SEISMIC RESPONSE ANALYSIS OF SURFACE LAYER
WITH IRREGULAR BOUNDARIES

by

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SYNOPSIS

The semi-infinite half-space consisting the base ground is taken into consideration in the seismic response analysis of a near surface ground by virtue of the discrete boundary equation which is deduced from an integral equation. Illustrative examples are given to indicate the applicability of the method and to examine the accuracy of numerical computation program.

INTRODUCTION

In analysing the seismic response of surface layer, the multiple reflection and refraction theory has been frequently used[1]. In the theory it is assumed that the system is consisted of horizontally stratified layers overlying homogeneous half-space and excited by vertically incident, plane shear waves. However this simplified model can not always represent the actual response of surface layer. On the other hand, transforming the surface layer into a finite system, it is possible to analyze the seismic response of surface layer with complex geological properties. But there are many difficulties in finite element idealization of actual system[2,3]. In the method the response of system is considerably affected by the position of boundary which encloses the finite area in an infinite media. Furthermore it is hard to take into account of the radiation-damping, because the energy of incident wave can not radiate through the boundary as closed form solutions in an infinite solid. The purpose of this investigation is, therefore, to present an effective analysing procedure for the surface layer with general irregular boundary. The first part of the study is devoted to the development of the analysing procedure and the derived method is then applied to a few actual examples to show the usefulness of the method.

METHOD OF ANALYSIS

Considering the homogeneous and isotropic half-space, the fundamental solutions of this media are defined by the displacement U_{jk} or force P_{jk} in x_j direction at point Q, which are induced by the concentrated force $\delta(x_k-r_k)\cdot exp(i\omega t)$ acting in x_k direction at point R. The solution of boundary value problem of dynamic elasticity based on the integral method are obtained by determining the fictitious force intensity acting upon the auxilliary surface S^* (Fig.1) as satisfying the boundary condition on S[4]. If the surface S and S^* are replaced with the polyhedron of N vertices, then the equations determining the fictitious force intensity Φ on those N nodal points are given as follows:

(i) when the surface traction is given by $t\cdot exp(i\omega t)$

$$\mathrm{T} = \mathrm{L}\,\Phi \qquad (1)$$

where T is a vector of order $3N$ determined from the surface traction on the frequency domain t, Φ is a vector of order $3N$ and L is a square matrix of order $3N$ defined by

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$$[L]_{lm} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}_{lm} \cdot \Delta S_m^* \qquad (l,\ m = 1,\ 2, \cdots\cdots,\ N)$$

(ii) when the surface displacement is given by $u\,exp(i\omega t)$

$$U = D\ \Phi \tag{2}$$

where U is a vector of order $3N$ determined from the surface displacement in frequency domain u and D is a square matrix of order $3N$ defined by

$$[D]_{lm} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix}_{lm} \cdot \Delta S_m^* \qquad (l,\ m = 1,\ 2, \cdots\cdots,\ N)$$

The equation of motion for surface layer A (Fig.2a) in matrix form is

$$(-\omega^2 M + i\omega C + K)\ \delta = F \tag{3}$$

in which M, C and K are the mass, damping and stiffness matrix for a finite element system, and δ is a vector of the nodal point displacement. For the purpose of combining the boundary equation, it is necessary to express Eq.(3) in partitioned form

$$\left(-\omega^2 \begin{bmatrix} M_b & M_c \\ M_c^T & M_A \end{bmatrix} + i\omega \begin{bmatrix} C_b & C_c \\ C_c^T & C_A \end{bmatrix} + \begin{bmatrix} K_b & K_c \\ K_c^T & K_A \end{bmatrix} \right) \begin{Bmatrix} \delta_b \\ \delta_A \end{Bmatrix} = \begin{Bmatrix} F_b \\ F_A \end{Bmatrix} \tag{4}$$

in which δ_b and F_b are the deformation and force of nodal points on the boundary S, respectively.

