A PROBABLISTIC APPROACH TO ESTIMATE DESIGN EARTHQUAKE FOR A SITE IN TERMS OF MAGNITUDE, EPICENTRAL DISTANCE AND RETURN PERIOD

by

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SYNOPSIS

A method is proposed to obtain damage potential earthquake for design purpose in terms of magnitude, epicentral distance and return period based on probablistic treatment of earthquake data. Gumbel's model is applied to simulate the occurrence of the maximum magnitude earthquake in a given time and in a given epicentral distant zone. As an example, earthquake data near Osaka, Japan are studied and the results of the analysis are dicussed.

1. INTRODUCTION

Several methods are available to estimate characteristics of a design earthquake for a site. The most commom method is to obtain relationship between return period and the expected magnitude in a region or the expected maximum ground motion for the site(Kawasumi 1951, Donovan and Valera 1972).

In some cases it is much more relevant to give informations about design earthquake with not only return period, magnitude or intensity but also epicentral distance and type of earthquake from which important characteristics of the design earthquake such as maximum ground motion, predominant period and duration of motion may be derived(Seed et al.1968).

2. METHOD OF ANALYSIS

Magnitude distribution The relationship between the number of occurrence of earthquake and its magnitude M is expressed by Gutenberg-Richter equation as follows,

$$\log N_m = a - bM \tag{1}$$

The relation between the number of earthquake and its magnitude from 1926 to 1962 in Japan are shown in Fig.1. For the earthquake of greater magnitude than 5, equation (1) is seen to be satisfied. But in the smaller magnitude range, the departure from the linearity is notified due to the lack of observation ability for the smaller earthquakes of long distance by J.M.A.(Japan Meteorological Agency) net work. In general the probability density function of number of magnitude can be assumed as an exponential function expressed by equation (1).

Extreme value method The occurrence of earthquakes in a given area may be assumed as a stochastic process of $F(M,t)$, where M is magnitude and t is time. If a homogeneous earthquake process is further assumed, $F(M,t)$ becomes independent of t and the cumulative magnitude distribution in a given period may be derived from equation (1) as

$$F(M) = \alpha(1 - e^{-\beta M}) \tag{2}$$

where $\alpha = exp(2.3a)/2.3b$
$\beta = 2.3b$

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If the time scale is devided into equally spaced interval T_i, the maximum value of M in an interval time, called as an extreme value M_e, forms a stochastic process and its characteristics have been studied by Gumbel(Gumbel,1958). For exponential distribution given by equation (2), the extreme value M_e follows the Gumbel´s "Type I distribution" expressed by

$$G(M_e) = exp(-\alpha e^{-\beta M_e}) \qquad (3)$$

One of the most important aspect of the above Gumbel´s distribution is that the cumulative distribution $G(M_e)$ depends upon only the edge portion of the magnitude distribution, which is practically useful to estimate probablistic character based on the historically described data likely to have little information about smaller portion of the earthquake magnitude.

Let the M_e in each interval time be expressed as $M_1, M_2 \cdots M_i \cdots M_n (M_1 < M_2 < \cdots < M_i < \cdots < M_n)$, then the parameters α and β may be estimated through curve fitting method based on the following equation(Lomnitz 1974).

$$\ell n(-\ell n G(M_i)) = \ell n \alpha - \beta M_i \qquad (4)$$

where $G(M_i) = i / (n + 1)$

α is the number of earthquakes with magnitude greater than 0 in an interval time, β is an index which shows tendency of decrease of earthquake number with magnitude. The number of earthquakes N_m with magnitude greater than m in an interval time is expressed as

$$N_m = \alpha exp(-\beta m) \qquad (5)$$

and the return period T_r is the inverse of N_m

$$T_r = 1 / N_m \qquad (6)$$

The area to be analysed may be divided into small sections according to its seismicity and the distance from a given site. If the small section is selected as ring section or a part of ring section(fan section) with outer and inner diameter of d_o, d_i and angles of the fan of θ_1, θ_2. The parameters α and β can be estimated for each element section. If α is divided by the area of the element and denoted as α_u which is the number of earthquakes greater than magnitude 0 in unit area, the parameters of α_u, β can be given in each element forming a spatial distribution as function of d and θ. The return period of the earthquakes greater than a given magnitude m in an area within epicentral distances from d_i to d_o is given as

$$T_{r,\ d_i - d_o} = T_i / N_{m,\ d_i - d_o} \qquad (7)$$

where $N_{m,\ d_i - d_o} = \iint \alpha_u(d,\theta)\ exp(\beta(d,\theta)\ m)\ d\theta\ dd$

