STRUCTURAL OPTIMIZATION IN ASEISMIC DESIGN

N.C. Nigam[I] and S. Narayanan[II]

SYNOPSIS

The structural optimization problem is formulated in deterministic and probabilistic framework for earthquake excitation. The problem is reduced to a nonlinear programming problem. The proposed algorithm is applied to the minimum weight design of a truss structure supporting an elevated water tank.

INTRODUCTION

The aseismic design of structures may be undertaken within a deterministic or a probabilistic framework. The occurence of earthquakes during the service life of a structure and the nature of ground motion during such an event are essentially random phenomena. In view of this, a probabilistic formulation of aseismic design problems is necessary. However,due to lack of statistically significant information regarding earthquake phenomena and relative complexities of the probabilistic approach, aseismic design has been largely undertaken within a deterministic framework. In this paper the deterministic and probabilistic methods of aseismic design are extended to the domain of structural optimization. The optimization problem is formulated with constraints on the dynamic characterstics of the structure and its response during an earthquake. The problem is reduced to a nonlinear programming problem by eliminating time from the inequality constraints (3,4). In this form, optimization may be carried out by one of the algorithms of nonlinear programming (5). The proposed formulation is applied to the minimum weight design of a truss structure supporting an elevated water tank using both the probabilistic and deterministic approaches.

OPTIMIZATION PROBLEM

A typical aseismic design optimization problem can be stated in probabilistic framework as; Minimize $W(\bar{d})$ (1)

Subject to $$P\left[\bigcup_{i=1}^{m_j}\left\{s_{ji}(\bar{x}(\bar{d},t)) \geq r_{ji}\right\}\right] \leq p_j \quad j=1,2\ldots n \qquad (2)$$

$$0 \leq t \leq T$$

I. Associate Professor of Aeronautical Engineering, Indian Institute of Technology, Kanpur, (INDIA).

II. Graduate Student, Indian Institute of Technology, Kanpur.

$$g_k(\bar{d}) \le \alpha_k \qquad k=n+1, \dots n_1 \qquad (3)$$

$$\omega_l^L \le \omega_l(\bar{d}) \le \omega_l^U \qquad l=1,2, \dots n_2 \qquad (4)$$

where $\bar{d}$ is the design vector; $\bar{x}(\bar{d},t)$ is the dynamic response due to earthquake; r_{ji} is the constraint on the function s_{ji}, p_j is the specified upperbound on the probability of failure; and $\omega_l(\bar{d})$ is the lth natural frequency of the system. In a deterministic formulation, the inequalities (2) are replaced by a set of inequalities of the form

$$s_h(\bar{x}(\bar{d},t)) \le r_h \quad h=1,2, \dots (m_1+m_2 \dots +m_n) \qquad (2a)$$

The occurence of time in the inequalities (2) and (2a) adds a dimension of difficulty to the optimization problem. In order to obtain a solution of the optimization problem through available algorithms of mathematical programming, the time dependence must be eliminated from these inequalities. Let

$$P\left[s_{ji}(\bar{x}(\bar{d},t)) \ge r_{ji}\right] \le 1-P\left[\max_{0\le t\le T}\{s_{ji}(\bar{x},(\bar{d},t))\} \le r_{ji}\right] = U_{ji}(\bar{d}) \qquad (5)$$

$$\text{If } U_{ji}(\bar{d}) \ll 1,\ P\left[\bigcup_{i=1}^{m_j}\{s_{ji}(\bar{x}(\bar{d},t)) \ge r_{ji}\}\right] \le \sum_{i=1}^{m_j} U_{ji}(\bar{d}) = q_j(\bar{d}) \qquad (6)$$

$$\text{Let} \quad q_h(\bar{d}) = \max_{0\le t\le T}\left[s_h(\bar{d},t)\right] \qquad (7)$$

Inequalities (2) and (2a) may, therefore, be replaced by time independent inequalities of the form

$$q_j(\bar{d}) \le b_j \qquad j = 1,2, \dots n \text{ or } (m_1+m_2 \dots +m_n) \qquad (8)$$

which reduce the optimization problem to a nonlinear programming problem, which can be solved by one of the several available algorithms.

