DIRECT AND INDIRECT CONVERSION FROM POWER SPECTRA TO RESPONSE SPECTRA

by

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SYNOPSIS

The earthquake ground motion is considered as a time-parameter Gaussian stationary stochastic process defined by a power spectral density function, psd , $S(\omega)$. Based on the properties of the stochastic processes and on the behaviour of linear systems, one obtains the response spectra of a one degree of freedom system for the given $S(\omega)$. An iterative technique allows the conversion of a response spectrum into the corresponding power spectral density.

RESPONSE OF LINEAR SINGLE DEGREE OF FREEDOM SYSTEMS

Response spectra and power spectral density functions have been widely used to represent earthquake ground motion in terms of frequencies. However, the two concepts are entirely different. While the power spectral density is related to the Fourier analysis of the ground motion, the response spectra considers the peak response of a one degree of freedom system.

It seems very useful for reasons of comparison, to have some technique that allows the conversion of one representation into the other.

To compute the response spectrum $R(\eta, T_r, \omega_o)$ of a ground motion described by the time history of acceleration $\ddot{x}_o(t)$, $o \leq t \leq T_r$ one should proceed as follows(1)

$$R_{\alpha}(\eta, T_r, \omega_o) = \max_{o \leq t \leq T_r} \int_o^t h_{\alpha}(\tau)\, \ddot{x}_o(t-\tau)\, d\tau \qquad 1)$$

where:

- T_r - total duration of the recorded earthquake;
- $\ddot{x}_o(t)$ - one component of the base acceleration;
- ω_o - natural angular frequency of the oscillator;
- η - damping ratio of the oscillator;
- $h_{\alpha}(t)$ - unit impulse response of the oscillator relative to the displacement, velocity or acceleration (α= d, v, a).

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Assuming that $\ddot{x}_o(t)$ is a realization of a stochastic process, then R_α is a random variable with a suitable statistical distribution.

In the particular case that $\ddot{x}_o(t)$ belongs to a stationary Gaussian stochastic process with a power spectral density function $S(\omega)$, the power spectral density function of the response of a one degree of freedom oscillator, $S_{Z_\alpha}(\omega)$, under the action of $\ddot{x}_o(t)$[2] is

$$S_{Z_\alpha}(\omega) = \left| H \ (i\omega) \right|^2 S(\omega) \qquad 2)$$

where H_α is the complex frequency response of the oscillator. In case of absolute acceleration, $H_a(i\omega)$ has the form

$$H_a(i\omega) = \frac{\omega_o^2 + i\tau\eta\omega\omega_o}{(\omega_o^2 - \omega^2) + i\tau\eta\omega\ \omega_o} \qquad 3)$$

The peak or extreme value distribution of the response Z is approximately given by[3]

$$F_{Z_\alpha}(z) = \Pr\left[Z < z\right] = \exp\left\{-\frac{T}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_o}} \exp\left(-\frac{z^2}{2\lambda_o}\right)\right\} \qquad 4)$$

where

$$\lambda_{2n} = \int_o^\infty \lambda^{2n}\, S_Z(\lambda)\, d\lambda \qquad 5)$$

For the values common to the earthquake processes this distribution is very narrow. The mean value of Z which is close to the median and to the mode, and the variance of Z are approximately[4]

$$E\left[Z_\alpha\right] \simeq \left[2\ \lambda_o \ln\left(\frac{T}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_o}}\right) - \ln(\ln 2)\right]^{1/2} \qquad 6)$$

$$\mathrm{var}\left[Z_\alpha\right] \simeq 2\lambda_o\left[E_1\left(\frac{T}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_o}}\right) + \gamma - \ln(\ln 2)\right] \simeq 2\lambda_o\,(0.21068) \qquad 7)$$

where $E_1(x)$ is the exponential integral function and γ the Euler constant $\simeq 0.57721$[5].

In this paper, the response spectrum $E[R_\alpha]$ is viewed as the mean value of the extreme value distribution $E[Z_\alpha]$ of the response of a one degree of freedom system. One should emphasize at this point that extreme-value distribution given by expression 4) takes into account the randomness of the response for a fixed intensity of excitation measured through $S(\omega)$. It does not include the randomness in the intensity of excitation. For low risks ($<10^{-3}$) the final distribution considering both the randomness of intensity and the randomness of response, has to be computed using the distributions of intensity and response and no longer it is valid to consider only the distribution of intensity and the mean value of maximum response[6].