Earthquake distribution to give a specified Intensity to a site It is sometimes useful to consider not only the earthquake occurrence itself but also the earthquake distribution which will give the ground motion larger than a given intensity at the site. The intensity of the ground motion at the site may be expressed as multiplying the intensity of the standard ground by an amplification factor of the site. If the intensity of the standard ground is assumed as a function of magnitude and epicentral distance $I=I(m, d)$, the magnitude distribution to give a specified intensity range and its return period for the standard ground is

$$T_{r,\ I(m)} = T_i / N_{m,\ d_{Ii} - d_{Io}} \qquad (8)$$

where d_{Ii}, d_{Io}; epicentral distance range where intensity I is expected by earthquake with magnitude m.

3. DAMAGE POTENTIAL EARTHQUAKE STUDY FOR OSAKA AREA

Data analysed The earthquake data analysed here are from two sources; one is historically described damage earthquake from 599A.D. to 1973 compiled by Usami(Usami 1975) and another is instrumentally observed earthquake data by J.M.A. from 1925 to 1970, hereinafter referred as historical and observed data respectively(Figs.2 and 3). Figs.4 and 5 show the distribution of the earthquake magnitude and epicentral distance from Osaka for both data. The historical data consist of earthquakes with magnitude 5 to 8.5 while the observed data consist of mainly much smaller earthquakes of magnitude 4 to 6. In the figures equi-intensity curves for standard ground expressed by J.M.A. scale are shown for reference.

Ring section analysis Assuming the homogeneous activity within a ring section zone of the same epicentral distance, stochastic anlysis for the both earthquake data is applied by equation (7) in the area within 200 km from Osaka taking the width of each ring as 10 km and the interval time as 3 years. Some of the results are shown in Figs.6 - 9 indicating the extreme value of the magnitude in the ring section vs. its cumulative probability distribution. For the shorter epicentral distances of $d = 40\text{-}50$ and *70-80 km* the results of both data show the same tendency of occurrence. However for the longer distance of $d = 140\text{-}150$ and *180-190 km*, the tendency of the both results shows rather differnt each other. Possible reasons to cause the difference may be 1. earthquake trend is not constant for long period of time. 2. the period of collecting observed data is not long enough to extrapolate the stochastic tendency and 3. the seismicity within the ring section is not uniform. The earthquake activity over several thousands years in these area are considered to be rather constant by geotectonic evidences(Sugimura 1967). The followings are some considerations on the seismic activity related with geotectonic structures with careful treatment of the earthquake data.

Geotectonic structure related with earthquake In the area within 100km from Osaka, earthquakes are caused by mainly by strike-slip type faulting system with very shallow depth of less than 30 km, called as inland type, while in the area more than 100 km away from Osaka the earthquake mechanism consist of two kinds of differnt geotectonic systems; one is above mentioned inland type in northern part and another is thrust type faulting system in southern part due to the intrusion of philipine-sea plate into Japan island arc, called as off-shore type.

Fan section analysis based on geotectonic zoning The ring section area more than 100 km away from Osaka which shows different tendency between historical and observed data are divided into two fan sections according to geotectonic structures mentioned above. The earthquake data in each separate section are analysed respectively and some of the results are shown in Figs.10-11. It is evident in the figures that the historical and observed data is seen to have much more consistent tendency for each earthquake type than those in the previous single ring section analysis.

