FORCE PULSE ON A CIRCULAR AREA AS AN EARTHQUAKE MODEL FOR NEAR-FIELD GROUND MOTION

by

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SYNOPSIS

Near-field ground motion due to an earthquake is modelled in terms of tangential stress pulse applied over a circular area. The formulation is mathematically tractable and physically reasonable. Finite rupture velocity and couple sources may also be incarporated. The case of suddenly applied tangential stress is considered in detail and numerical results are given.

INTRODUCTION

Success of the model where an instantaneous effective stress is applied over an infinite area[1,2,3] in predicting near-field ground motion comes from its simplicity and from the fact that it relates the motion to a physical quantity, the effective stress, which can be roughly estimated. A more satisfactory model should also incorporate finite rupture velocity and finite area of the faulting. Self-similar dynamic shear crack solutions[4-7] in 3-dimensions take in to account the rupture velocity but contain no intrinsic length; the fault grows to infinity. Numerical solution to the problem of a circular fault which grows and stops has been attempted[8]. Experimental studies on faulting in foam rubber[9] show good agreement with self-similar solutions at early times. An analytic solution to dynamic circular fault problem in which rupture grows and stops has not been obtained as yet due to mathematical difficulties. As an approximation we could also model an earthquake in the near field by an application of a stress pulse on the fault area (when the fault is infinite and stress application is instantaneous, we get the simple model[1,2,3]).The model would have the advantage of being mathematically tractable and physically reasonable. The effect of the half-space and couple sources may be considered[10] and variable stress on the fault may be included. As a first step towards such a model we give solution to the problem of a tangential stress applied suddenly over a circular area and present some numerical results.

FORMULATION AND RESOLUTION

From the general formulation of the response of a homogeneous, isotropic, elastic infinite space under the action of body forces[11], we specialize to the case of a circular area of radius a lying in the plane $z=0$ with its center as the origin of the coordinate system (fig. 1) on which a constant tangential stress σ is applied in the x-direction as Heaviside function of time. The corresponding body force X is given by

$$X = \frac{\sigma}{\rho_d}\,\delta(z)\,H(\tau)\,;\quad (x^2+y^2)^{1/2} = r<a, \qquad\qquad (1)$$
$$= 0 \qquad\qquad ; \qquad r>a,$$

where ρ_d = density of the medium, $\tau = c_1 t$, $\delta(z)$ = Dirac delta function, $H(\tau)$ = Heaviside function, and c_1 = P-wave velocity. The displacement u in the x-direction (omitting other components for brevity) is given by

$$u = QL^{-1}\left[\frac{1}{s}\iiint_{-\infty}^{\infty}\left\{\frac{\beta^2(\gamma_1^2+s^2)-(\beta^2-1)\xi^2}{(\gamma_1^2+s^2)(\gamma_1^2+\beta^2 s^2)}\right\}\frac{J_1(a\rho)}{\rho}\,e^{-i(\xi x+\eta y+\zeta z)}\,d\xi\,d\eta\,d\zeta\right] \qquad (2)$$

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where $Q = \sigma a / 4\pi^2 c_1^2 \rho_d, \rho^2 = \xi^2 + \eta^2, \gamma_1^2 = \rho^2 + z^2$, $\beta = c_1/c_2, c_2 =$ S-wave velocity, $J_\nu(x) =$ Bessel function of order ν, $s =$ Laplace transform parameter, and $L^{-1} =$ inverse Laplace transform. u can also be written as

$$u = Q\pi^2 L^{-1} \Big[2\beta^2 I(1,0,1,\beta^2) + I(1,0,3,\beta^2) - I(1,0,3,1) - \cos 2\alpha \Big\{ I(1,2,3,\beta^2) - I(1,2,3,1) \Big\} \Big] \quad (3)$$