EXAMPLE

An elevated water tank is supported on a truss structure as shown in Fig.1. The optimization problem is formulated as:

Minimize weight of the truss $W(\bar{d})$, subject to

Stress constraint: $P\left[\bigcup_{i=1}^{16}\{|s_i(\bar{d},t)| \ge r_i\}\right]_{0\le t\le T} \le 10^{-4}$ (9)

Acceleration constraint: $P\left[|a(\bar{d},t)| \ge \alpha g\right]_{0\le t\le T} \le 10^{-4}$ (10)

Frequency constraint: $5 \le \omega(\bar{d}) \le 30$ (11)

Side constraint: $d_i \ge 0 \quad i = 1,\dots\dots 12$ (12)

where $\bar{d}$ is the design vector with elements $A_1, A_2, A_3, A_6, A_7, A_8, A_{11}, A_{12}, A_{16}, L_2, L_7$; A_i and L_i are area of crosssection and length of ith member; s_i is the stress in the ith member and r_i is the yield stress; $a(t)$ is the absolute acceleration of the water tank. In the deterministic formulation, constraints (9) and (10) are replaced by

Stress constraint: $\max_{0 \leq t \leq T} |s_i(\bar{d},t)| \leq r_i \quad i = 1,2, \ldots 16$ (9a)

Acceleration constraint: $\max_{0 \leq t \leq T} |a(\bar{d},t)| \leq \alpha g$ (10a)

The dynamic response of the system is obtained by treating it as a single degree of freedom system with water tank as rigid mass. The ground acceleration is treated as a stationary random processes with power spectral density (1)

$$G(\omega) = (2.9)^2 \times .01238 \; (1+ \omega^2/147.8)/((1- \omega^2/242)^2 + \omega^2/147.8) \quad (13)$$

In the deterministic formulation the maximum response is obtained by using the velocity spectra given by (1)

$$S_V(\zeta,\omega,T) = 2.9 \times 1.7976 \left[\pi \, G(\omega)(1- e^{-2\zeta\omega T/2.44})/2\zeta\omega\right]^{1/2} \quad (14)$$

Assuming the response to be stationary Gaussian, we have (2)

$$P\left[\max_{0 \leq t \leq T} \{|s_i(t)|\} \geq r_i\right] = 1-\exp\left[-T/\pi \frac{\sigma_{\dot{s}_i}}{\sigma_{s_i}} \exp\left(-\frac{1}{2}(r_i/\sigma_{s_i})^2\right)\right] \quad (15)$$

The constraints on frequencies and acceleration are chosen so as to keep the first three natural frequencies of free water oscillations less than the natural frequency of the structure and to limit the displacement of the free water oscillation. Stress constraints limit the stress within the yield limit.

The reduced nonlinear programming problem is solved by the SUMT of Fiacco and McCormick (5) incorporating a Fletcher Powell algorithm.

CONCLUSIONS

The structural optimization problem for aseismic design is formulated as a nonlinear programming problem. Optimization leads to considerable saving in weight. The results for the probabilistic formulation and deterministic formulation with factors of safety of 2.0 and 3.0 are given in Tables I, II and III respectively.

The probabilistic design has the advantage of providing an upper bound on the reliability of the design.

REFERENCES

1. Jennings, P.C.; 'Response of Simple Yielding Structures To Earthquake Excitation'. EERL, CALTECH, June 1963.
2. Amin, M. and I.Gungar; ASCE National Structural Engg.Meeting, Bultimore, Maryland, April 19-23, 1971, preprint 1409.
3. Fox, R.L., Kapoor, M.P.; AIAA J, Oct.1970, pp 1798-1804.
4. Nigam, N.C.; AIAA J, April 1972, pp 551-553.
5. Fiacco, A.V., McCormick, G.P.; 'Nonlinear Programming:- The Sequential Unconstrained Minimization Techniques', Newyork Wiley, 1968.