NUMERICAL IMPLEMENTATION

Response spectra using expression 6) were developed for any shape of $S(\omega)$. To compute the integrals in expression 5) a numerical technique was used[7],[8]: $S(\omega)$ is represented by $S'(\omega)$ which is described by the ordinates at n points $_i$ and linear segments between those ordinates. The expression 5) can be written as

$$\lambda_{2n} = \int_0^{\infty} \lambda^{2n} S'(\lambda)\, d\lambda = \sum_{k-1}^{n-1} \int_{\omega_k}^{\omega_{k+1}} \lambda^{2n} S'_k(\lambda)\, d\lambda \qquad 8)$$

where according to the definition of $S'(\cdot)$

$$S'_k(\omega) = \begin{cases} S(\omega_k)+(\omega-\omega_k)\; S(\omega_{k+1})-S(\omega_k)\;/(\omega_{k+1}-\omega_k) & \text{if } \omega_k \leq \omega < \omega_{k+1} \\ 0 & \text{elsewhere} \end{cases} \qquad 9)$$

From here onwards we work in terms of absolute acceleration. True values of spectral velocity and displacement can be obtained using the complex frequency response for velocity and displacement in expression 2).

The values of the integrals on the right hand side of 8) can be found in references [7],[8] and are not transcribe here.

Figure 1 illustrates the direct conversion of $S(\omega)$ into response spectra for power spectral density represented on the top right corner of the same figure. This psd is intended to represent the Taft earthquake (N-S, July 21/1952)[9]. The response spectra was drawn for an oscillator with 5% damping ratio and a 15 sec duration. It also shows the 5% and 95% fractiles of the extreme value distribution as well as the response spectra of Taft earthquake computed by expression 1)[10]. Agreement between the probabilistic model and actual values can be appreciated.

An iterative procedure developed to carry out the inverse transformation from response spectra $E[Ri]$ into power spectral density $S(\omega)$ can be found in detail in reference [11]. Briefly, one choose a set of frequencies $\omega_1 \ldots \omega_n$ with adequate bandwith given by

$$\omega_{i+1} = \omega_i (1 + 2\eta)$$

A flat spectral density (white noise $S(\omega_1)_1 = S(\omega_2)_1 = S(\omega_n)_1$ provides a first estimate of the psd. The spectral ordinates $E\ Z_{i1}$ are computed for the frequencies ω_i and compared with actual response spectra $E[Ri]$. Then the psd of acceleration in the neighbourhood of ω_i is multiplied by a factor proportional to the square root of $E[Ri]/E[Zi1]$, providing a new estimate of the psd of acceleration $S(\omega_1)_2, S(\omega_2)_2 \ldots S(\omega_n)_2$.

After a few iterations, j , one can get estimations $E[Z_{ij}]$ which are only 1% off the prescribed values $E[R_i]$. Figure 2 shows the inverse conversion from response spectrum into power spectral density. The crosses are the prescribed response spectrum and the full line the response spectra obtained for the computed power spectral density shown on the top right corner. The prescribed values of the response spectrum correspond to the mean values of four earthquakes given by Housner and Jennings(12).

Figure 3 presents the comparison between the four power spectral density of acceleration $S'_H(\omega)$ mentioned in conection with Figure 2 and the common representation of $S(\omega)$

$$S(\omega) = \frac{11.5\,(1 + \omega^2 / 147.8)}{(1 - \omega^2 / 242)^2 + \omega^2 / 147.8} \qquad S - cm^2/sec^3 \quad \omega - rad\ sec^{-1}$$

CONCLUSIONS

Based on some results from the theory of stochastic processes, the idealization of a strong motion earthquake as a sample of a time duration T , of a stationary Gaussian stochastic process with zero mean value and suitable power spectral density, allows to compute the response spectrum of a single degree of freedom system under the action of that earthquake.

Conversely, it is also possible, given a prescribed response spectrum to find a corresponding psd of acceleration.

In spite of the limitations, not discussed herein, of the model of earthquake ground motions used in this work (stationarity, normality, duration), the relationships presented have been used to improve design rules(13),(14) and are also useful in solving such problems as the influence of soil conditions on response spectra and the generation of time series with prescribed response spectra(11),(15).

