THE TIME VARIATION OF THE PREDOMINANT FREQUENCY OF EARTHQUAKE MOTIONS

J.G. Joannon (I), A. Arias (II) and G.R. Saragoni (III)

SYNOPSIS

A new method to estimate the time variation of the predominant frequency of earthquake acceleration and velocity records is presented. The method uses the ratios between the mean square functions of velocity, acceleration and derivate of acceleration. It has been applied to 32 earthquake accelerograms from U.S.A., México, Perú and Chile. The determined functions for the predominant frequency of acceleration records are in agreement with functions estimated by other authors assuming a non homogeneous Poisson process of exponential decay trend for the zero-crossings. This time variation is characterized by a chi-square decay trend which is particularly significant in some 1971 San Fernando earthquake records.

REPORT

1. Introduction.

The characterization of the variation of the frequency content of earthquake records is important in earthquake engineering since it has been shown that ductility requirements of stiffness degrading structures can be strongly affected by the time variation of the frequency content, (1), (2).

2. The Method.

The proposed method characterizes only the variation of the predominant frequencies $\omega_a(t)$ and $\omega_v(t)$ corresponding respectively to acceleration and velocity records by assuming that earthquake records are stationary processes in small interval of time Δt :

$$\omega^2_a(t) = \frac{\int_{t-\Delta t}^{t} E\{\dot{a}^2(\tau)\}\, d\tau}{\int_{t-\Delta t}^{t} E\{a^2(\tau)\}\, d\tau} \approx \frac{E\{\dot{a}^2(t)\}}{E\{a^2(t)\}} \tag{1}$$

(I) Civil Engineering Student, Department of Civil Engineering, University of Chile, Casilla 5373, Santiago, Chile.

(II) Visiting Professor, Soil Dynamics Project, Instituto de Ingeniería, UNAM, Apartado Postal 70-474, Ciudad Universitaria, México 20, D.F., México.

(III) Professor of Earthquake Engineering, Department of Civil Engineering, University of Chile, Casilla 5373, Santiago, Chile.

$$\omega^2_v(t) = \frac{\int_{t-\Delta t}^{t} E\{a^2(\tau)\}\,d\tau}{\int_{t-\Delta t}^{t} E\{v^2(\tau)\}\,d\tau} \approx \frac{E\{a^2(t)\}}{E\{v^2(t)\}} \tag{2}$$

where $E\{\cdot\}$ denotes expected value, $a(t)$ and $v(t)$ are the acceleration and velocity of the ground, the dot denotes time derivate and t is time. Eqs. (1) and (2) estimate the average predominant frequencies in the vecinity of t.

The mean square functions considered in Eqs. (1) and (2) can be replaced by chi-square functions of the type (2), (3).

$$E\,\{v^2(t)\} = \beta_1\, t^{\gamma_1}\, e^{-\alpha_1 t} \tag{3}$$

$$E\,\{a^2(t)\} = \beta_2\, t^{\gamma_2}\, e^{-\alpha_2 t} \tag{4}$$

$$E\,\{\dot{a}^2(t)\} = \beta_3\, t^{\gamma_3}\, e^{-\alpha_3 t} \tag{5}$$

where the α_i, β_i and γ_i are real constants, with $i = 1,2$ and 3.

Finally the functions $\omega_v(t)$ and $\omega_a(t)$ become

$$\omega_v(t) \approx \sqrt{\frac{\beta_2}{\beta_1}}\; t^{\frac{\gamma_1-\gamma_2}{2}}\; e^{\frac{-\alpha_2+\alpha_1}{2} t} \tag{6}$$

$$\omega_a(t) \approx \sqrt{\frac{\beta_3}{\beta_2}}\; t^{\frac{\gamma_3-\gamma_2}{2}}\; e^{\frac{-\alpha_3+\alpha_2}{2} t} \tag{7}$$

3. Results and Conclusions.

The parameters α_i, β_i and γ_i were estimated for 32 earthquake accelerograms from U.S.A., México, Perú and Chile using a time moment technique (2). These results are given in Table 1. The chi-square approximation was applied for the first time to velocity and derivative of acceleration records and it was found that it also gives good results as in the case of acceleration records. The functions $\omega_v(t)$ and $\omega_a(t)$ are shown in Figs. 1 and 2 for both components of El Centro 1940 and 15250 Ventura Blvd., Los Angeles, 1971. In these figures the total duration of the records has been normalized equal to one. The figures for $\omega_v(t)$ and $\omega_a(t)$ for all the accelerograms of Table 1 are given in reference (4).

