ANALYSIS OF PROPAGATION OF SHOCK-WAVES BY FINITE-ELEMENT METHOD

by

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SYNOPSIS

A finite-element method is proposed to study the propagation of waves generated by man-made blasts in underground media. An infinite medium having a buried source with six types of forcing functions, and a semi-infinite medium, with or without layers, having the source either on the surface or buried, are evaluated.

INTRODUCTION

Mathematically the wave-propagation problem is an initial-value problem. It is approximated to a boundary-value problem by considering the conditions initially for the whole region and after a time interval Δt for the region which is yet to be traversed by the wave. The present formulation is based on a variational principle in which an energy functional, based on the displacement model, is defined. The expressions for the potential and kinetic energies are evaluated in terms of the admissible known functions to define a single-valued scalar energy functional, the variation of which in a given time-domain for a constrained conservative system under a dynamic loading, attains a stationary value leading to the well-known integro-differential equations for the system. The shapes of the elements to suit the geometry of the structure are selected and these are annular spherical-sector, annular cylindrical-sector, and the triangular, as shown in Fig.1.

FORMULATION

The governing equations of motion in final form are obtained by considering the variation of Lagrangian, L, which is the total energy of the system

$$L = T - (V + U) \qquad \ldots (1)$$

where T is the total kinetic-energy, U is the total strain-energy and V is the potential-energy of the external loads. For an annular spherical-sector element the strain-energy is,

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$$U_{sp} = \frac{1}{2} E' \iiint_V \left[(1-\nu) \left[\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{u}{r}\right)^2 \right] + 2\cdot\nu\cdot\frac{u}{r}\cdot\frac{\partial u}{\partial r} \right.$$

$$\left. + 4\cdot\nu\left(\frac{u}{r}\right)^2 \right] r^2 \, dr \, d\theta \, d\phi \qquad \ldots (2)$$

The kinetic-energy is $T_{sp} = \frac{\rho}{2} \iiint_V (\dot{u})^2 \, r^2 \, dr \, d\theta \, d\phi \quad \ldots (3)$

and the potential-energy is $V_{sp} = - \iint_S \left[p(r,\theta,\phi)\cdot u \right] r d\theta d\phi \quad (4)$

where u, v, w denote the displacements of a point in r, θ and z directions, respectively, in cylindrical-polar coordinates, or in r, θ and ϕ directions, respectively, in spherical coordinates.

And $$E' = E/(1+\nu)(1-2\nu) \qquad \ldots (5)$$

where E is the modulus of elasticity, ν is the Poisson's ratio and ρ is the mass density. Equations for energies for an annular cylindrical-sector element can also be derived as above. The variation of the Lagrangian, L, gives the equation of motion as

$$[M] \{\ddot{x}\} + [K] \{x\} = \{p\} \qquad \ldots (6)$$

where [M] is the mass matrix, [K] is the stiffness matrix and $\{\ddot{x}\}$ is the acceleration vector, $\{x\}$ is the displacement vector and $\{p\}$ is the load vector. The displacement function assumed for an annular spherical-sector element is,

$$u = \alpha_1 + \alpha_2 r \qquad \ldots (7)$$

while for an annular cylindrical-sector element are,

$$u = \alpha_1 + \alpha_2 r + \alpha_3 \theta + \alpha_4 r \theta \qquad \ldots (8a)$$

$$v = \alpha_5 + \alpha_6 r + \alpha_7 \theta + \alpha_8 r \theta \qquad \ldots (8b)$$

Stiffness matrices for the annular spherical-sector element and the plane-strain annular cylindrical-sector element are evaluated following a standard procedure.

For deriving the lumped-mass matrix the interpolation function ϕ is assigned a unit value over an area contributing to that particular node and zero values elsewhere. The expression for the pressure generated due to explosives is generally assumed as of an exponential form.

The governing equations of motion are solved using a step-by-step numerical integration procedure based on the assumption of a linear variation of acceleration between any two consecutive time-intervals. The time interval Δt should be such that the successive cycles of iteration converge and the resulting solution is stable and accurate. Wilson (1) and Lysmer (2) recommend $\Delta t \lesssim 0.10\ T_s$, where T_s is the shortest period of free-vibration of the structure. In wave propagation problems it is neither possible to find out the shortest period of vibration of the media, nor it is easy to obtain the shortest period of the wave in advance. Hence a convenient way to know whether the time-interval is sufficiently small to give numerical stability and the desired degree of accuracy is to attempt different time intervals and compare their results.

