LOW-CYCLE FATIGUE UNDER RANDOM EARTHQUAKE MOTIONS

by

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SYNOPSIS

The safety of a randomly excited bilinear hysteretic system is discussed from the viewpoint of low-cycle fatigue process as well as of first excursion over a specified barrier. The rasponse under a quasi-nonstationary filtered shot noise is evaluated through a linearization technique. The reliability function is computed by assuming a Poisson type for level crossings and the Palmgren-and-Miner's hypothesis for fatigue accumulation. Comparison is made between the above different types of failure to investigate the nonlinear effect.

INTRODUCTION

Among structural failure mechanisms due to earthquake motions the first passage failure or the response excess probability over a specified level has extensively been studied. However, the repeated structural response beyond yield level may result in the so-called low-cycle fatigue damage. It depends both on the nonlinearity and nonlinear response level either of these two typical criteria governs the structural failure. For the about investigation a structure whose restoring force is characterized by the bilinear hysteresis of Fig. 1 is subjected to earthquakelike piecewise stationary (quasi-nonstationary) shot noise. The response analysis is carried out through the linearization technique in random motion [5] since the analytical solution for such a system has yet to be found.

RESPONSE ANALYSIS

The motion of a bilinear hysteretic structure is described, when subjected to base acceleration $\ddot{z}(t)$, by

$$\eta'' + 2\beta_o\eta' + Q(\eta) = -\frac{\ddot{z}(\tau)}{\omega_o^2 Y} \qquad (1)$$

in which a nondimensional form is taken by using $\eta = \frac{x}{Y}$ and $\tau = \omega_o t$; the prime denotes the time derivative with respect to τ, and $\omega_o = \sqrt{K/M}$ is the natural circular frequency of small response amplitude, $\beta_o = C/2\sqrt{KM}$ is the small amplitude fraction of critical viscous damping, M = mass, C = viscous damping coefficient, K = initial stiffness, $Q(\eta)$ = force-deformation characteristic of Fig. 1 and y = yield level.

Modelling of the input acceleration is made in such a way that the frequency contents appear similar to those of earthquake motions. Mostly for this purpose a white noise n(t) is passed through a certain linear filter L so that

$$L[y(t)] = n(t) \qquad (2)$$

in which

$$E[n(t)] = 0 \qquad E[n(t)n(t+\Delta\tau)] = 2\pi S_n(t)\delta(\Delta t) \qquad (3)$$

and E[ ] = mathematical expectation, $\delta(\ )$ = Dirac's delta function and $S_n(t)$, giving the variation of mean-square intensity, is assumed as shown in Fig. 2.

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The 2nd order differential operator is used herein for L.

$$L = \frac{d^2}{dt^2} + 2\beta_g\omega_g\frac{d}{dt} + \omega_g^2 \qquad (4)$$

and the absolute acceleration is taken as the input acceleration to the structure. By letting $\eta_o = y/\omega_o^2 Y$ one can get

$$\frac{\ddot{z}_o(\tau)}{\omega_o^2 Y} = -2\beta_g(\frac{\omega_g}{\omega_o})\eta_o' - (\frac{\omega_g}{\omega_o})^2\eta_o \qquad (5)$$

Since the analytical solution of Eq. 1 due to random excitations is not available, the equivalent linearization technique [5] is adopted to evaluate the response of original system. Such obtained linear system is governed by

$$\eta'' + 2\beta_{eq}(\frac{\omega_{eq}}{\omega_o})\eta' + (\frac{\omega_{eq}}{\omega_o})^2\eta = -\frac{\ddot{y}(\tau)}{\omega_o^2 Y} \qquad (6)$$

in which

$$\beta_{eq} = \beta_o(\frac{\omega_o}{\omega_{eq}}) + (\frac{\alpha}{2\pi})^{\frac{1}{2}}(1-\alpha)[1-\mathrm{erf}(\frac{1}{2^{\frac{1}{2}}\sigma_\eta})](\frac{1}{2\sigma_\eta})(\frac{\omega_o}{\omega_{eq}})^3 \qquad (7)$$

$$(\frac{\omega_{eq}}{\omega_o})^2 = 1 - (1-\alpha)\exp(1 - \frac{1}{2\sigma_\eta^2}) \qquad (8)$$

and σ_η = rms displacement response and erf( ) = error function.