Possible maximum magnitude in the geotectonic zone In the case where the sampling period of the earthquake data is too short to cover the occurrence of larger magnitude range, it would be realistic not to extend the probablistic estimate beyond the maximum magnitude expected in the region. The possible maximum magnitude may be estimated from the relationship between length of fault and earthquake(Tocher 1958,Iida 1965 and Matsuda 1975). Matsuda(1975) have proposed the possible maximum magnitude M expected from a fault of length L for inland zone in Japan as

$$M = 1.67\log L + 4.85 \qquad (9)$$

In the area within 100 km from Osaka, the maximum fault length is 50 km which yields the maximum magnitude of 7.68 based on the equation(9). This may correspond to the historically described maximum magnitude of 7.4.

Return period in terms of magnitude and epicentral distance The return period is derived for earthquake of magnitude greater than M in a ring section with 10 km width shown in Figs.12-13. If parameters α_u and β are constant for the region, the occurrence probability should increase with epicentral distance due to the enlargement of the area involved. However the results show the minimum return periods at several epicentral distances.

Earthquake magnitude which will cause more than a given intensity The results are further rearranged to show the return period of the given magnitude earthquake which will cause a given intensity to the standard ground in Osaka given by equation(8) and shown in Fig.14 for two earthquake types. For the inland type earthquake, the damage potential earthquake of, say, $M=$ 7.5 is expected $T_r{}' \doteqdot$ *1000 years* in a single ring section with 10km width(Fig 12). However the overall effect from surrounding area to Osaka is estimated that the magnitude 7.5 could cause the ground intensity V_- every 250 years.

4. CONCLUSION

A new method is introduced by the authors to estimate characteristics of design earthquake based on stochastic procedure of analysis. As an example, Osaka area is studied and the results of the probability analysis show that 1. for the area where the earthquake mechanism and its geotectonic system is considered to be uniform, the magnitude-probability relationship is found relatively in good coincidense for historical and observed data. 2. the probability shows different tendency for historical and observed data where the geotectonics to cause earthquakes are considered not to be uniform. 3. However through the separate treatment of the data based on each geotectonic system, the both of historical and observed data are seen to give consistent relationship for each zone. It would be further concluded that the design earthquake or risk map based on any probablistic approach should be considered on the geotectonic systems ralated with earthquakes in the area. The method introduced here is demonstrated of its usefulness to clarify the earthquake characteristics of magnitude and epicentral distance as well as its return period which are important factors for aseismic design and urban planning near seismic active area.

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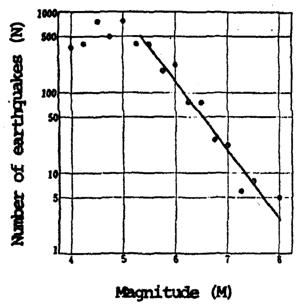


Fig.1 Relationship between Magnitude vs. its number near Japan(1926-1962)

Fig.2 Distribution of Observed Earthquakes

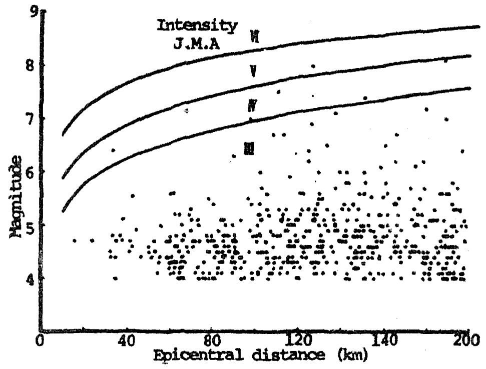


Fig.4 Magnitude and Epicentral Distance Distribution at Osaka for Observed Earthquakes

