PROBABILISTIC ASSESSMENT OF MAXIMUM RESPONSE ACCELERATION

by

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SYNOPSIS

The empirical distribution of the acceleration magnification factor for the typical alluvial ground in Japan was obtained. By combining this with the probability distribution of the maximum ground acceleration for a given site for a given future period, the distribution of the maximum response acceleration was calculated. Discussions were made for several points of interest regarding the problems of earthquake resistant design indicated by the results of example computations.

THEORY

Denote the maximum ground acceleration of an earthquake expected for a given site during a given interval of future time by α, and the acceleration magnification factor (AMF) by β, which is defined as the ratio of the maximum response acceleration of a one-DOF linear system with given damping and natural period to the maximum ground acceleration. Assume that these two quantities be independent. Then, the maximum response acceleration α_R can be determined by

$$\alpha_R = \alpha \cdot \beta \qquad (1)$$

By denoting the probability density of α by $p(\alpha)$ and that of β by $q(\beta)$, the probability that the maximum response acceleration exceeds α_R can be expressed as

$$P(\alpha_R) = \iint_{\alpha\beta>\alpha_R} p(\alpha)q(\beta)d\alpha d\beta \qquad (2)$$

The two-dimensional probability density $p(\alpha)q(\beta)$ has a distorted bell-shape and its volume in the region specified by $\alpha\beta>\alpha_R$ is the probability $P(\alpha_R)$.

PROBABILITY DENSITY FUNCTIONS $p(\alpha)$ AND $q(\beta)$

Goto and Kameta[II] have calculated the probability distributions of the maximum ground acceleration for various places in Japan expected in 75 years by using a probabilistic model incorporating the irregularities of both earthquake occurrences and characteristics of motions. Though the distributions do not show smooth curves mainly because of the lack of data, the probability density $p(\alpha)$ was computed by numerically differentiating the probability distributions obtained by Goto and Kameta. Fig.1 shows the probability distributions and densities for Tokyo and Kyoto for a period of 75 years. The expected values are 332 gal for Tokyo and 258 gal for Kyoto.

The distribution of AMF was obtained from 42 components of accelero-

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[II] H.Goto and H.Kameta; "Statistical Inference of the Future Earthquake Ground Motion", Proc. 4th WCEE, 1969, pp.A-1-28~ A-1-38c.

grams recorded in Japan. Though a number of factors may influence the AMF spectrum, it was assumed that the ground condition and the analyzed record length are the two most important factors. Regarding the ground condition, only those records obtained on the typical alluvial ground in Japan were analyzed, by discarding the records obtained on extremely soft or hard ground. The effect of analyzed record length was taken into account by using the empirical modification factors shown in Table 1. From the 12 horizontal components of the 1968 Tokachi-oki earthquake and its aftershock recorded at three stations, average acceleration response spectra were constructed for durations of 5, 10, 15, 20, 25, and 30 seconds, from which the factors in Table 1 were estimated.

The distributions of β for a damping of 5% were studied at 17 different natural periods between 0.1 and 4 seconds. The range between β_{min} and β_{max} was divided into seven equal intervals and the numbers of data in each interval were counted. Examples of the results are shown in Fig.2 by bar charts. Several trial calculations were made to find the most suitable relation to represent the observed distributions. It was found that, when a new variable is defined as

$$x = \sqrt{\beta - \beta_{min}} \qquad (3)$$

its distribution is approximately normal. Table 2 summarizes β_{min}, $\bar{x}$ (mean value of x), $\beta_{0.5}$ (value of β corresponding to $\bar{x}$), and σ_x (standard deviation of x) for the 17 values of natural period. Here, $\beta_{0.5}$ is the value of AMF with 50% probability of being exceeded. Fig.2 also shows by line diagrams the frequency distributions computed by assuming that x obeys the normal distribution. By knowing the probability distributions of β for various natural periods, the AMF spectra can be obtained for specified values of probabilities of being exceeded, as shown in Fig.3. The width of distribution is very large indicating the inadequacy of the present design method of using the average spectrum alone. The spectra of the ratios of each spectrum in Fig.3 to the spectrum of 50% probability of being exceeded are shown in Fig.4. It can be seen that the scatter of AMF at periods longer than say 1 second is larger than those at periods shorter than 0.5 seconds.

ACCELERATION RESPONSE SPECTRA WITH PROBABILITIES OF BEING EXCEEDED

By means of Eq.(2), the maximum response acceleration of a linear one-DOF system for a given probability of being exceeded can be determined. The acceleration response spectra thus obtained for Tokyo and Kyoto are shown in Figs.5 and 6, respectively. As expected, the acceleration values corresponding to small probabilities of being exceeded are seen to be very large. For Tokyo, when the natural period is about 0.5 seconds, the probability that the maximum response acceleration exceeds 0.2g in 75 years is 94% and the corresponding value for 0.5g is 64%. For the probability of being exceeded to be 10%. the maximum response acceleration is as high as 1.2g. Of course, the failure of a structure cannot be directly related to the instantaneous maximum acceleration computed for a linear one-DOF system. However, if a design were to be based on the maximum response for which the probability of being exceeded is say 10%, then the safety margin a structure is usually supposed to possess may not be enough for the design to be safe.

