WAVE PROPAGATION IN SOILS

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SYNOPSIS

The general theory of viscœlasticity suggests that the characteristics of S waves transmitted across a bedrock-soil or a soil-soil interface differ significantly from those that have been considered previously in the earthquake engineering literature. The general theory predicts that type-II S waves transmitted across such boundaries are in general inhomogeneous with velocities and attenuations that depend on the incident angle of the incoming wave, directions of maximum energy flow that differ from those of phase propagation, and energy flow and dissipation due to interaction of the waves. Current numerical models of soil response in general do not account for these physical characteristics of the waves. Numerical results derived for an alluvium-shale interface and a shale-granite interface suggest that these theoretically predicted characteristics are significant for problems concerned with the in-situ measurement of seismic amplitudes, but that they are probably not significant for seismic traveltimes measured using present technology.

INTRODUCTION

The anelastic behavior of earth materials plays an important role in changing the characteristics of seismic waves. Such behavior is especially important in earthquake engineering for defining the dynamic response of local geologic deposits for purposes of earthquake-resistant design.

The general physical characteristics of body waves in anelastic media inferred by Borcherdt[3] and Buchen[7] have been utilized to solve the reflection-refraction problems associated with a general (either homogeneous or inhomogeneous) P, type-I S, and type-II S wave incident on a plane boundary[5,6]. This paper summarizes these theoretical results for a type-II S wave incident on a plane boundary and presents numerical results for interfaces between alluvium, shale, and granite.

Previously, anelasticity has been considered for seismic waves by introducing an attenuation factor of the form $e^{-\alpha x}$ where $\alpha = \omega/2vQ$. Introduction of such a factor implicity implies that body waves are homogeneous and that except for attenuation and a weak dependence of velocity on frequency other physical characteristics of the waves are the same as those predicted by elasticity theory. These previously published assumptions will be referred to here as the low-loss approximation.

SUMMARY OF THEORETICAL CHARACTERISTICS

The physical characteristics of type-II S waves are described by expression derived[3,6] for the velocity, attenuation, mean energy flux, mean kinetic density, mean potential energy density, total mean energy density, velocity of energy transport, mean rate of energy dissipation per unit volume, loss in energy density per cycle, peak energy density stored during a cycle, and Q^{-1}. These expressions show that for inhomogeneous type-II S waves;

1) the phase velocity and maximum attenuation depend on the angle

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[I] Geophysicist, U.S. Geological Survey, Menlo Park, California 94025 between the directions of propagation and maximum attenuation, and that they approach zero and infinity, respectively, as the angle approaches 90 degrees,

2) the direction of maximum energy flow differs from that of phase propagation,

3) the mean kinetic energy density is in general not equal to the mean potential energy density,

4) the velocity of energy transport is not equal to the phase velocity, and

5) the mean rate of energy dissipation depends on the component of the mean intensity in the direction of the attenuation vector.

Each of these characteristics for type-II S waves differs from those usually assumed with the low-loss approximation for SH waves in anelastic media.

The preceding physical characteristics were utilized in deriving a theoretical solution to the problem of a general (either homogeneous or inhomogeneous) type-II S wave incident on a welded boundary between viscoelastic materials[6]. The solution, valid for boundaries between both elastic and anelastic media, shows that the reflected and refracted waves are both of type II and that their physical characteristics are significantly different from those previously derived with the low-loss approximation.

The anelastic attenuations of materials at most interfaces in the earth, such as at a soil-soil, soil-bedrock, or crust-mantle interface, are not the same. For such boundaries the theoretical solution[6] predicts that the transmitted wave is in general inhomogeneous for non-normal angles of incidence; the velocity and attenuation may approach zero and infinity, respectively, depending on characteristics of the incident wave; the direction and velocity of energy flow are not equal to those of phase propagation; and, in general, there are no critical angles of incidence, that is, there are no angles of incidence for which the transmitted wave propagates exactly parallel to the boundary. Each of these characteristics derived for body waves in anelastic media differ from those assumed as part of the low-loss approximation.

