STOCHASTIC NETWORK FORMULATION AND SOLUTION OF EARTHQUAKE RISK IN STRUCTURAL DESIGN

by

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SYNOPSIS

The risk of earthquake damage to a structure at a given site is determined by simulation of a stochastic network representing the chain of uncertainties which begins with the occurrence of an earthquake and results in economic loss.

INTRODUCTION

Seismic risk studies should give insight into the possible consequences of design decisions in earthquake prone regions. Seismic codes are only minimum criteria for reasonable protection, but they are no guarantee against failure and they do not indicate accepted risk levels. Since seismic design is a compromise between cost and potential earthquake loss of a structure, it seems essential that the entire problem be formulated and solved in a quantitative fashion. Herein, the earthquake risk problem, with its economic consequences, is presented in network form using GERT(1) (Graphical Evaluation and Review Technique) symbolism. Such a pictorial representation of the problem, and its solution by digital simulation, should be of real practical value as a means of communication between experts, insurance underwriters, and engineers finally responsible for design decisions.

GERT SYMBOLISM

The basic elements of a GERT network can be viewed as events (nodes) related to each other by precedence relations (branches). Two random variables are associated here with each event; the maximum structural response and the corresponding cost. The parameters associated with a node, shown in Fig. 1, are: event number A; the type of probability distribution of the structural response defined by a selection number t_d and a list number t_p (e.g., $t_d = 1$ for response at specified level t_p and $t_d = 7$ for response from a beta distribution with parameters defined in the list t_p); the cost associated with the occurrence of the event, c_s, and a cost per unit response c_v; the index F to specify that averages, standard deviations and histograms of both structural response at first occurrence of the event and corresponding costs are determined after several simulations of the network; and the number R of preceding branches which must be taken before letting event A occur (releasing A). The output side of the event represents a branching operation where, if the branching is DETERMINISTIC (semi circle at the output), all branches emanating from the activity will be activated or, if it is PROBABILISTIC (lazy V), a selection of one of the branches emanating from the event is made with the probability p that it is a given branch.

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SEISMIC RISK NETWORK

The following qualitative description of the network shown in Fig.2 illustrates an hypothetical earthquake risk situation. Assume that the situation involves 50 years of the life of a structure of given strength at a given site. Releasing node 1 (source node) represents the beginning of the 50 year period. Realization of node 20 (sink node) represents the end of the period. Since $t_p=c_s=c_v=0$ and $t_d=1$ for nodes 1 to 7 and 20, no structural displacement or cost is associated with the occurrence of the corresponding events. After node 1 has been realized, nodes 2, 18, and 19 are released. Nodes 18 and 19 set limits of structural response, X and Y, beyond which permanent damage and catastrophic failure will occur, respectively. Node 2 is a dummy node used to initiate a probabilistic branching. The probability of .5 of taking branch 2-20 after realization of node 2 means that there is a 50 percent chance that during the 50 year period the largest earthquake to hit the site will have no engineering significance. Thus, the branch leading to node 20 will not incur structural displacement or cost in the period. Realization of node 3 indicates a significant earthquake has occurred. Realization of node 4 indicates that the largest earthquake experienced in 50 years was very intense; intensity being understood as an average measure of energy released per unit time. Nodes 5, 6, and 7 are similar to node 4, but they correspond to lower intensity earthquakes which have a greater probability of occurrence (.08, .15, and .75, respectively). One of the two branches emanating from nodes 4, 5, 6, or 7 is taken according to whether the earthquake which had a certain intensity lasted a long time (top branch) or short time (bottom branch), say 30 or 10 sec. The consequences of an earthquake of given intensity and duration are given as numerical values of the random variable response and its associated cost at the realization of any one of nodes 8 to 15. The beta probability distribution ($t_d=7$) with its parameter sets ($t_p=1$ through 8, are lists defining each beta distribution but not included here) is the conditional distribution of maximum response, given an earthquake of specified properties has occurred. Node 16 is realized simultaneously with the end of the period (node 20) only if the maximum structural response (from any node, 8 through 15) is greater than X (from node 18), and node 17 is realized only if that response is greater than Y (from 19).

Costs are defined according to earthquake losses which are separated into four levels: (1) type 1 - a minimum loss at each earthquake level given in nodes 8 through 15 as c_s=C, H, K, or L in Fig.2; (2) type 2 - a loss proportional to the maximum structural response given in nodes 8 through 15 as c_v=D; (3) type 3 - complete loss of the structure because of permanent damage if the maximum response is greater than a certain level X given in node 16 as c_s=A; (4) type 4 - catastrophic additional cost if the structure collapses and life is lost which occurs when the maximum response is greater than Y and given in node 17 as c_s=B.

One simulation of the network of Fig. 2 determines one path between node 1 and node 20 with the possible realization of nodes 16 and 17 (not included in the path). If the network is simulated a large number of times (here 6000), the statistics collected at nodes marked F are quantitative descriptions of the risk involved. For example, statistics on cost at node 20 include all the costs incurred prior to that node realization; that is, the largest cost over a 50 year period. The average of that cost should be the minimum insurance premium for the building. Statistics collected at nodes 8 through 15, both in terms of maximum structural displacement and cost, indicate what stages are contributing most to the overall risk.