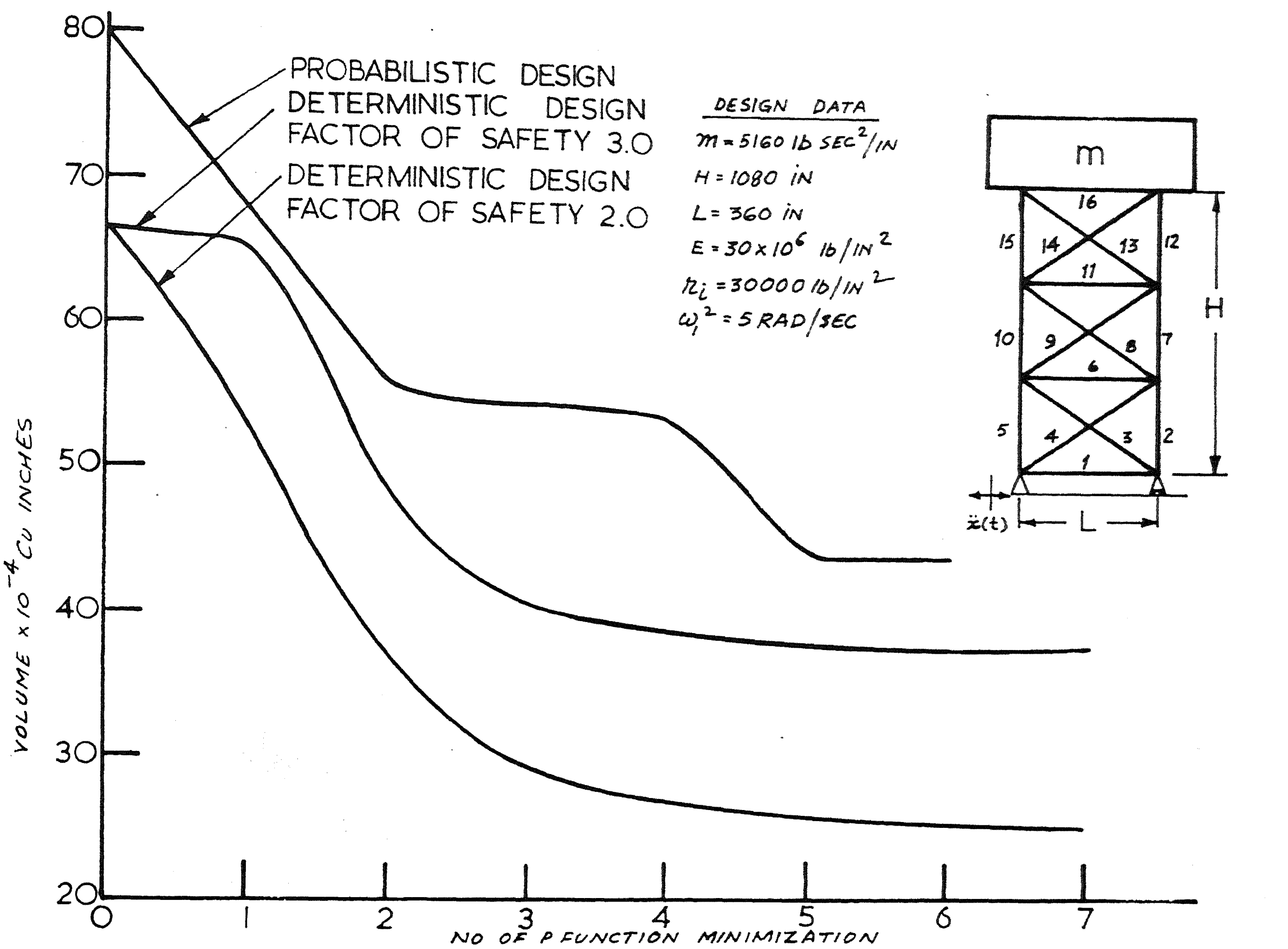

FIG.1_SEQUENCE OF UNCONSTRAINED OPTIMA CONVERGING TO CONSTRAINED OPTIMUM

TABLE I _ PROBABILISTIC DESIGN

	A_1	A_2	A_3	A_6	A_7	A_8	A_{11}	A_{12}	A_{13}	A_{16}	L_2	L_7	VOLUME	% RED.
INITIAL	120	120	120	120	120	120	120	120	120	120	360	360	798564	IN VOLUME
FINAL	52.7	77.13	71.43	100.9	84.45	65.39	89.08	42.56	52.98	48.74	325.7	333.2	439533	45

ACTIVE CONSTRAINTS AT OPTIMUM STRESS CONSTRAINT.

TABLE II _ DETERMINISTIC DESIGN FACTOR OF SAFETY – 2

	A_1	A_2	A_3	A_6	A_7	A_8	A_{11}	A_{12}	A_{13}	A_{16}	L_2	L_7	VOLUME	% RED
INITIAL	100	100	100	100	100	100	100	100	100	100	360	360	665470	IN VOLUME
FINAL	29.33	53.16	39.48	61.96	33.3	45.65	52.72	22.52	30.18	20.10	324	353	252098	62

ACTIVE CONSTRAINTS AT OPTIMUM. STRESSES IN MEMBERS 1, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,

TABLE III. DETERMINISTIC DESIGN FACTOR OF SAFETY – 3

	A_1	A_2	A_3	A_6	A_7	A_8	A_{11}	A_{12}	A_{13}	A_{16}	L_2	L_7	VOLUME	% RED.
INITIAL	100	100	100	100	100	100	100	100	100	100	360	360	665470	VOLUME
FINAL	43.58	81.11	58.58	92.24	50.80	67.01	78.91	34.87	46.12	30.19	324	340	378159	43

ACTIVE CONSTRAINTS AT OPTIMUM STRESSES IN MEMBERS 1, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,

ALL DIMENSIONS IN INCH UNITS.