The equations of motion Eqs. 2 through 5, can be expressed, by introducing a state vector $\{u\}^T = \{\eta_o', \eta_o ; \eta', \eta\}$, as

$$\{u'\} + [D]\{u\} = \{Q\}n(t) \qquad (9)$$

in which

$$[D] = \begin{bmatrix} \begin{bmatrix} 2\beta_g(\frac{\omega_g}{\omega_o}) & (\frac{\omega_g}{\omega_o})^2 \\ -1 & 0 \end{bmatrix} & [0] \\ -\begin{bmatrix} 2\beta_g(\frac{\omega_g}{\omega_o}) & (\frac{\omega_g}{\omega_o})^2 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 2\beta_{eq}(\frac{\omega_{eq}}{\omega_o}) & (\frac{\omega_{eq}}{\omega_o})^2 \\ -1 & 0 \end{bmatrix} \end{bmatrix} \qquad \{Q\} = \begin{Bmatrix} \frac{1}{\omega_o^2 Y} \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

In order to compute the response covariance matrix $[R_u] = E[\{u\}\{u\}^T]$, which plays an important role for the probabilistic response investigation, the following method [4] is taken. Applying a unitary transformation to Eq. 9

$$\{u\} = [\Phi]\{r\} \qquad (10)$$

with $[\Phi]$ composed of complex eigenvectors for [D], the response covariance in complex modes is calculated by

$$E[\{r\}\{r\}^T] = [R_r] = \int_o^\tau [e^{-\lambda_j(\tau-\mu)}][G(\mu)][e^{-\lambda_j(\tau-\mu)}]d\mu \qquad (11)$$

and

$$[G(\tau)] = 2\pi S_n(\tau)[\Phi]^{-1}[\{R\}\{Q\}^T]([\Phi]^{-1})^T \qquad (12)$$

When a staircase function is used to approximate the variation of $S_n(\tau)$, Eq. 11 can be performed step-by-step analytically and the result is

$$[R_r]_{\tau=\tau_{i+1}} = [e^{-\lambda_j \Delta\tau_i}][\tilde{R}_r]_{\tau=\tau_i}[e^{-\lambda_j \Delta\tau_i}] + [\frac{1}{\lambda_\ell+\lambda_m}([G(\tau_i)) - [e^{-\lambda_j \Delta t_i}][G(\tau_i)][e^{-\lambda_j \Delta t_i}])_{\ell m}] \tag{13}$$

in which $\Delta\tau_i = \tau_{i+1} - \tau_i$ and the initial condition is given from the response continuity by

$$[\tilde{R}_r]_{\tau=\tau_i} = [\Phi]^{-1}_{\tau=\tau_{i+1}} [R_u]_{\tau=\tau_i} ([\Phi]^T)^{-1}_{\tau=\tau_{i+1}} \tag{14}$$

The response covariance in the original coordinates is obtained from

$$[R_u] = [\Phi][R_r][\Phi]^T \tag{15}$$

LOW-CYCLE FATIGUE

The low-cycle fatigue test on yielding materials [3] suggests a certain relationship between strain amplitude and number of loadings. Presuming a similar relationship between structural deformation and number of loadings, one may get

$$4\left(\frac{\bar{\eta}}{\eta_F}\right)^a = \frac{1}{N_c} \tag{16}$$

in which $\bar{\eta}$ = ductility response amplitude, η_F = ductility factor and a = constant to be determined from experiments and structural analysis. This repeated loading result is extended into random loading due to earthquake motions. Assuming N_c is a continuous function of $\bar{\eta}$, the contribution of dN_c cycles with amplitude in $(\bar{\eta}, \bar{\eta}+d\bar{\eta})$ to damage can be evaluated from

$$D_{\bar{\eta}} = \frac{dN_c}{N_c(\bar{\eta})} \tag{17}$$

in which

$$dN_c = \frac{\partial[M(\bar{\eta},\tau)]}{\partial\bar{\eta}} d\bar{\eta} = -E[M_T(\tau)]p(\bar{\eta},\tau) \tag{18}$$