If the displacement and force at nodal points on the surface S induced by the incident wave g in an infinite homogeneous and isotropic half-space with imaginary surface l-l' (Fig.2b) are determined as d and f, respectively, then the fictitious force intensity distributed on the nodal points of surface S^* satisfies the conditions, considering the continuity of stress and displacement on the boundary S:

$$\delta_b = D\ \phi + d \quad (5), \qquad F_b = -f - L\ \Phi \quad (6)$$

Eliminating Φ from Eqs.(5) and (6), the relationship between the deformation and force of nodal points on the boundary S are obtained as follows:

$$F_b = -(K^*_b + i\omega C^*_b)\delta_b + F^*_b \tag{7}$$

in which

$$LD^{-1} = K^*_b + i\omega C^*_b \quad , \qquad F^*_b = -f + LD^{-1}d \tag{8}$$

Eqs.(4) and (7) can be combined to give

$$\left(-\omega^2 \begin{bmatrix} M_b & M_c \\ M_c^T & M_A \end{bmatrix} + i\omega \begin{bmatrix} C_b + C_b^* & C_c \\ C_c^T & C_A \end{bmatrix} + \begin{bmatrix} K_b + K_b^* & K_c \\ K_c & K_A \end{bmatrix} \right) \begin{Bmatrix} \delta_b \\ \delta_A \end{Bmatrix} = \begin{Bmatrix} F_b^* \\ F_A \end{Bmatrix} \tag{9}$$

APPLICATION TO ONE DIMENSIONAL SH WAVE PROPAGATION

In the case of one dimensional harmonic SH wave propagation in y direction in Fig.3, the fundamental solutions for the domain $y>\eta$ are given as follows:

$$kU(y,\eta) = i\,exp(-iky)cos(k\eta),\quad kP(y,\eta) = \rho c\omega\ exp(-iky)cos(k\eta) \tag{10}$$

where $k=\omega/c$, c is shear wave velocity, ω is circular frequency, y and η are position of the observation and that of the source point, respectively. If the incident harmonic plane wave is given by $A\,exp(iky)$, the expressions for harmonic displacement d and shear stress f have the representations

$$d(y) = 2A\cos(ky), \qquad f(y) = -2\rho c\omega A\sin(ky) \tag{11}$$

Considering the surface layer with depth H as shown in Fig.3, Eq.(8) is rewritten in the scalar form

$$\begin{aligned} LD^{-1} &= P(H,\eta)/U(H,\eta) = i\rho c\omega \\ F_b^* &= 2i\rho c\omega A' \qquad (A' = A\exp(ikH)\ ) \\ C_b^* &= \rho c \ , \qquad K_b^* = 0 \end{aligned} \tag{12}$$

Substituting Eq.(12) into Eq.(9), the equation of motion for a layered system on an elastic half-space with vertically incident wave is obtained in a matrix form. This is similar to the equation of motion for layered medium obtained by Tsai and Housner[5]. In order to examine the accuracy of the calculation, solution of Eq.(9) under the restriction of Eq.(12) is compared with exact solution obtained by the multiple reflection theory. The result is shown in Fig.4, in which full line designates the exact solution and m is the number of nodal points in a surface layer. The ground parameters used for calculation are listed in Table 1. It is seen from this figure that for lower frequency range, even m=3 yields answers of acceptable accuracy, although more point needed for heigher frequency range.

This approximate computational method is also applied to a surface layer composed of the material with non-linear properties. In this case, the motion of equation being transformed into time domain is used, instead of Eq.(9). As an example, Fig.5 shows the distribution with depth of the maximum shear stress and strain in a subsurface ground, for which surface material is considered to have bi-linear properties listed in Table 1. The prescribed acceleration is NS component of the El Centro, 1940 and it is so modified as to have the maximum acceleration of 100, 200 and 300 gal. The result reveals that the ratio of shear strain to input acceleration level is dependent to the input amplitude and that, in the contrary, the ratio of shear stress to input level is not so significantly affected.