The results for $\omega_a(t)$ are compared in Figs. 1b. and 2b. with the functions obtained in references (2) and (5) assuming a non homogeneous *Poisson process of the exponential decay trend for the zero-crossing.* This comparison was always found satisfactory. However it was also found that the proposed method does not give satisfactory estimates for $\omega_v(t)$ and $\omega_a(t)$ in the first 20% of the record due apparently to the use of time moment technique for the estimates of α_i, β_i and γ_i.

From Figs. 1 and 2 and Eqs. (6) and (7) it is concluded that the time variation of the predominant frequencies is characterized by chi-square functions which gives the same results that the exponential decay trend for the last 80% of the records. These time variations are particularly relevant in some 1971 San Fernando, California records.

In addition, it is also concluded that chi-square approximations give satisfactory results in very small intervals of the time since predominant frequencies are obtained as ratios between chi-square functions.

4. Bibliography.

1. Saragoni, G.R. and G.C. Hart, (1973), "Time Variation in Earthquake Frequency Content-Characterization and Relevance", 5WCEE, Paper N°156, 5 pages, Rome, Italy.

2. Saragoni, G.R., (1972), "Nonstationary Analysis and Simulation of Earthquake Ground Motion", Doctoral Dissertation, University of California, Los Angeles, June 1972.

3. Saragoni, G.R., (1977), "The αβγ Method for the Characterization of Earthquake Accelerograms", 6WCEE, New Delhi, India.

4. Joannon, J.G., A.Arias and G.R. Saragoni, (1975), (In Spanish), "Time Variation of the Frequency Content of Earthquake Accelerogram", Department of Civil Engineering, Structure Division, Report SES I-1/75, Santiago, Chile.

5. Saragoni, G.R. and G.C. Hart, (1974), "Simulation of Artificial Earthquakes", Earthquake Engineering and Structural Dynamics, Vol. 2, 149-268.

TABLE 1

CHISQUARE FUNCTIONS PARAMETERS

STATION	DATE	COMPONENT	α_1	β_1	γ_1	α_2	β_2	γ_2	α_3	β_3	γ_3
			seg^{-1}	$0.01g^2 seg^{2-\gamma_1}$	—	seg^{-1}	$0.01g^2 seg^{-\gamma_2}$	—	seg^{-1}	$0.01g^2 seg^{-(2+\gamma_3)}$	—
El Centro, California	20-Dec-1934	N-S	0.211	0.111 E-1	0.522	0.286	0.184 E 0	1.267	0.392	0.217 E 2	2.449
		E-W	0.251	0.820 E-2	0.887	0.372	0.109 E-1	2.937	0.356	0.161 E 2	2.482
El Centro, California	18-May-1940	N-S	0.090	0.585 E-1	-0.033	0.119	0.132 E 1	0.077	0.151	0.131 E 4	0.176
			0.085	0.440 E-1	0.038	0.148	0.343 E 0	0.752	0.176	0.179 E 3	0.971
Olympia, Washington	13-Apr-1949	N10°W	0.461	0.360 E-6	6.208	0.351	0.109 E-1	3.081	0.265	0.274 E 3	1.268
		N80°E	0.273	0.681 E-3	2.383	0.365	0.298 E-2	3.870	0.279	0.130 E 3	1.776
Taft, California	21-Jul-1952	N21°E	0.088	0.456 E-3	1.130	0.150	0.101 E 0	0.916	0.192	0.569 E 2	1.156
		S69°E	0.078	0.818 E-3	0.882	0.142	0.190 E 0	0.622	0.179	0.919 E 2	0.920
Golden Gate Park	22-Mar-1957	N10°E	0.677	0.827 E-3	0.290	1.253	0.277 E 0	1.388	1.166	0.443 E 3	1.410
		S80°E	0.660	0.414 E-3	0.420	1.568	0.650 E 0	2.073	1.223	0.802 E 3	1.626
8244 Orion	9-Feb-1971	N-S	0.453	0.522 E-4	4.615	0.573	0.451 E-2	4.726	0.633	0.609 E 2	3.371
		E-W	0.659	0.247 E-8	9.560	0.376	0.959 E-2	3.171	0.496	0.258 E 2	2.759
15250 Ventura		N11°E	0.390	0.476 E-5	5.212	0.298	0.315 E-1	2.412	0.477	0.180 E 2	3.257
		N79°W	0.328	0.854 E-7	6.654	0.218	0.163 E-1	2.086	0.369	0.127 E 2	2.618
15910 Ventura		S81°E	0.376	0.255 E-8	7.494	0.211	0.693 E-2	2.210	0.384	0.146 E 2	2.652
		S09°W	0.436	0.720 E-7	6.843	0.328	0.799 E-3	3.834	0.397	0.102 E 2	2.934
250 E First Street		N36°E	0.396	0.529 E-3	2.641	0.240	0.234 E 0	0.332	0.220	0.287 E 3	-0.012
		N54°W	0.334	0.489 E-2	1.292	0.276	0.225 E 0	0.512	0.255	0.298 E 3	0.153