$M(\bar{\eta},\tau)$, representing the number of peaks in that range, is given from the heuristic approach, $p(\bar{\eta},\tau)$ = peak distribution and $E[M_T(\tau)]$ = total expected number of peaks per unit time which is approximated by the zero crossings rate as below.

$$E[M_T(\tau)] = \frac{1}{2\pi}\frac{\sigma_{\eta'}}{\sigma_\eta}(1-\rho^2)^{\frac{1}{2}} \tag{19}$$

in which $\sigma_{\eta'}$ = rms velocity response and $\rho = E[x\dot{x}]/\sigma_\eta\sigma_{\eta'}$ is the correlation coefficient. The total fatigue accumulation during the response time T is then summed up from the Palmgren-Miner hypothesis and is expressed, in view of Eqs. 16 through 19, as

$$E[D(T)] = \int_o^T \frac{2}{\pi}\frac{\sigma_{\eta'}}{\sigma_\eta}(1-\rho^2)^{\frac{1}{2}} \int_o^\infty \left(\frac{\bar{\eta}}{\eta_F}\right)^a p(\bar{\eta},\tau)\, d\bar{\eta}\, d\tau \tag{20}$$

RELIABILITY ANALYSIS

The reliability function is defined for judging the structural survival against earthquake motion. The first passage failure has been extensively investigated for linear structures, and the corresponding reliability with unit initial condition is provided by

$$R(\tau) = \exp[-\int_0^{\tau} \gamma(\tau)d\tau] \tag{21}$$

in which $\gamma(\tau)$ *is the rate of crossing of a specified response level. If the* Poisson process is assumed for crossing over a certain level B, as well as the symmetric nature of response,

$$\gamma(\tau) = 2E[N^+(|B|,\tau)] = \frac{1}{\pi}\frac{\sigma_{\eta'}}{\sigma_\eta}\exp[-\frac{1}{2}(\frac{B}{\sigma_\eta})^2][(1-\rho^2)^{\frac{1}{2}}\exp\{-\frac{\rho^2}{2(1-\rho^2)}(\frac{B}{\sigma_\eta})^2\} + \frac{\pi^{\frac{1}{2}}\rho}{2}(\frac{|B|}{\sigma_\eta})\{1+\mathrm{erf}(\frac{\rho}{[2(1-\rho^2)]^{\frac{1}{2}}}\frac{|B|}{\sigma_\eta})\}] \tag{22}$$

In this study the magnitude $|B|$ is chosen as the γ times of the stationary rms displacement σ_s, i.e.

$$|B| = \gamma\sigma_s = \gamma\left[\frac{\pi}{4\beta_o}\frac{1+4\beta_g^2(\frac{\omega_o}{\omega_g})^2}{\{1-(\frac{\omega_o}{\omega_g})^2\}^2+4\beta_g^2(\frac{\omega_o}{\omega_g})^2}\right]^{\frac{1}{2}}(\frac{N}{Y}) \tag{23}$$

When the effect of low-cycle fatigue is taken into account, the available ductility is decreased from η_F to η_R by

$$\eta_R = \eta_F(1-E[D])^{1/a} \tag{24}$$

The crossing rate $\gamma(\tau)$ is found from

$$\gamma(\tau) = \int_{\eta_R'}^{\infty}(\eta'-\eta_R')p(\eta_R,\eta';\tau)d\eta' - \int_{-\infty}^{-\eta_R'}(\eta'+\eta_R')p(-\eta_R,\eta';\tau)d\eta' \tag{25}$$