APPLICATION TO TWO DIMENSIONAL SH WAVE PROPAGATION

The fundamental solutions for outgoing waves in a half-space are expressed by the Hankel function of the second kind as follows[6],

$$\begin{aligned} U(x,y/\xi,\eta) &= 0.25i(H_0^2(k((x-\xi)^2+(y-\eta)^2)^{12}+H_0^2(k((x-\xi)^2+(y-\eta)^2)^{12})) \\ P(x,y/\xi,\eta) &= \mu\partial U(x,y/\xi,\eta)/\partial n \end{aligned} \tag{13}$$

where n is the unit normal at (x,y), μ is the shear modulus, (x,y) and (ξ,η) are the position of the observation and source point, respectively. An incident harmonic plane wave of unit magnitude, making a counterclockwise angle θ with respect to x axis, has the form

$$u(x,y) = \exp(ik(-x\cos\theta + y\sin\theta)) \tag{14}$$

Then the expressions for harmonic displacement d and total shear stress f at the point $p(x,y)$ are given by

$$\begin{aligned} d(x,y) &= 2\exp(-ikx\cos\theta)\cos(ky\sin\theta) \\ f(x,y) &= \mu\partial d(x,y)/\partial n \end{aligned} \tag{15}$$

Numerical computations for two example models are performed to show the applicability of the method to various dynamic response problems. The first example is prepared to test the computational program. Fig.6 is the schema of the finite element mesh used in computations, in which the nodal points on the interface are positioned on a semi-circle with radius a. Fig.7 shows the displacement and phase angle response of the interface for SH wave having incident angle 60° and nondimensional frequency $\omega a/\pi c=0.75$. In this figure the curves represent the exact solutions and dots the results obtained by the proposed method. The case A shows the interface displacement when the base ground and the semi-circular region have the same elastic constants. The case B shows the response of a half-space from which semi-circular region is removed. The exact solution for this case is given by Trifunac[7]. Comparison of both results shows a high degree of accuracy of the method.

The second example is prepared to investigate the influence of incident angle and frequency on the response of ground and/or super-structures. The finite element idealization of an embankment are shown in Fig.8. The material properties for the base ground and the embankment are given in Table 2. Figs.9 and 10 show the displacement amplitude along the surface of embankment due to an incident harmonic wave with unit amplitude and angle of incident 0°, 30°, 60° and 90°. Shear wave velocity of the embankment is assumed to be 160 and 320m/sec in case of Figs.9 and 10, respectively. From these figures, it is concluded that the distribution of surface displacement depends on the direction of arriving wave and on the ratio of shear wave velocity of the base ground to that of embankment, and that the influence of frequency on the response becomes more noticeable when the wave length of the incident motion is comparable with the length of the embankment base.

CONCLUSION

The method studied in this paper is applicable for calculating the seismic response of a system with arbitrary shaped boundary. The degree of accuracy of the method is examined by comparing the response of ground motion calculated by this method with that done by the exact solution for several numerical models. Moreover the method is applied to an embankment with actual shape in order to examine the effect of the angle of incident and of the material properties of the ground.

Although the analysis presented herein considers only simplified SH wave, it is not difficult to expand the method for the case that another kind of body waves and surface waves are incorporated provided the fundamental solutions of layered ground are given.

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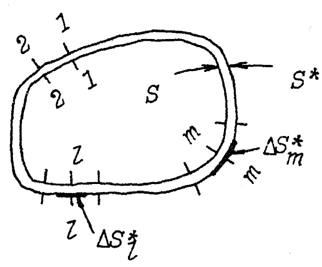


Fig.1 Schematic representation of actual and auxiliary boundary.

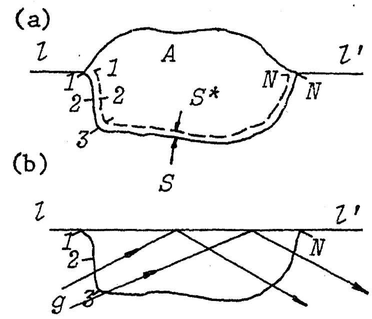


Fig.2 Physical model for near surface and base ground.

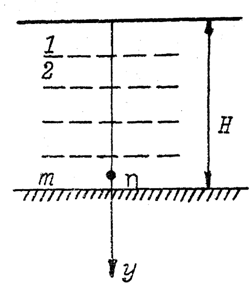


Fig.3 Surface layer

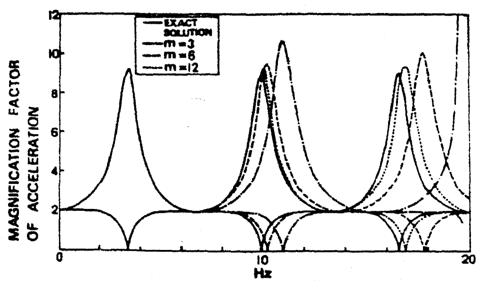


Fig.4 Frequency response of the ground surface and the base ground.