TABLE 1. (Cont.)

CHISQUARE FUNCTIONS PARAMETERS

STATION	DATE	COMPONENT	α_1	β_1	γ_1	α_2	β_2	γ_2	α_3	β_3	γ_3
			seg^{-1}	$0.01g^2seg^{2-\gamma_1}$	—	seg^{-1}	$0.01g^2seg^{-\gamma_2}$	—	seg^{-1}	$0.01g^2seg^{-(2+\gamma_3)}$	—
Santiago, Chile	13-Sep-1945	N10°W	1.096	0.799 E-4	4.402	0.867	0.333 E-1	2.916	1.021	0.642 E 2	3.486
		S80°W	0.373	0.214 E-2	0.108	0.507	0.175 E 0	0.691	0.592	0.150 E 3	1.431
Santiago, Chile	28-Mar-1965	N10°W	0.346	0.302 E-3	1.967	0.476	0.228 E-1	2.777	0.479	0.833 E 2	2.794
		S80°W	0.310	0.115 E-3	1.109	0.483	0.350 E-1	2.744	0.458	0.194 E 3	2.607
Lima, Perú	10-Oct-1966	N08°E	0.364	0.103 E-1	1.143	0.579	0.674 E 0	2.041	0.629	0.120 E 4	2.736
		N82°W	0.737	0.165 E-7	8.461	1.171	0.810 E-8	13.000	1.025	0.520 E-3	11.215
Latino. Tower, Mexico	11-May-1962	N09°E	0.107	0.169 E-4	2.069	0.114	0.290 E-3	1.852	0.124	0.103 E-1	1.725
		N81°W	0.158	0.590 E-7	4.326	0.174	0.170 E-5	4.021	0.154	0.646 E-3	2.876
Alameda Park, Mexico	11-May-1962	N10°46'W	0.124	0.182 E-2	1.457	0.148	0.617 E-3	2.307	0.114	0.326 E 0	1.028
		N79°14'E	0.144	0.163 E-4	2.735	0.194	0.165 E-4	3.734	0.128	0.708 E-1	1.507
Latino. Tower, Mexico	19-May-1962	N09°E	0.218	0.960 E-9	6.137	0.199	0.160 E-6	4.810	0.177	0.798 E-4	3.630
		N81°W	0.198	0.122 E-7	5.325	0.197	0.750 E-6	4.457	0.207	0.588 E-4	4.061
Alameda Park, Mexico	19-May-1962	N10°46'W	0.132	0.875 E-4	2.532	0.138	0.732 E-4	2.809	0.140	0.160 E-1	2.135
		N79°14'E	0.101	0.605 E-3	1.164	0.175	0.243 E-4	3.376	0.161	0.712 E-2	2.441