NUMERICAL WORK

The numerical cases considered are : (i) comparative study of the wave propagation in an infinite medium by a classical method and by finite-element technique, assuming a spherical cavity and having the following types of shock loadings.

$$p(t) = p_o,\ p(t) = p_o\, e^{-\omega t/\sqrt{2}},\ p(t) = p_o\, e^{-\sqrt{2}\,\omega t}$$

$$p(t) = p_o(1-e^{-\omega t/\sqrt{2}}),\ p(t) = p_o(1-e^{-\sqrt{2}\,\omega t}),\ \text{and}$$

$$p(t) = p_o(e^{-\omega t/\sqrt{2}} - e^{-\sqrt{2}\,\omega t})$$

(ii) propagation of stress-waves due to shock in semi-infinite medium with and without layers, and having surface/buried sources, (iii) influence of variation of depth of buried-source on the propagation of waves, (iv) evaluation of stress-fields developed in the media for all such cases.

NUMERICAL RESULTS

The displacements and stresses are found to converge rapidly as the element size is reduced. The time-interval and the velocity of wave propagation automatically fixes the one dimension of the element parallel to the direction of wave-propagation. These numerical results, shown in Figs. 2 confirm that the finite-element method predicts very satisfactorily the displacement-time histories.

BIBLIOGRAPHY

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2. Lysmer, J. and Darke, L.A. : A finite element method for seismology, Methods in Computational Physics, Vol. II, Academic Press, Inc. New York, pp 181-216, 1972.

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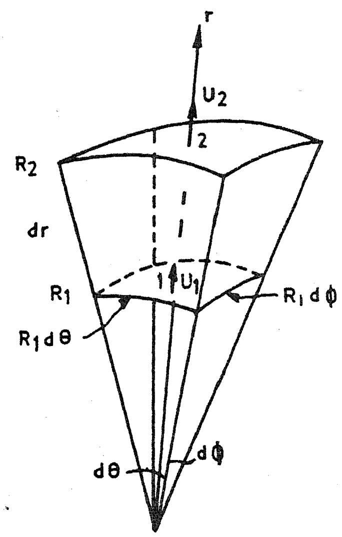
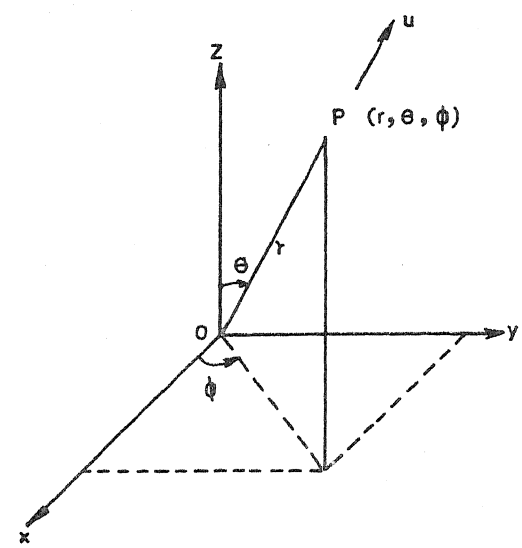
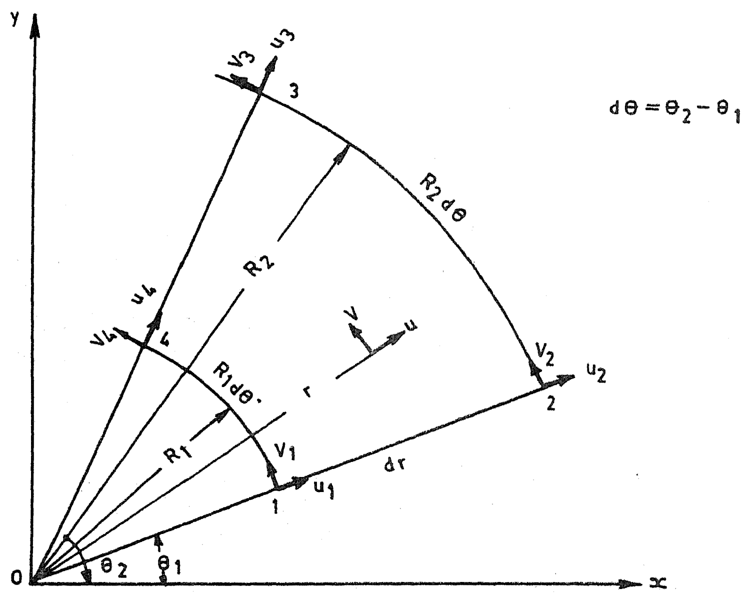