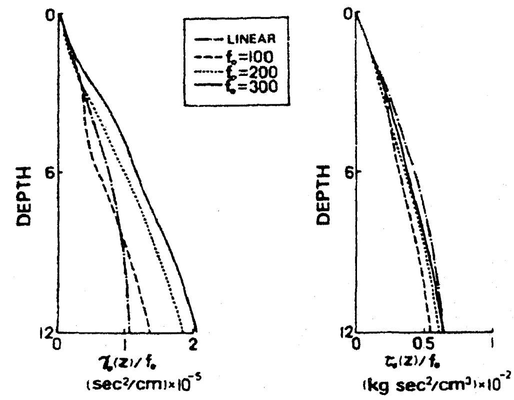


Fig.5 Distribution of shear strain and stress with depth.

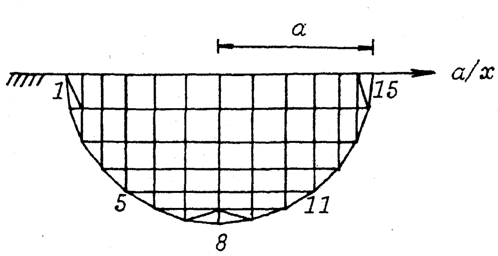


Fig.6 Finite element mesh for semi-circular boundary.

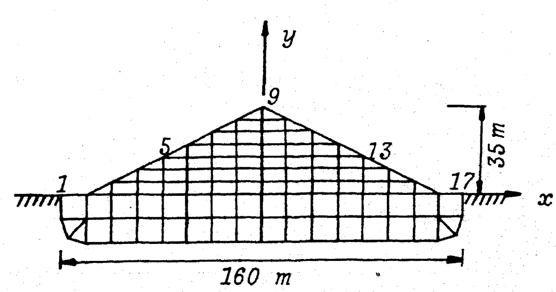


Fig.8 Finite element idealization of an embankment and supporting ground.

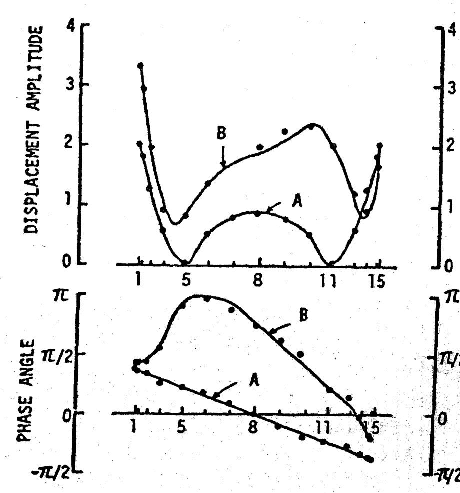
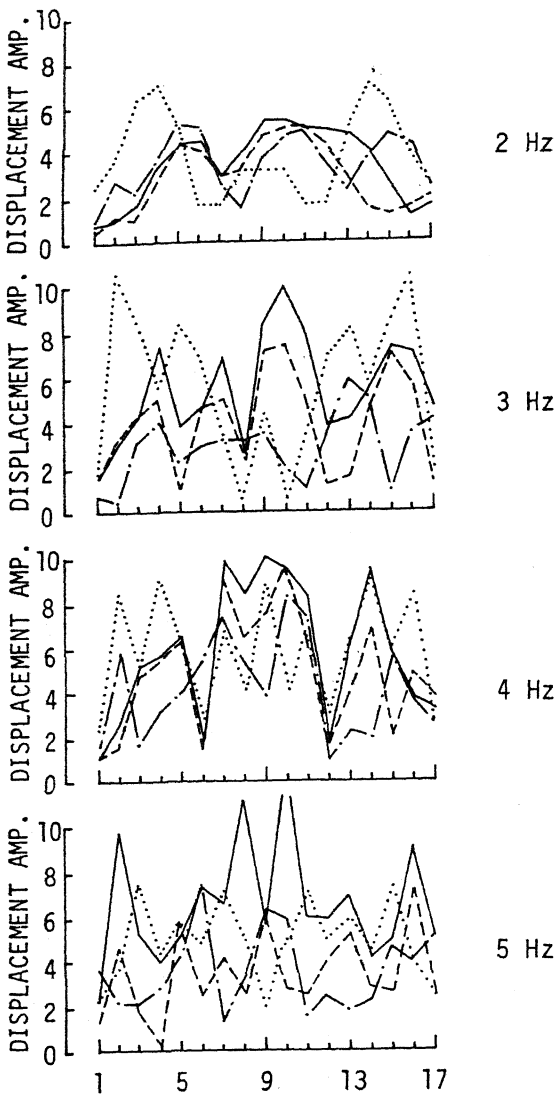