A detailed account of energy flow and dissipation[6] shows that the normal component of the mean energy flux is continuous across anelastic boundaries with the sum of the normal components carried to and from a boundary by the incident wave, the reflected wave, and the interactions of the incident and reflected waves being equal to that carried away from the boundary by the transmitted wave. In contrast, the low-loss approximation assumes no energy propagates toward or away from a boundary due to interaction of the waves.

NUMERICAL RESULTS

The theoretical results summarized here and presented in detail by Borcherdt[6] are readily adaptable to exact numerical calculations of the physical characteristics of body waves transmitted across interfaces between material in the Earth.

For illustration purposes a homogeneous S wave was assumed incident upon a plane interface separating alluvium and shale (see Fig. 1 for material parameters). Physical characteristics of the resulting transmitted and reflected waves have been computed as a function of the angle of incidence utilizing the general theory of anelasticity (Fig. 2).

The theoretical results predict that a homogeneous type-II S wave incident on an interface between alluvium and shale will transmit an inhomogeneous type-II S wave at all angles of incidence except for normal incidence. That is, for the transmitted wave a non-zero angle exists between the direction of propagation and maximum attenuation. The numerical calculations illustrate the magnitude of this predicted angle (see Fig. 2A) and show that this angle increases rapidly from 0 to an asymptotic value of 90 degrees as the angle of incidence increases. In contrast, the usual low-loss approximation assumes that the transmitted wave is homogeneous for angles of incidence less than some critical angle (that is $\gamma = 0°$) and for angles greater than this critical angle, γ becomes exactly 90°.

The reflection and transmission coefficients computed assuming low loss and those computed on the basis of general anelasticity are compared in figure 2B. The coefficients computed on the basis of the two theories are approximately the same for angles of incidence less than the critical angle defined by the low-loss approximation. However, for angles of incidence greater than this critical angle, the two sets of coefficients differ significantly. The general theory predicts that a finite amount of energy is transmitted as well as reflected from the boundary for all angles of incidence. In contrast, the low-loss approximation predicts that for angles of incidence greater than critical all of the incident energy is reflected.

Both the general theory and the low-loss approximation predict that homogeneity of the incident wave implies homogeneity of the reflected wave. Hence, the low-loss approximation predicts the angle of reflection and the specific attenuation factor for the reflected wave. This is substantiated by the numerical calculations (see Figs. 2C and 2E). However, since the general theory predicts that the transmitted wave is inhomogeneous, the low-loss approximation incorrectly predicts the transmission angle by as much as 8 percent, the phase velocity by as much as 0.5 percent, and Q^{-1} by as much as 50 percent (see Figs. 2C, 2D, and 2E). The numerical calculations show that the phase velocity of the transmitted wave does indeed depend on the angle of incidence as predicted theoretically, however, the dependence is small and not likely to be of consequence in most field experiments. The errors in calculating the transmission angle and specific attenuation factor with the low-loss approximation are more significant and are probably detectable with present technology (Figs. 2C and 2E).

Since the transmitted wave is inhomogeneous for all non-normal angles of incidence (Fig. 2A), it is interesting to consider how it interacts with another boundary. For illustration purposes, a boundary between anelastic shale and anelastic granite was presumed (see Fig. 1 for parameters of the materials). Physical characteristics of the waves reflected and refracted at the shale-granite boundary are shown in figure 3. The calculations for the general theory of anelasticity assume that the incident wave is the inhomogeneous wave transmitted across the overlying alluvium-shale boundary. The calculations based on the low-loss approximation assume that the incident wave is homogeneous.