Now that $\eta_R = -|\eta|$ may hold in view of the fatigue accumulation process [1,2], one can get

$$R(\tau) = \exp[-4\{\int_0^{\min(\tau,\tau^*)}E[N^+(\eta_R,\tau)]d\tau + \int_{\min(\tau,\tau^*)}^{\tau}E[N^+(0,\tau)d\tau\}] \tag{26}$$

in which τ^* is the time when $E[D(\tau^*)]=1$ is attained.

NUMERICAL EXAMPLE AND DISCUSSION

The response analysis of a representative case of $\omega_o = 2\pi$, $\beta_o = 0.02$ and $\alpha = 0.1$ is carried out step-by-step through linearization process for a shot noise passed through a filter L characterized by $\omega_g = 4\pi$ and $\beta_g = 0.6$. The time increment is taken as $\Delta t = 0.025$. Figs.3 and 4 show the corresponding rms responses.

The fatigue accumulation is investigated by using the data $a = 2$ and $\eta_F = 15$. The computation results for $E[D]$ and $R(\tau)$ are shown in Figs. 5 and 6. Figure 7 compares the reliability of the structure with and without considering the fatigue accumulation. For the latter case the barrier is defined by Eq. 23 in which γ is changed as 2, 2.5 and 3. It may be concluded from this figure that the effect of low-cycle fatigue grows and becomes significant as the response level increases much beyond Y, whereas the first passage failure which disregards this effect determine the structural failure for small nonlinear response level.

REFERENCES

1. Lardner, R.W., "A Theory of Random Fatigue", Journal of Mechanical Pysics Solids, Vol. 15, 1967, pp. 205-221.

2. Minai, R., "On the Aseismic Safety of Building Structure", Bulletin of Disaster Prevention Research Institute, Kyoto University, No. 13A, 1970, pp. 1-17.

3. Ohji, K., Miller, W.R., and Marin, J., "Cumulative Damage and Effect of Mean Strain in Low Cycle Fatigue of a 2024-T351 Aluminum Alloy", J. of Basic Eng., Vol. 88, 1966, pp. 801-810.

4. Takemiya, H., "Stochastic Seismic Response Analysis of A Multi-Degree-of-Freedom Hysteretic Structure", Proceedings of Japan Society of Civil Engineers, No. 245, 1976, pp. 17-26.

5. Takemiya, H. and Lutes, L.D., "Stationary Random Vibration of Hysteretic Systems", Submitted to the Journal of the Engineering Mechanics Division, ASCE.