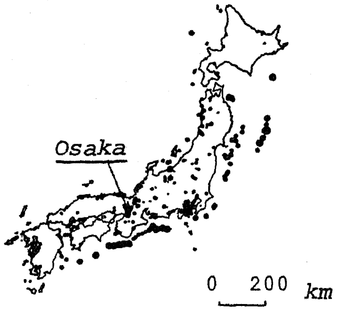


Fig.3 Distribution of Historical Earthquakes

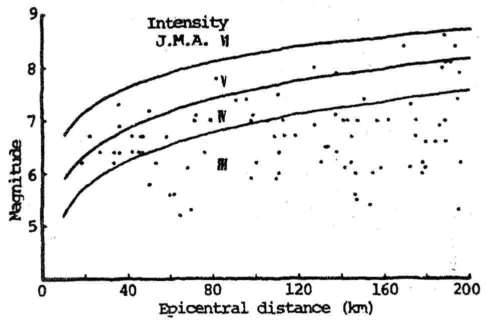
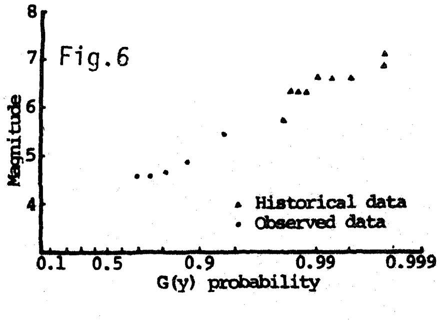


Fig.5 Magnitude and Epicentral Distance Distribution at Osaka for Historical Earthquakes



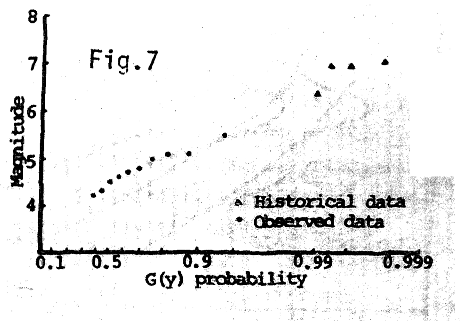


Fig.6-7 Cumulative Probability Distribution of Maximum Earthquake in every three years, Ring section Analysis

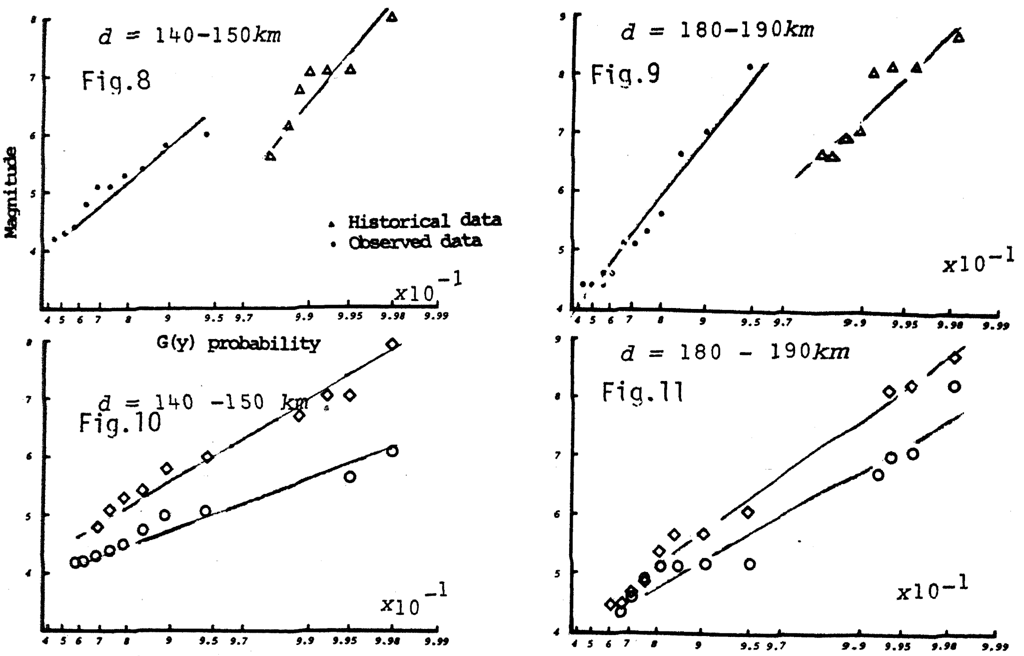


Fig.8-11 Cumulative Probability Distribution of Maximum Earthquake in every three years. Ring section Analysis (Fig.8-9), Fan section Analysis according to different geotectonic systems related with earthquake (Fig.10-11). o *inland type* ◇ *offshore type*