The general theory predicts that the wave transmitted across the shale-granite boundary is extremely inhomogeneous for angles of incidence differing only slightly from normal incidence (Fig. 3A). For example at an angle of incidence of 10 degrees the general theory predicts that the angle between the directions of propagation and maximum attenuation is 78 degrees as opposed to an angle of zero predicted by the low-loss approximation. This large differen suggests that amplitude measurements based on the assumption amplitudes are constant along a plane of constant phase could be significantly in error.

For angles of incidence greater than critical, the transmission and reflection coefficients computed on the basis of the general theory differ significantly from those computed using the low-loss approximation (Fig. 3B). In addition, the calculations based on the general theory show that the percent of the energy reflected from the shale-granite boundary does not approach 100 percent as the angle of incidence approaches grazing incidence. The calculations predict that the maximum amount of energy reflected is less than about 95 percent. The remaining energy flux is in part associated with the transmitted wave and in part associated with the interaction of the velocity and stress fields of the incident and reflected waves. The coefficients show that the energy flux associated with the interaction of the waves increases significantly as grazing incidence is approached.

Errors associated with using the low-loss approximation to calculate other characteristics of the reflected and transmitted waves are shown in figures 3C, 3D, and 3E. The errors appear to be of most significance for waves propagating near the critical angle and near grazing incidence.

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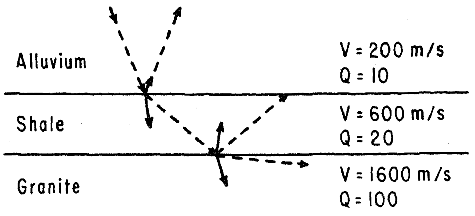

Fig. 1 Velocities and specific attenuation factors for homogeneous shear waves in alluvium, shale, and granite. The propagation vectors (dashed) and attenuation vectors (solid) are illustrated for the incident, reflected, and refracted waves. The wave, presumed incident on the anelastic alluvium-shale boundary, is a homogeneous type-II S wave. The wave, presumed incident on the anelastic shale-granite boundary, is the inhomogeneous type-II S waves transmitted across the alluvium-shale boundary.