where $\cos\alpha = x/r$ and $I(m\ n,\ p,\ \delta^2) = \int_0^\infty J_m(a\rho)\, J_n(r\rho) \Big\{ e^{-(\rho^2+\delta^2 s^2)^{1/2} z}\ \rho^{p-1} / s^p (\rho^2+\delta^2 s^2) \Big\}\, d\rho.$
We first take the inverse Laplace transform using complex variables theory and then perform the integration over ρ . Function $f_{p,\delta}(s) = \Big\{ e^{-(\rho^2+\delta^2 s^2)^{1/2} z} \big/ s^p \cdot (\rho^2+\delta^2\ s^2)^{1/2} \Big\}$ has pole at $s = 0$ (which gives the static part of the solution) and branch points at $s = \pm\, i\rho/\delta$ (fig2). To make $f_{p,\delta}$ (s) single valued, branch cuts are introduced from $i\rho/\delta$ to $i\infty$ and $-i\rho/\delta$ to $-i\infty$. $L^{-1}\big[f_{p,\delta}(s)\big]$ can be expressed in terms of residue at $s = 0$ and branch line integrals. u at any point can be reduced to finite integrals involving elliptic functions. Here we consider the simpler case of points on the axis of the circle ($r = 0$). Writing $u|_{r=0} = u_s|_{r=0} + u_d|_{r=0}$, where u_s is the static part and u_d is the dynamic part of the solution, it can be shown that

$$u_s|_{r=0} = Q\pi^2 \frac{(a^2+z^2)^{1/2}}{a} \left[(3\beta^2+1) - \frac{4\beta^2 z}{(a^2+z^2)^{1/2}} + \frac{(\beta^2-1)z^2}{a^2+z^2} \right] , \quad (4)$$

$$u_d|_{r=0} = Q\pi^2 \left[2\beta^2 K_{1,1} - K_{1,3} - K_2 \right] , \quad (5)$$

where

$$K_{1,p} = -\frac{1}{\pi} \int_{1/\beta}^{\infty} \frac{dy}{q_2 y^p} \int_0^\infty J_1(a\rho) \left[\cos\{\rho(y\tau + q_2 z)\} + \cos\{\rho(y\tau - q_2 z)\} \right] \frac{d\rho}{\rho} , \quad (6a)$$

$$K_2 = \frac{1}{\pi} \int_1^\infty \frac{dy}{q_1 y^3} \int_0^\infty J_1(a\rho) \left[\cos\{\rho(y\tau + q_1 z)\} + \cos\{\rho(y\tau - q_1 z)\} \right] \frac{d\rho}{\rho} , \quad (6b)$$

and $q_1^2 = y^2 - 1,\ q_2^2 = \beta^2 y^2 - 1.$ Integrals in(6)are reduced to a single integral by nothing that

$$\int_0^\infty J_1(a\rho) \cos c\rho \frac{d\rho}{\rho} = \begin{cases} \dfrac{(a^2-c^2)^{1/2}}{a} & ; c \leq a \\ 0 & ; c \geq a \end{cases} \quad (7)$$

From(7)it is clear that the inner infinite integrals in(6)have values only when $(y \pm q_{1,2} z) \leq a$. This then limits the range of integration in(6)over y The details are discussed by Eason[12], which we use to derive the following results:

$$\int_1^\infty \frac{dy}{q_1 y^3} \int_0^\infty J_1(a\rho)\cos\{\rho(y\tau+q_1 z)\}\frac{d\rho}{\rho} = \begin{cases} 0 & ; \quad \tau \geqslant a > 0 \\ {}^1/a \displaystyle\int_1^{y_1^*} \frac{\left[a^2-(y\tau+q_1 z)^2\right]^{1/2}}{q_1 y^3}\, dy ; & \tau < a \end{cases}$$

$$y_1^* = \frac{a\tau - z\left[(a^2+z^2)-\tau^2\right]^{1/2}}{\tau^2 - z^2}$$

$$\int_1^\infty \frac{dy}{q_1 y^3} \int_0^\infty J_1(a\rho)\cos\{\rho(y\tau-q_1 z)\}\frac{d\rho}{\rho} = \begin{cases} 0 & ; \quad \tau \geqslant (a^2+z^2)^{1/2} \\ {}^1/a \displaystyle\int_b^{y_1^{**}} \frac{\left[a^2-(y\tau-q_1 z)^2\right]^{1/2}}{q_1 y^3} ; & \tau < (a^2+z^2)^{1/2} \end{cases}$$

$$b = 1 \text{ if } \tau \leqslant a;\ b = y_1^* \text{ if } a < \tau < (a^2+z^2)^{1/2},\ y_1^{**} = \frac{a\tau + z\left[(a^2+z^2)-\tau^2\right]^{1/2}}{|\tau^2 - z^2|} .$$

