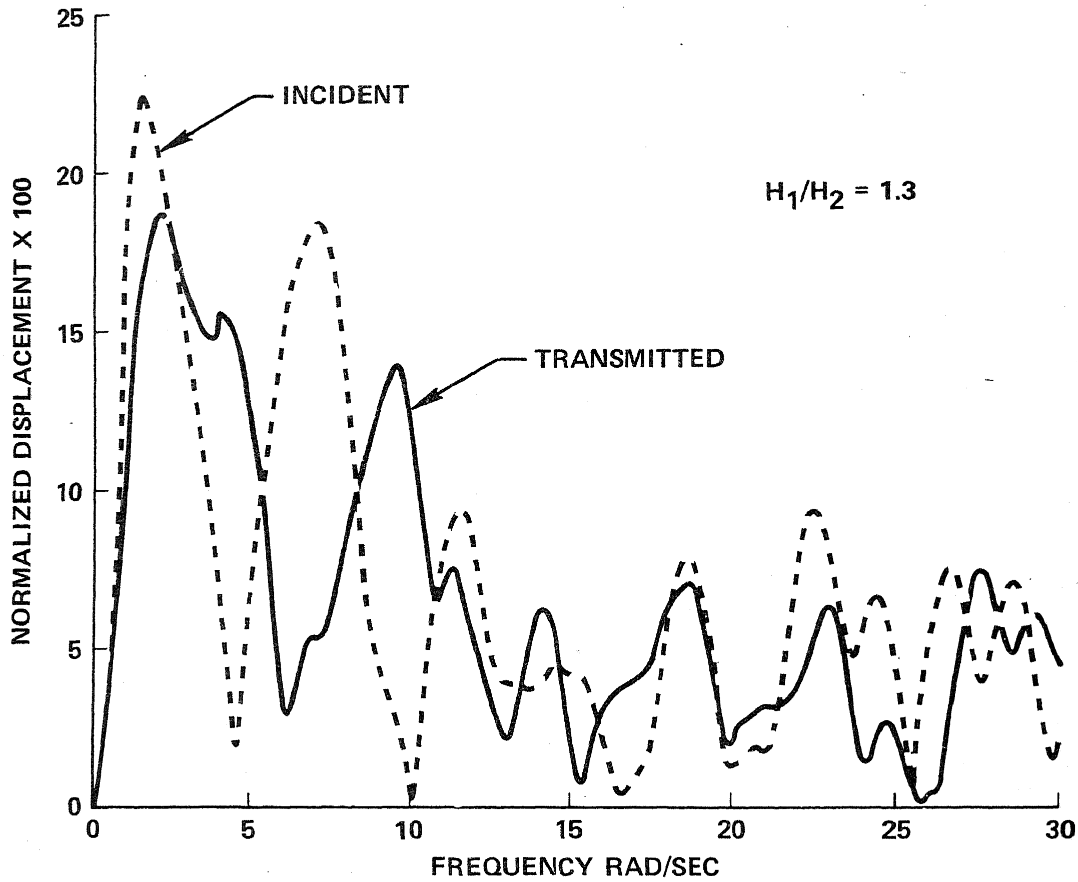

Figure 3. Normalized Love Wave Spectra $|\beta_1 u/D\delta\ell|$ Due to a Near-Surface Point Dislocation at Sites on Either Side of the Discontinuity.