Fig.7 Displacement amplitude and phase angle of the interface between base ground and enclosed region



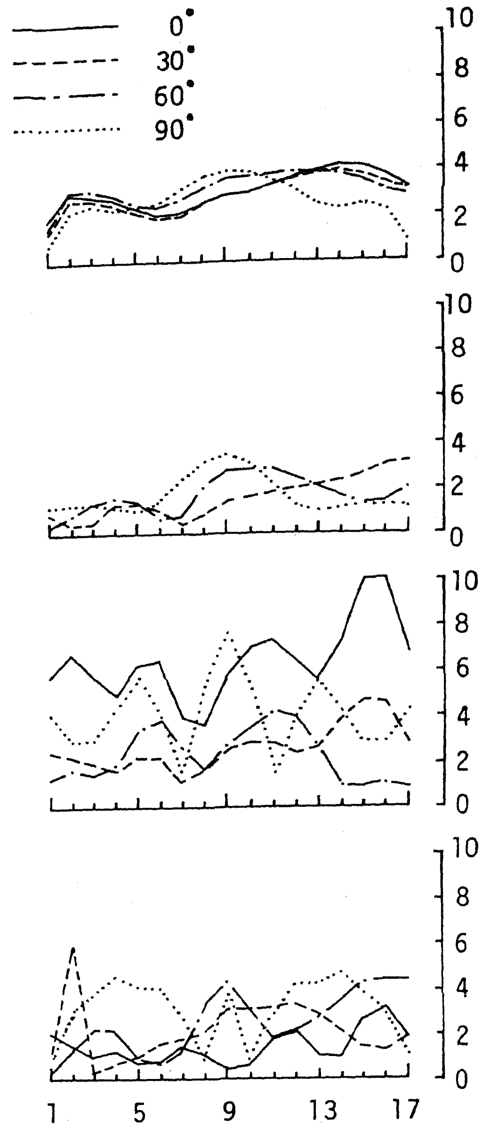


Fig.9 Displacement amplitude along the surface of embankment. (C_e/C_b = 0.5)

Fig.10 Displacment amplitude along the surface of embankment. (C_e/C_b = 1.0)

Table 1 Layer parameters.

	Linear	Bi-linear
Surface layer	C=160 m/s ρg=2.24t/m^3 H=12 m	n=0.5 $\gamma_y=5\times10^{-4}$
Base layer	C=720 m/s ρg=2.30t/m^3	------

Table 2 Parameters for base ground and embankment

	Base ground	Embankment
Density (t/m^3)	1.8	1.8
Shear wave velocity	C_b=320m/s	C_e=160m/s
		C_e=320m/s

ρg : density, n : elasto-plastic parameter, γ_y : yield strain

DISCUSSION

J.M. Mulay (India)

What is the functional you have considered and what is the shape of the finite element you have considered ?

Author's Closure

With regard to the question of Mr. Mulay, we wish to state that from the standpoint of the variational theory, it would be better to define the general functional including also the domain B which is enclosing the discretized domain A and transmits the waves radiated from the irregular domain A. In such a case the functional dose not demand the subsidiary condition. In this analysis, however, we used the classical functional which is obtained by the method developed by Washizu, which is defined within the domain A, and from which we can easly obtaine the equation of motion such as Eq. (4). Therefore the subsidiary condition of Eq. (7) is considered to combine the analytical domain B with the domain A.

Triangular and square element are used in this analysis.