$$\int_{1/\beta}^\infty \frac{dy}{q_2 y p} \int_0^\infty J_1(a\rho)\cos\{\rho(y\tau+q_2 z)\}\frac{d\rho}{\rho} = \begin{cases} 0 & ; \quad \tau \geqslant \beta a \\ {}^1/a \displaystyle\int_{1/\beta}^{c} \frac{\left[a^2-(y\tau+q_2 \tau)^2\right]^{1/2}}{q_2 y p}\, dy ; & \tau < \beta a \end{cases}$$

$$c = y_2^* \text{ if } y_2^* > {}^1/\beta ;\ c = {}^1/\beta \text{ if } y_2^* < {}^1/\beta ;\ y_2^* = \frac{a\tau - z\left[\beta^2(a^2+z^2)-\tau^2\right]^{1/2}}{\tau^2 - \beta^2 z^2} .$$

$$\int_{1/\beta}^\infty \frac{dy}{q_2 y p} \int_0^\infty J_1(a\rho)\cos\{\rho(y\tau-q_2 z)\}\frac{d\rho}{\rho} = \begin{cases} 0 & ; \quad \tau \geqslant \beta(a^2+z^2)^{1/2} \\ {}^1/a \displaystyle\int_b^{y_2^{**}} \frac{\left[a^2-(y\tau-q_2 z)^2\right]^{1/2}}{q_2 y p}\, dy ; & \tau < \beta(a^2+z^2)^{1/2}, \end{cases}$$

$$b = y_2^{**} \text{ if } y_2^{**} < {}^1/\beta ;\ b = y_2^* \text{ if } \beta a \leqslant \tau < \beta(a^2+z^2)^{1/2} \text{ and } y_2^* > {}^1/\beta ;$$

$$b = {}^1/\beta \text{ if } \beta a \leqslant \tau < \beta(a^2+z^2)^{1/2} \text{ and } y_2^* \leqslant {}^1/\beta ,\ y_2^{**} > {}^1/\beta ;\ b = {}^1/\beta \text{ if } \tau < \beta a \text{ and } y_2^{**} > {}^1/\beta ;$$

$$y_2^{**} = \frac{a\tau + z\left\{\beta^2(a^2+z^2)-\tau^2\right\}^{1/2}}{|\tau^2 - \beta^2 z^2|}$$

In fig 3, u at $r=0$ is plotted as a function of $c_1 t/a$ at $z/a = 0, 1/2$ and 1 ($\lambda = \mu$). The displacement at first increases linearly, *i.e.*, the particle velocity is constant. This particle velocity is approximately $\sigma c_2/2\mu$ which is half the predicted value by the simple model [1,2,3] where μ = rigidity. This is so because we have considered the force in an infinite medium. Assuming the fault surface to be totally reflecting during rupture we get the predicted particle velocity of $\sigma c_2/\mu$. The velocity decreases after the arrival of *P*-wave from the edge of the fault and displacement reaches its static value after the arrival of *S*-wave. From plots at $z/a = 1/2$ and 1 it is noted that the initial contribution to the displacement u from *P*-wave is negligible. Unlike the numerical solution of instantaneous circular fault problem where an overshoot of displacement (above the static value) appears[8], we find no such overshoot in the present case. The plots in fig 3 are very similar to results from foam rubber experiment[9].

The case considered here confirms the predictions of Brune[3] on the near-field ground motion. The motion is directly related to the effective stress available to accelerate the sides of the fault. Effective stress is usually taken to be equal to the stress drop (Orowan model) which is determined from the seismic moment M_0 and the radius of the fault a (often estimated from teleseismic body wave spectra) and assuming constant stress drop on the fault. A variable stress drop may be more reasonable for earthquakes in which case the stress drop, and thus ground motion estimates, obtained from M_0 and a would change[13]. Variable stress drop, couple forces, and finite rupture velocity is presently being incorporated in the model.

CONCLUSIONS

An earthquake model for predicting near-field ground motion has been developed in terms of body forces. For a stress pulse applied instanteneously over a circular area, the ground motion agrees with experiments and for short times, with the simple model in which a stress pulse is applied over an infinite area. The formulation of the problem in terms of body forces permits variable stress drop, couple sources and finite rupture velocity to be taken in to account.