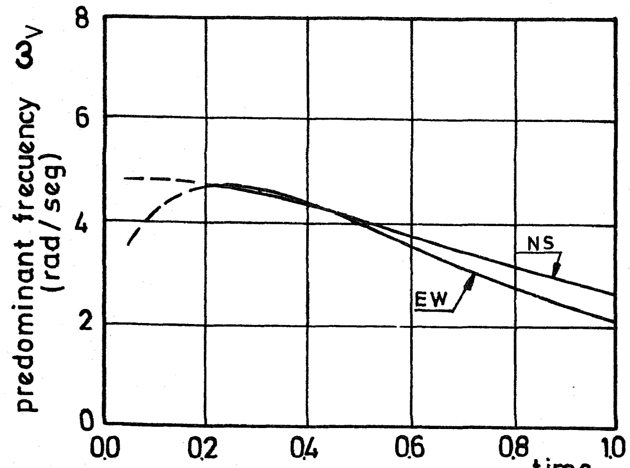

Fig 1a- EL CENTRO 1940

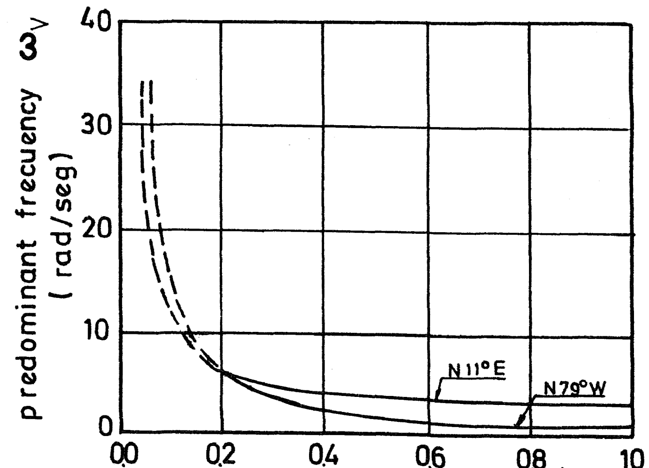

Fig 2a-15250 VENTURA

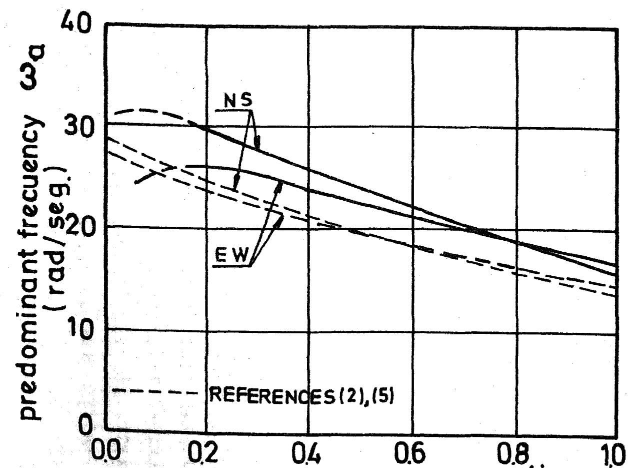

Fig 1b- EL CENTRO 1940

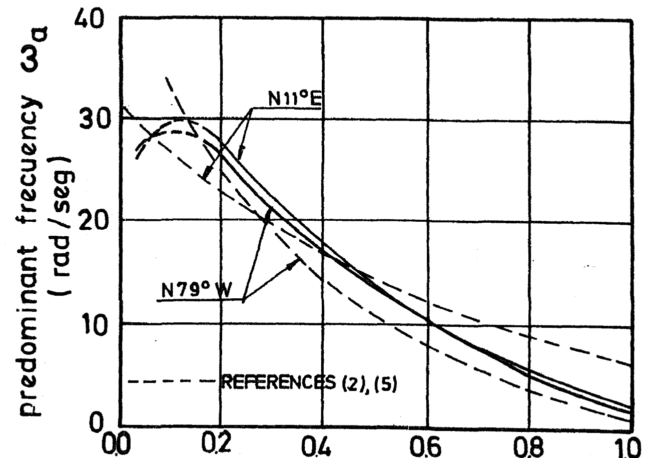

Fig 2b-15250 VENTURA

DISCUSSION

E. Faccioli (Mexico)

The discussor would like to know whether any correlation between seismotectonic characteristics and focal mechanisms on one side and time variation of W_a and W_v on the other side could be established in general for a given site or region.