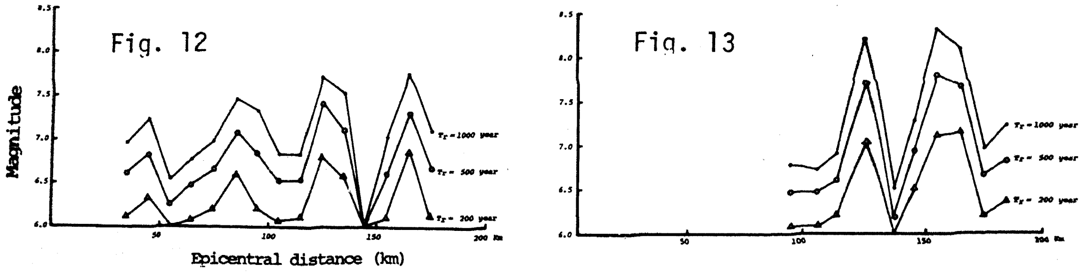


Fig.12-13 Equi-return Period Lines for each geotectonic zone. Inland Type(Fig.12) Offshore Type(Fig.13)

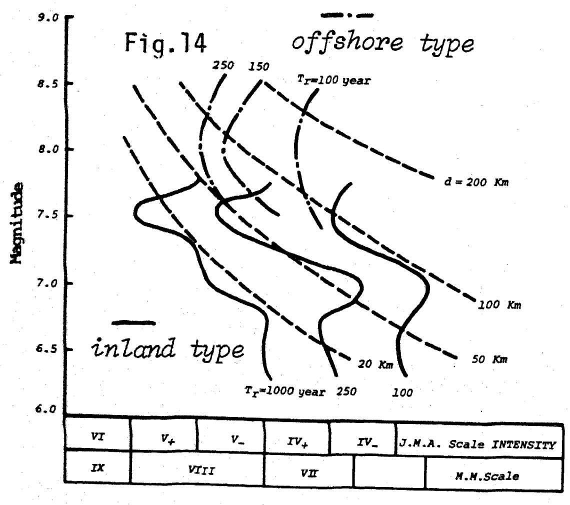


Fig.14 The Relation of the Return Period of the Earthquake Magnitude which will cause a Given Intensity to the Standard Ground at Osaka

DISCUSSION

J.F. Borges (Portugal)

Don't you think that some type of truncature has to be considered in the extreme distributions of magnitude you are dealing with ?

Vit Karnik and Zdenka Schenkova (U.S.S.R.)

In the paper of Yoshikawa, Iwasaki and Ishii the Gumbel's "type I distribution" is applied to simulate the occurrence of the maximum magnitude earthquake in a given time and in a given epicentral zone.

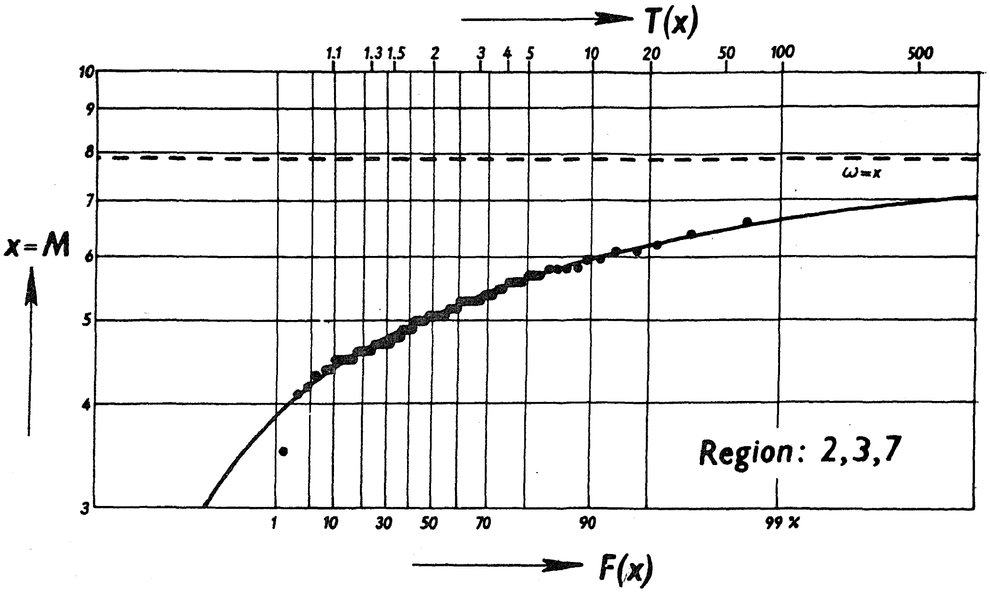
The stability postulate of the theory of largest values leads to three and only three asymptotic distribution of extremes and each assumes a specified behaviour of the absolute large values of the variable. In the occurrence of maximum magnitudes also the third distribution is considered which is defined by the relation