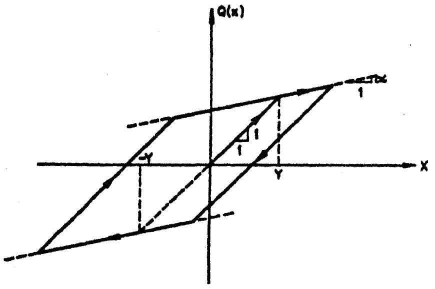

FIG.1.- RESTORING FORCE CHARACTERISTIC

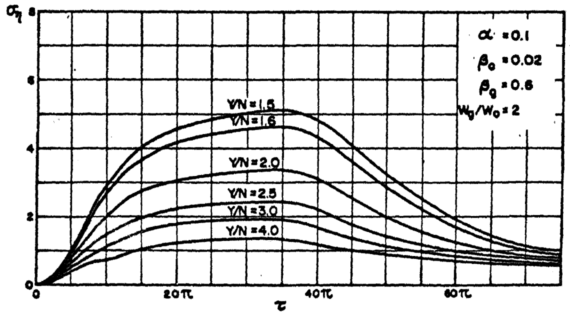

FIG. 3.- RMS DISPLACEMENT FOR FILTERED SHOT NOISE INPUT

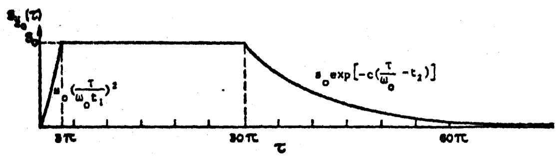

FIG.2.- INTENSITY OF $S_{\ddot{z}_g}(\tau)$

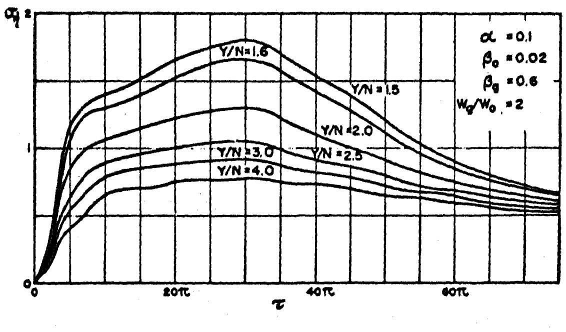

FIG.4.- RMS VELOCITY FOR FILTERED SHOT NOISE INPUT

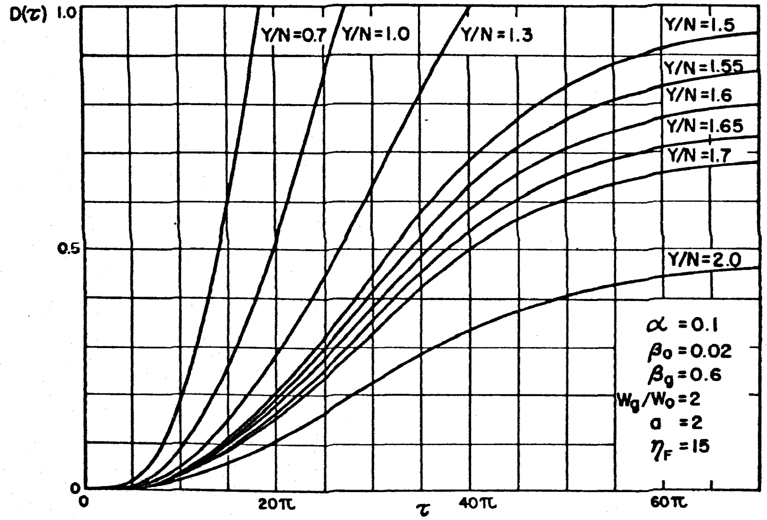

FIG. 5.—CUMULATIVE DAMAGE FOR FILTERED SHOT NOISE INPUT

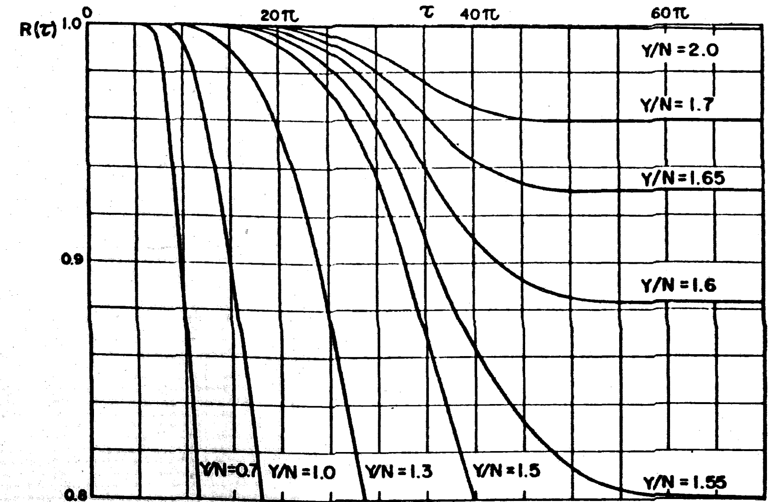

FIG. 6.—RELIABILITY FOR FILTERED SHOT NOISE INPUT

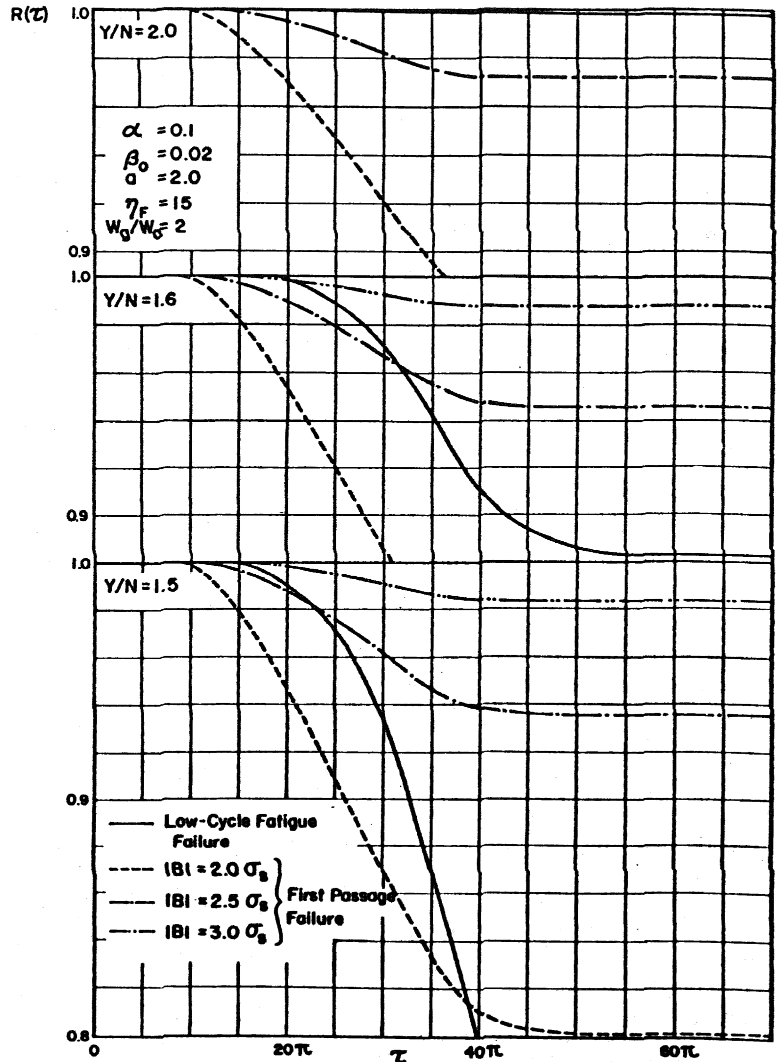

FIG. 7.—COMPARISON OF RELIABILITY FOR FILTERED SHOT NOISE INPUT

DISCUSSION

Rudolf Grossmayer (Austria)

The author has given a solution for the fatigue problem of hysteretic structures under nonstationary earthquake loadings. In order to get information about crossing statitics of the nonlinear structural response, the nonlinear equations of motions were solved by means of the equivalent linearization technique. Because this method is not very well settled for the nonstationary case. The discussor would like to comment on this point. What is the specified time interval, where the parameters are assumed to be equivalent, and how great was the time interval in the computation of the staircase approximation of $S_n(t)$?

It is well known that, in the case of a stationary input motion, the equivalent linearization method can predict the rms value of the structural response very well, if the nonlinearity is small or moderate. However, as the probability distribution of the response variable may particularly in the tail deviate significantly from the Gaussian distribution, the equivalent linearization method seems not to be adequate to compute statistics of sample properties like crossing rates. Therefore the discussor would like to know, if the author has checked his results by any other means like simulation techniques in order to verify the accuracy of his proposed method.

Author's Closure

Not received.