P.N. Agrawal (India)

Will your new method of estimating the time variation of predominant frequency permit studies of azimuthal effect if records for same earthquake in various azimuths were studied.

Ricardo Duarte (Portugal)

The discussor guesses that what you call predominant frequency are the frequency of zero-crossings of accelerations (and velocities). The question is: How they are related to the frequency of maximum power spectral density of accelerations (and velocities) as determined in your previous papers ?

Author's Closure

With regard to the question of Mr. E. Faccioli, we wish to state that as far as we know no correlation has been developed yet between seismic wave path characteristics and focal mechanism parameters on one side and the time variation of $W_a(t)$ and $W_v(t)$ on the other side. Correlations among the physical parameters that you suggest and parameters that characterize time variation of frequency content of earthquake motions as for example $W_a(t)$ and $W_v(t)$ will very helpful to our understanding of earthquake ground motions in the future.

At this step of our research we have only tried to develop good statistical estimators for parameters which partially characterize the time variation of frequency content of earthquake ground motions considering that it has become an important aspect in the understanding of earthquake ground motions and in design technique which consider the nonlinear behavior of structures.

With regard to the question of Mr. Agrawal, we wish to state that yes, we think that our method would permit studies of effects due to the direction from the source if accelerograms at various azimuth were studied for the same earthquake.

In addition, since time variation of predominant frequencies were practically the same for the two horizontal accelerograms at each of the analyzed station, we think that it is possible to study for a given earthquake the influence of epicentral distance, azimuth, topography and soil effects if enough records are available for the earthquake. Such studies would permit to find the correlations that Dr. Faccioli ask us in the previous question.

With regard to the question of Mr. Duarte, we wish to state that we have called, as previous researcher predominant or characteristic frequency to the frequency of zero crossings with positive slope in attention that power spectral density function of most earthquake ground motion does not exhibit a narrow band character and are strongly skewed right. For this type of power spectra its mode or maximum it is not a predominant frequency. On the other hand, if the power spectral density function $\Gamma(\omega)$ can be considered of narrow band type in the vecinity of a central frequency or predominant frequency the expression for the frequency of zero crossings with positive slope ω_0 due to Rice (1)

$$\omega_0 = \sqrt{\frac{\int_0^\infty \omega^2 \Gamma(\omega)\, d\omega}{\int_0^\infty \Gamma(\omega)\, d\omega}}$$

can be approximated to

$$\omega_0 = \sqrt{\frac{\omega_c^2 \int_0^\infty \Gamma(\omega)\, d\omega}{\int_0^\infty \Gamma(\omega)\, d\omega}} \approx \omega_c$$

using an analogy with Laplace's method of evaluating integrals and considering that function ω^2 is a slowly varying function in the vecinity of ω_c.

In attention to these two previous reasons and its physical meaning we have called predominant frequency to the frequency of zero crossings with positive slope.

The relation between the frequency of the maximum of the power spectral density ω_m and the frequency of zero crossings with positive slope ω_0 for power spectral density of the type (Saragoni and Hart (2)).

$$\Gamma(\omega) = S_0 \, \omega^P \, e^{-Q\omega} \; ; \quad \omega \geq 0$$

are given by

$$\omega_m = P/Q$$

and

$$\omega_0 = \frac{1}{Q}\sqrt{(P+2)(P+1)}$$

<u>References</u>

1. Rice, S.O., "Mathematical Analysis of Random Noise", Selected Papers on Noise and Stochastic Processes, edited by N. Wax, pp. 133-204, Dover Publications, Inc., New York, 1954.

2. Saragoni, G.R. and G.C. Hart, "Nonstationary Analysis and Simulation of Earthquake Ground Motions", Earthquake Engineering and Structural Laboratory, University of California, Los Angeles, UCLA ENG-7238.