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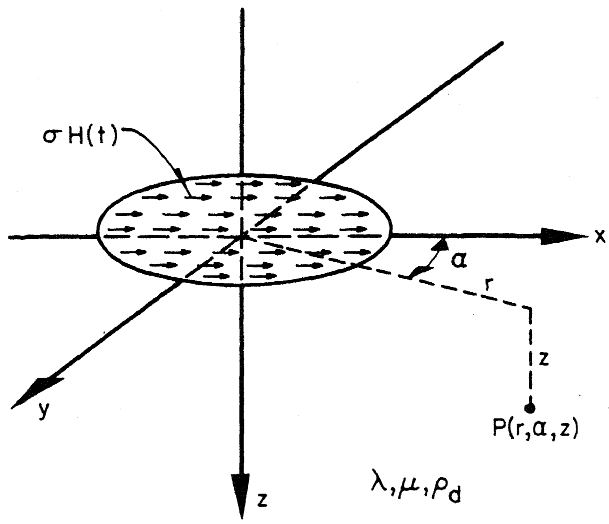

Fig 1. A tangential force acting on a circle of radius a in an infinite space

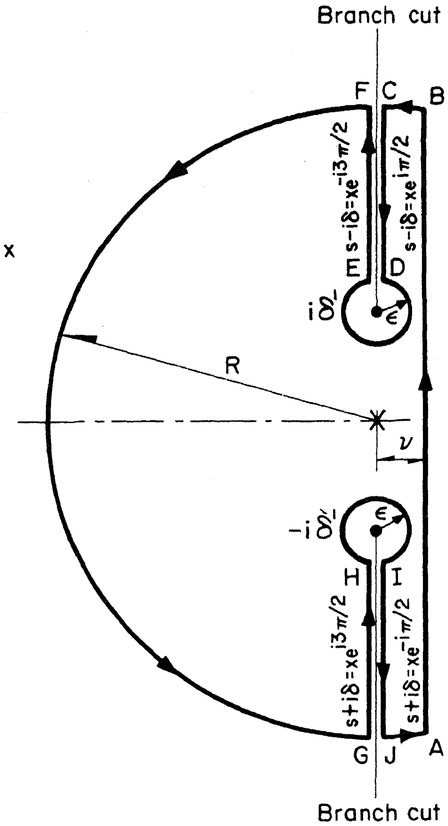

Fig 2. Contour in the complex s-plane. Pole (x), branch points (•) $\delta' = \rho$ or ρ/β

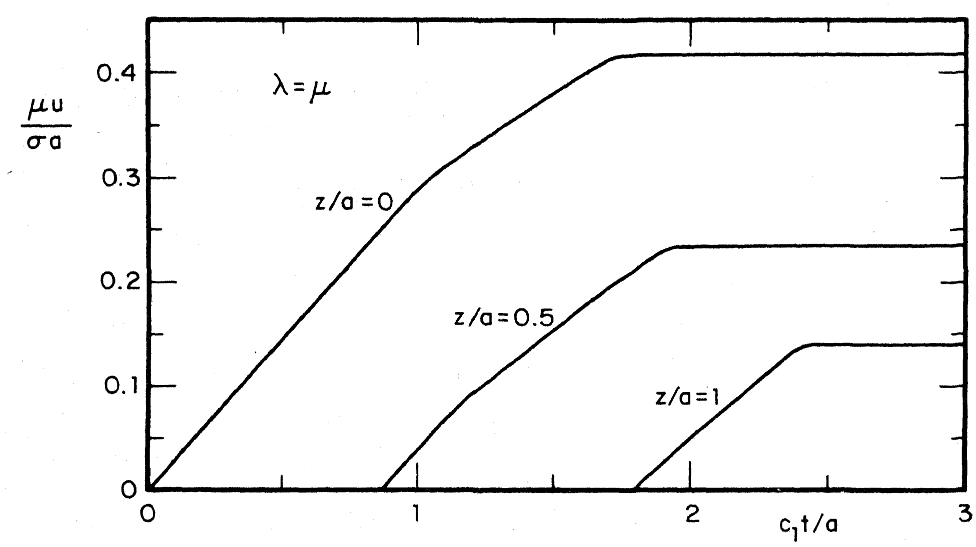

Fig 3. x component of displacement on the axis of the circle, at different distances, as a function of time