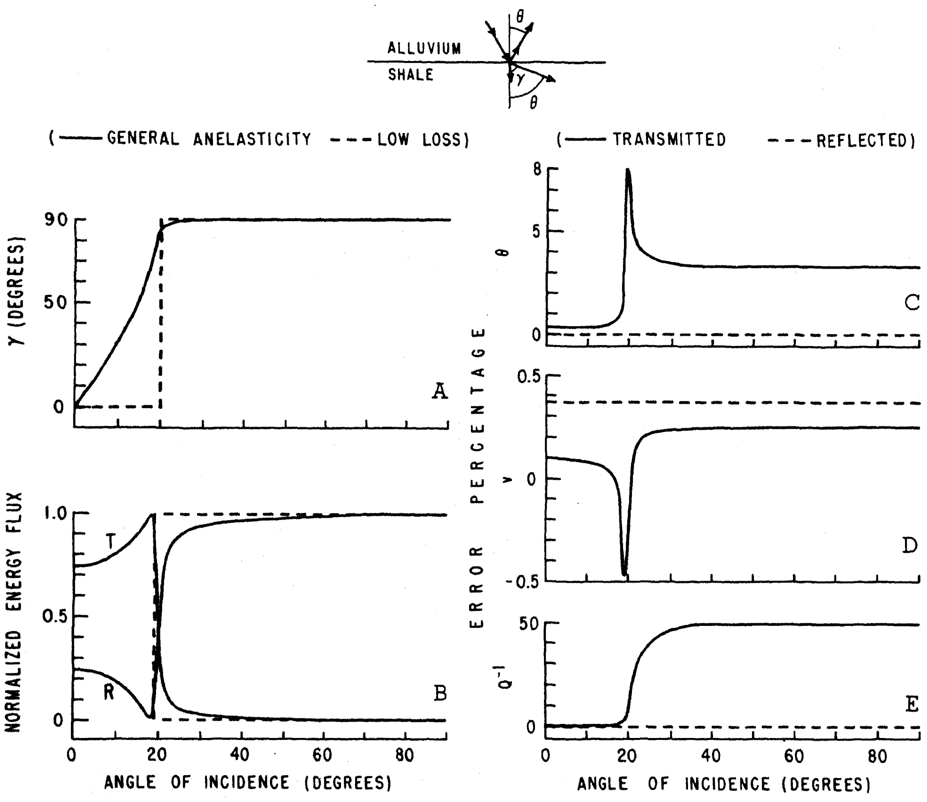

Fig. 2 Parameters for the transmitted and reflected waves at an alluvium-shale interface (see Fig. 1) assuming an incident homogeneous type-II S wave. A - Angle γ between the directions of propagation and maximum attenuation of the transmitted wave predicted by the general theory (solid) and the low-loss approximation (dashed). B - Reflection and transmission coefficients. C - Angles of reflection (dashed) and transmission (solid). D - Velocity of the reflected inhomogeneous wave and transmitted inhomogeneous wave. E - Specific attenuation factors for the reflected and transmitted waves.

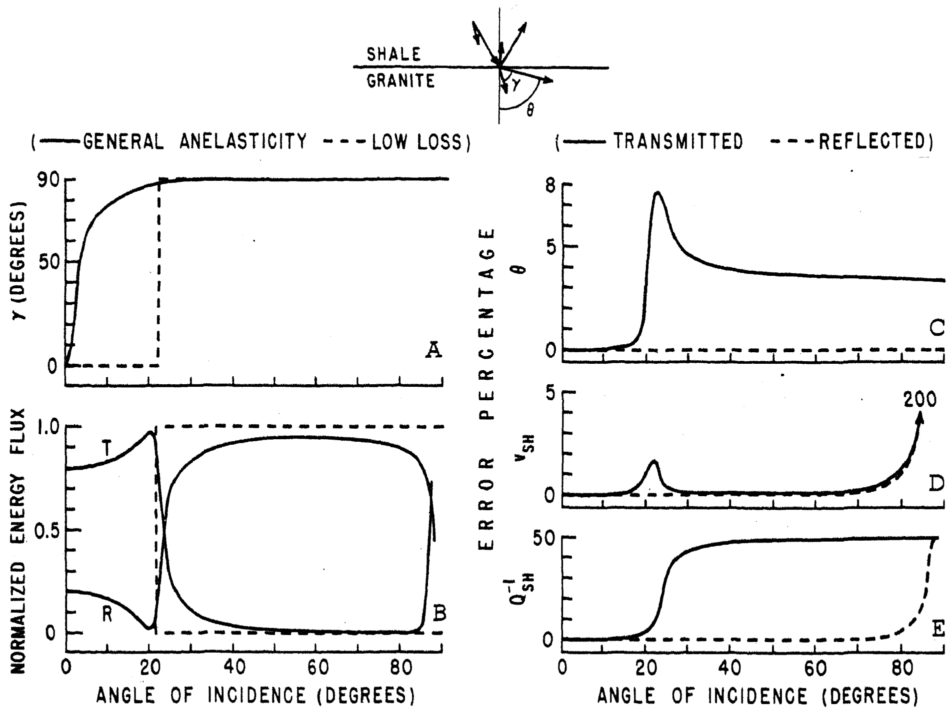

Fig. 3 Parameters for the transmitted and reflected waves at a shale-granite interface (see Fig. 1) assuming that the incident wave is inhomogeneous for the general theory and homogeneous for the low-loss approximation A. The directions of propagation and maximum attenuation of the transmitted wave predicted by the general theory (solid) and the low-loss approximation (dashed). B - Reflection and transmission coefficients. C - Angles of reflection (dashed) and transmission (solid). D - Velocity of the reflected inhomogeneous wave and transmitted inhomogeneous wave. E - Specific attenuation factors for the reflected and transmitted waves.