$$F(x) = \exp\left[- \left((\omega - x) / (\omega - u_n) \right)^{k_n} \right],$$

$k_n > 0$, $x \leq \omega$, $u_n < \omega$, where ω is the upper limit of largest values, k_n is the shape parameter, u_n is the characteristic largest value, and $F(u_n) = 1/e$ and $F(\omega) = 1$. It means that there exists an upper limit of the largest values whereas the first asymptotic distribution assumes an unlimited variable from the right. However, an upper magnitude threshold must exist in a given volume of material of certain physical properties and under the given stress distribution. The third asymptotic distribution is related to the first one by a logarithmic transformation.

We have applied the first and third asymptotic distribution to shallow focus earthquakes ($h \leq 50$ km) of the Balkan region from the period 1901 - 1970. Annual time intervals are adopted. Figure shows an example of the third asymptotic distribution for selected provinces Nos 2, 3, 7 (North-Western Yugoslavia). The observed probability function F(x) is traced on the extremal probability paper by plotting observed magnitudes x_i arranged in order of increasing magnitude versus their plotting positions p_i. It is natural to trace the earthquakes in increasing magnitude along the abscissa in decadic logarithmic scale. In the case $\omega > 0$, the logarithms of the earthquake magnitudes should lie on an ascending curve with a horizontal asymptote at x = ω which is the case in the majority of Balkan provinces.

We tested both the first and the third distriburions which provide the estimates of the probability with which individual large magnitudes will be exceeded in the future. However, the third distribution is closer to the reality. Using least sqaares method we get the important parameter $\omega \approx M_{max}$ which should not be exceeded. It should be noted that the third distribution leads to longer return periods in comparison with the first distribution, i.e. to a lower risk.



The third asymptotic distribution of the largest values for North-Western Yugoslavia

Author's Closure

J.F. Borges and Vit Karnik-Zdenka Schenkova noted the consideration of the maximum magnitude in the analysis of the earthquake occurrence. They correctly point out the physical existance of some maximum limit number of the earthquake magnitude. The writers also believe the upper limit of the nagnitude and consider that the limit value may be estimated by geotectonic study like distribution of faults and length in the area based upon the relationship between magnitude and fault length as mentioned in the paper.

Vit Karnik and Zdenka Schenkova showed their analysis to model the earthquake occurrence with limited magnitude based on the type III distribution and gave an interesting result. However it should be noted that the maximum limit magnitude given by the analysis may not be the same but rather greater than that in the real field situation. Because the upper limit magnitude earthquake obtained in the analysis has infinite large return period, while the real upper limit magnitude has finite value of the period. In this sense, the application of Type III distribution also has failed to cope with the real physical upper limit magnitude earthquake.

To satisfy the characteristics of limited magnitude and its finite return period, the range of the value above the limited magnitude in the results of the analysis based on any type of distribution model may be truncated and regarded as "non-realisable part in-situ".

Under the present state of knowledge on the mechanism of earthquake occurrence, it is rather difficult to take whole characteristics of stochastic process of earthquake phenomena into simple mathematical formulations. However the writers believe that the treatment described in the paper will give meaningful result of engineering assesment regarding to design earthquake to a specified site.