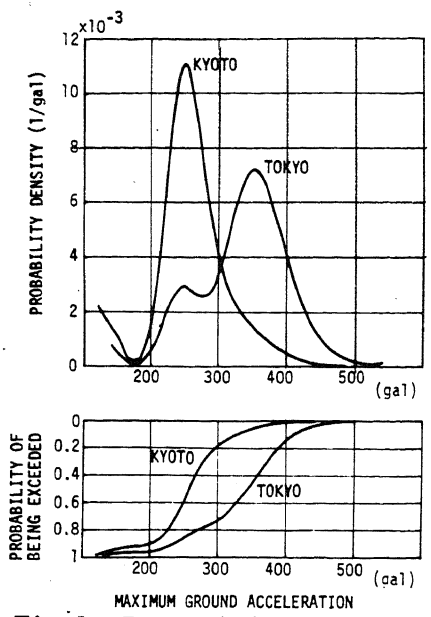

Fig.1 (↑) Probability Distribution and Density of Maximum Ground Acceleration in 75 Years for Tokyo and Kyoto.

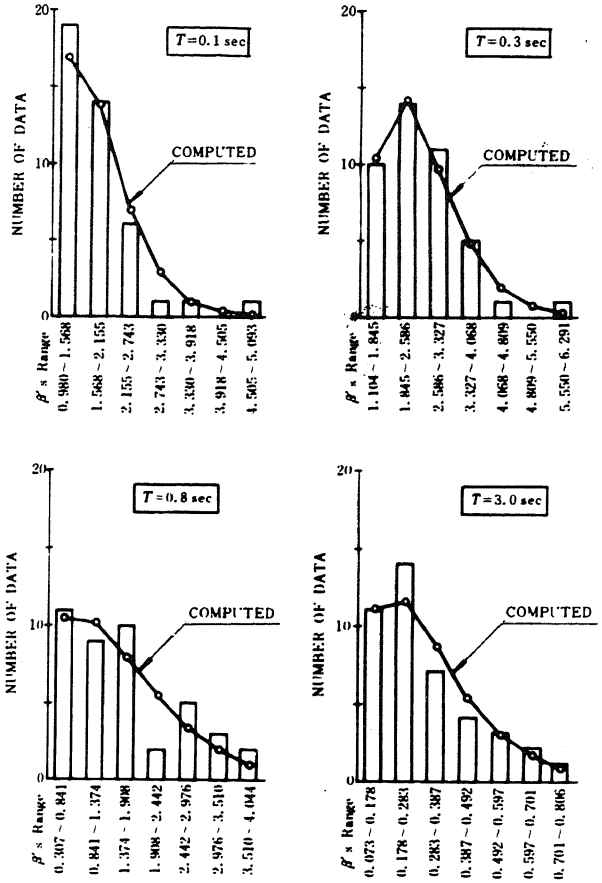

Fig.2 (→) Examples of Distribution of AMF at Given Natural Periods.

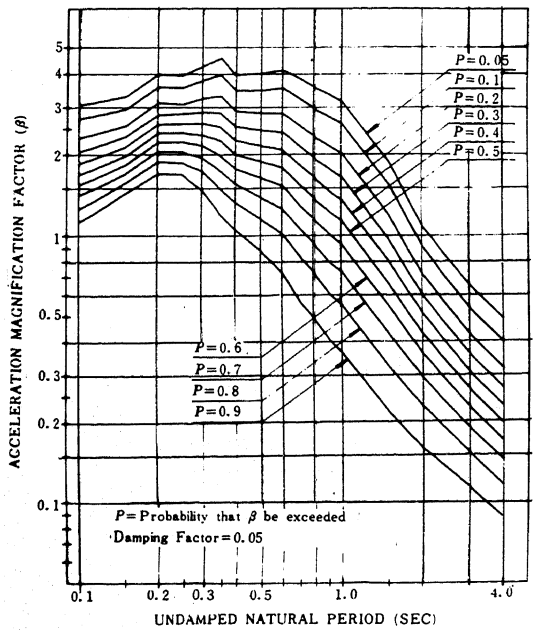

Fig.3 AMF Spectra with Specified Probabilities of Being Exceeded.