a) ELEMENT SHAPE AND NODE NUMBERS



b) POSITIVE DIRECTIONS IN SPHERICAL COORDINATES

FIG-1·1—AN ANNULAR SPHERICAL — SECTOR ELEMENT



GEOMETRY OF THE ELEMENT, NODE NUMBERS, POSITIVE DIRECTION OF NODAL DISPLACEMENTS IN CYLINDRICAL CO-ORDINATES

FIG-1·2—AN ANNULAR CYLINDRICAL — SECTOR ELEMENT

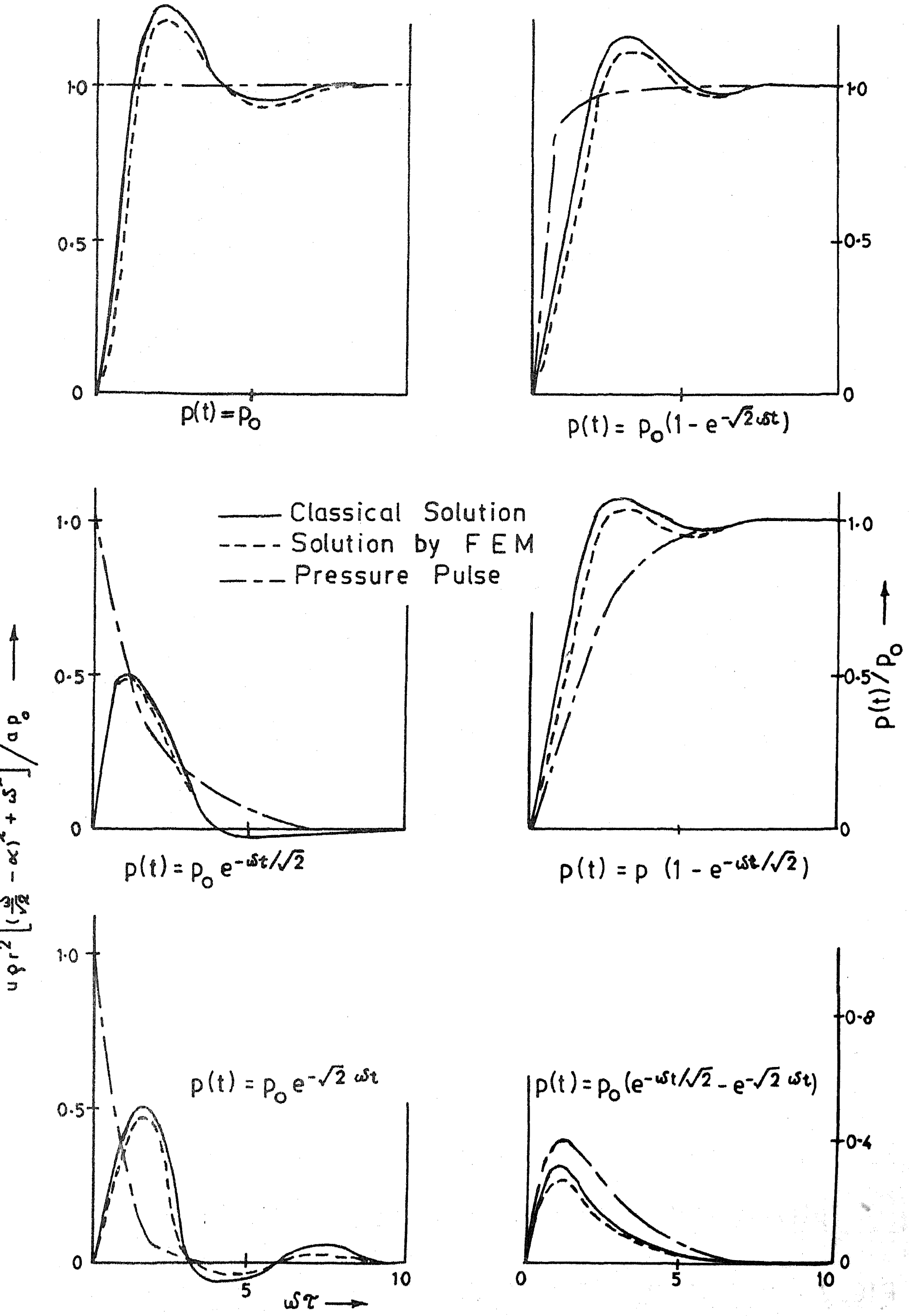
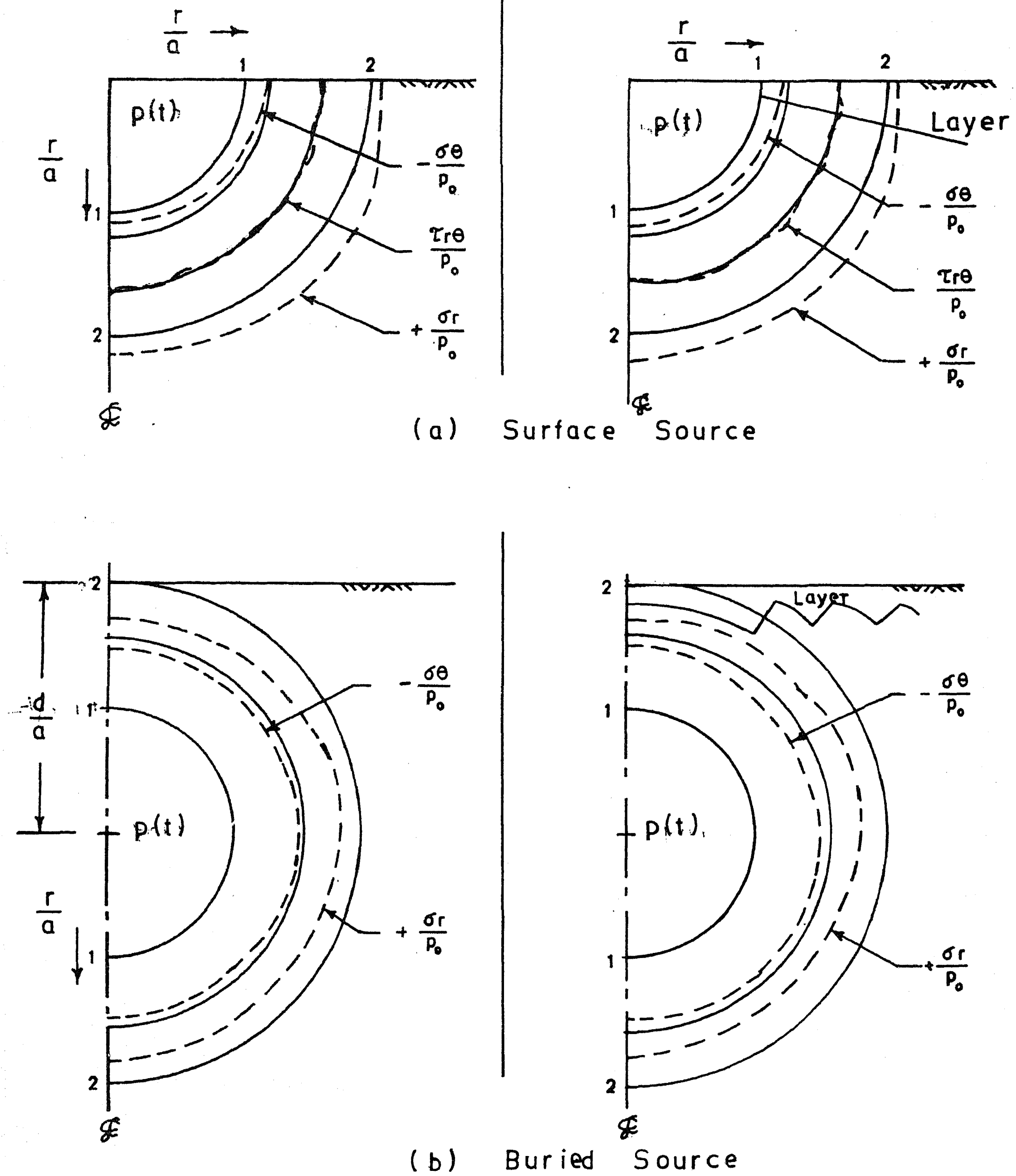


FIG. 2 – DISPLACEMENTS OF SURFACE OF SPHERICAL CAVITY IN AN INFINITE MEDIUM



$p(t) = p_o e^{-\omega t/\sqrt{2}}$, + Compression, - Tension

FIG. 3 - VARIATION OF STRESSES IN SEMI-INFINITE MEDIA, HOMOGENEOUS/LAYERED WITH SURFACE/BURIED SOURCE

DISCUSSION

D. Costes (France)

Is the perturbation, emitted in the half space by an hemispheric source, in dynamics or statics, given by a classical theory ?

Author's Closure

Not received.