Drawing the seismic risk problem in network form makes it much easier to identify the kind of information which should be obtained from various experts. It is clear that the joint distribution of intensity and duration of ground motion at the site is needed to assign branch probabilities. An engineering measure of intensity such as the power density of ground acceleration over a relevant range of frequencies is needed to identify nodes 4 to 7. With potential earthquakes having their statistical properties defined, it is possible for the engineer to find a probability distribution function which will predict the corresponding maximum structural response. Analytical work has been done in this direction for elastic structures, and Monte Carlo simulations have been used for some nonlinear cases[2].

EXAMPLE SIMULATION

Sample output from a computer simulation of the network in Fig.2 is shown in Table 1. Unity is used to denote maximum elastic displacement; thus, the expected value of maximum structural displacement for the most intense and longest hypothesized earthquake (node 8) is 17.32 times the elastic limit of the structure. A displacement of 10 times yield (X=10) was used to represent permanent damage level, and 20 times yield (Y=20) for the collapse level. Information about the total cost over the hypothesized 50 year period is given in Table 2. The same network could be used with different lists t_p to compare risks in alternate designs of the same structure.

CONCLUSION

A description of a GERT network representing an hypothetical earthquake risk situation was made to illustrate the advantages, both in term of formulation and solution, of the method. The stochastic network formulation of the problem gives a format in which experience from various disciplines can be used to help the structural designer make the best possible decisions.

REFERENCES

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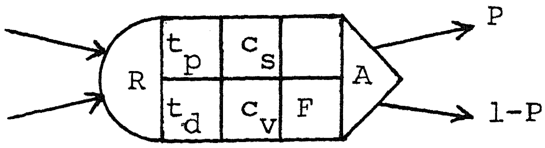

Fig. 1. P-GERT Node Symbolism - Probabilistic Output

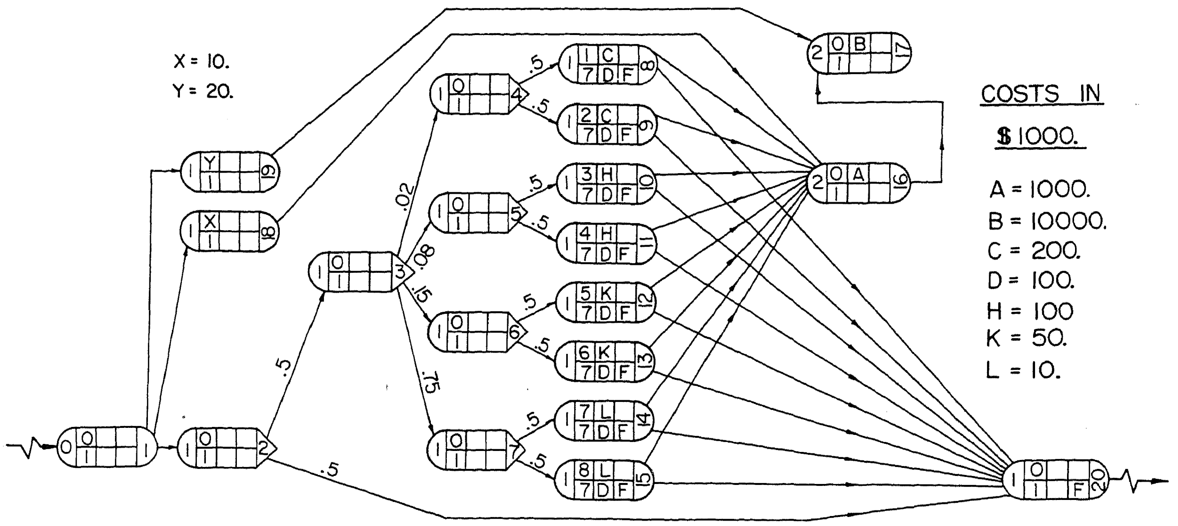

Fig. 2. P-GERT Network Representation of Seismic Risk

Table 1. Maximum Responses Recorded From 6000 Simulations

Node	8	9	10	11	14	20
No. of Obs.	21	24	118	128	1130	6000
Mean	17.32	13.27	5.65	3.56	.56	.57
Std. Dev.	5.45	5.03	1.82	1.67	.15	1.70
Minimum	10.89	5.92	3.08	2.00	.37	.00
Maximum	27.22	21.56	10.92	6.99	.99	27.22

Table 2. Histogram of Total Cost in $1,000,000. (Recorded at node 20 for 6,000 simulations)

Lower Cell limit	0.0	0.5	1.0	2.0	2.5	3.0	12.5
Upper Cell limit	0.5	1.0	1.5	2.5	3.0	3.5	13.5
No. of Obs.	5818	135	6	15	11	7	8

Mean Value = .0898   Minimum Value = .0000
Standard Deviation = .5435   Maximum Value = 13.9228