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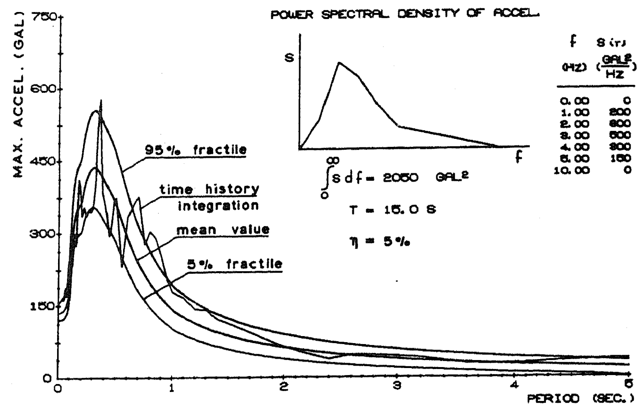

Fig. 1 - Transformation of the power spectral density of acceleration of Taft earthquake (N-S, July 21/1952) into a response spectrum. Comparison is made with the response spectrum computed directly from the time-history of acceleration.

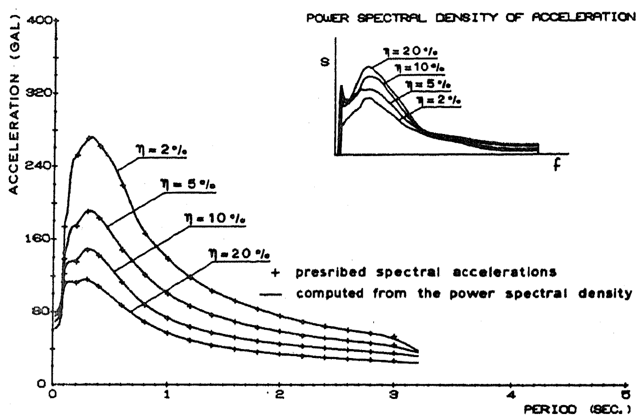

Fig. 2 - Transformation of a response spetrum into a power spectral density of acceleration, and verification of the inverse operation.

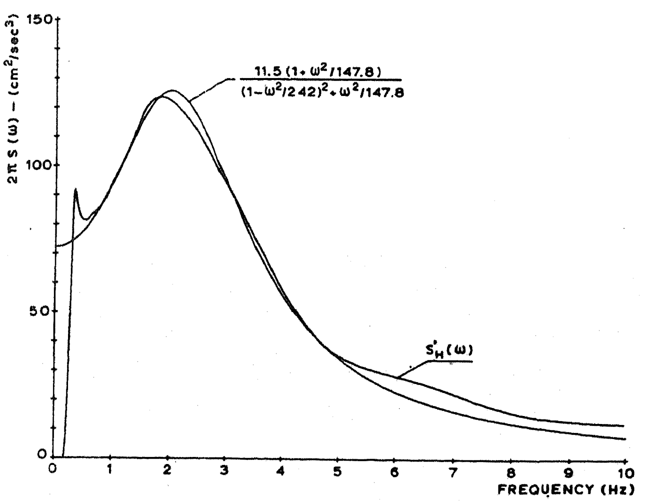

Fig. 3 - Comparison between the average of four power spectral density of acceleration $S'_H(\omega)$ and equation (15)

DISCUSSION

Rudolf Grossmayer (Austria)

The idea of converting a response spectrum into a power spectral density is a very fruitful one. As the discussor performed a similar procedure it would like to recommend the authors a correction of equation (4). It turns out that this equation does not describe the transient behaviour of the structural response, which is important for structures with long periods and relatively high damping values. In addition, it would like to ask the authors, if they have also tried to convert the response spectrum into a nonstationary spectral density ?

Author's Closure

With regard to the question of Mr. Rudolf Grossmayer, we wish to inform that unfortunately, it was not sufficiently emphasized in the paper that the mathematical analysis didn't contemplate the transient behaviour of the oscillator. Accordingly, equation 4, which is only valid for stationary processes, should not be corrected.

The subject of probabilistic non-stationary models for earthquake motion and associated transient response was dealt with in the paper "A PROBABILISTIC APPROACH TO THE STUDY OF LINEAR RESPONSE OF STRUCTURES UNDER MULTIPLE SUPPORT, NON-STATIONARY GROUND SHAKING", Preprint, Vol. 3, page 3-385, presented to this Conference.