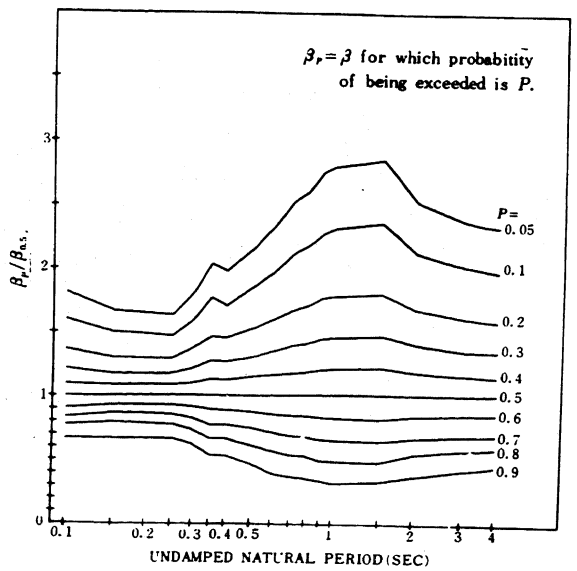

Fig.4 Values of $\beta_p/\beta_{0.5}$ at Different Periods.

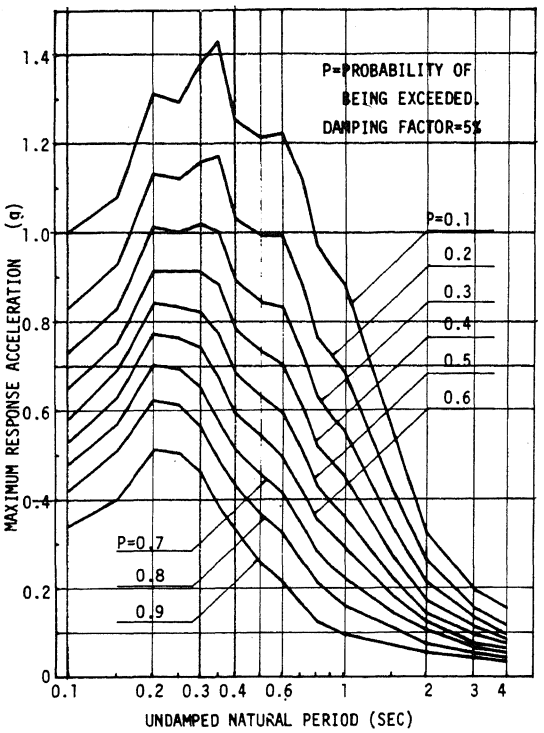

Fig.5 Acceleration Response Spectra for Tokyo in 75 Years.

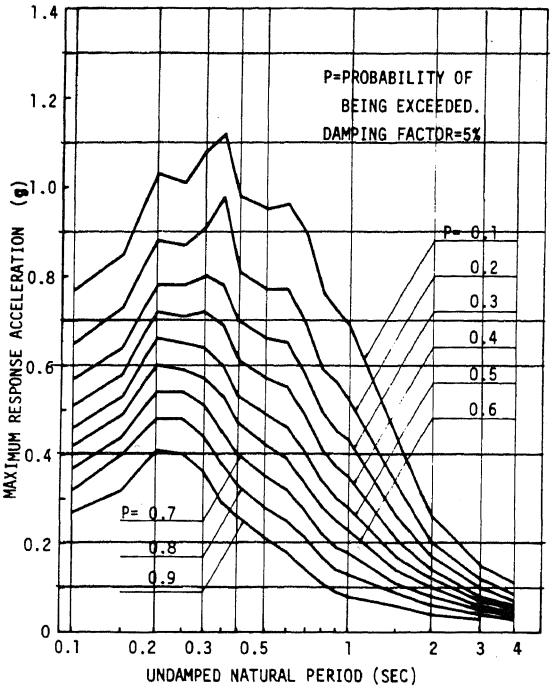

Fig.6 Acceleration Response Spectra for Kyoto in 75 Years.

Table 1 Multiplication Factor to Obtain Spectrum of 25 to 30 Sec Duration from Spectra of Shorter Durations.

		Ranges of Undamped Natural Period of One-DOF System (Sec)		
		.1<T≤.5	.5<T≤1	1<T≤4
Analyzed Record Length (Sec)	5	1.6	2.2	2.8
	10	1.1	1.3	1.7
	15	1.0	1.1	1.4
	20	1.0	1.1	1.2

Table 2 Summary of β_{min}, $\bar{x}$, $\beta_{0.5}$ and σ_x.

Undamped Natural Period(Sec)	β_{min}	$\bar{x}$	$\beta_{0.5}$	σ_x
0.1	0.980	0.846	1.696	0.367
0.15	1.312	0.813	1.974	0.363
0.2	1.452	0.978	2.409	0.374
0.25	1.267	1.073	2.418	0.343
0.3	1.104	1.129	2.378	0.398
0.35	0.796	1.208	2.254	0.449
0.4	0.609	1.177	1.995	0.398
0.5	0.451	1.192	1.872	0.424
0.6	0.302	1.217	1.783	0.450
0.7	0.309	1.106	1.532	0.472
0.8	0.307	1.023	1.353	0.468
0.9	0.274	0.972	1.219	0.476
1.0	0.253	0.936	1.128	0.467
1.5	0.163	0.708	0.664	0.370
2.0	0.093	0.578	0.427	0.252
3.0	0.073	0.437	0.263	0.186
4.0	0.063	0.366	0.197	0.160