DIGITAL SIMULATION OF GROUND ACCELERATIONS

by

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SYNOPSIS

In view of the recent trend in which numerical analysis of dynamic structural response, particularly to strong-motion earthquakes, is becoming more and more routine, development of techniques for digital simulation of ground accelerations appears to be an important engineering task. With this in mind, this study (a) reviews the current state of art in digital simulation of earthquake accelerations with a particular emphasis on a systematic classification of the stochastic simulation models proposed so far and (b) describes a simulation technique which is recently developed by the author and can generate sample functions of a multivariate process.

INTRODUCTION

A number of stochastic models for artificial ground accelerations have recently been proposed and applied successfully to a variety of structural problems arising from ground motion. All of these models simulate the ground accelerations as one-dimensional one-variate random process, some stationary and others nonstationary. They can also be classified either as the filtered white noise model or as the filtered Poisson process model. The former produces Gaussian processes because of the basic white noise being Gaussian whereas the latter is usually non-Gaussian. It will be pointed out that these two models, stationary or nonstationary, can be made identical up to second moment (the expected value and the covariance function) and in fact under certain conditions they can be made asymptotically identical.

The nonstationarity can be introduced into either of these models without difficulty. In case of the filtered white noise model, the nonstationarity can be achieved by multiplying either the basic (stationary) white noise or

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the filtered (stationary) process by a deterministic envelope function. Basically, the same procedure can be used to introduce the nonstationarity to the filtered Poisson process model. All the stochastic models for artificial earthquakes proposed so far belong to these models. Two exceptions are (a) the model with evolutionary spectral density and (b) the model consisting of a sum of sinusoidal functions with random phase and deterministic envelope functions.

One major difficulty associated with all the models mentioned above lies in the fact that these do not have capabilities to generate vector processes with specified statistical characteristics that are to be maintained among the component processes. The ground acceleration is basically a three-variate process involving two horizontal components (e.g. N-S and E-W components) and the vertical component. As the structure becomes more complex and the structural analysis becomes more sophisticated, the necessity arises to generate sample functions of these component processes, for example, for the purposes of the Monte Carlo analysis. A technique will be described to accomplish this when the cross-correlation function matrix or equivalently cross-spectral density function matrix of the process is known. The nonstationarity can also be introduced in this case without much difficulty.

ONE-VARIATE ONE-DIMENSIONAL MODELS

With only few exceptions, the stochastic models proposed for nonstationary ground acceleration g(t) caused by strong-motion earthquakes can be classified either as the filtered white noise model;

$$g_1(t) = \int_{-\infty}^{\infty} h(t-\tau)\ \Psi(t)\ n(\tau)d\tau = \Psi(t)\ x_1(t) \tag{1}$$

or

$$g_2(t) = \int_{-\infty}^{\infty} h(t-\tau)\ \Psi(\tau)\ n(\tau)d\tau \tag{2}$$

or as the filtered Poisson process model (1);

$$g_3(t) = \sum_{n=-\infty}^{\infty} A_n \Psi(t) h(t-t_n) = \Psi(t) x_2(t) \tag{3}$$

or

$$g_4(t) = \sum_{n=-\infty}^{\infty} A_n \Psi(t_n) h(t - t_n) \tag{4}$$

In Eqs. 1 and 2, n(t) is a Gaussian white noise with spectral density S, h(t) is the impulse response function of a linear, time-invariant filter whose Fourier transform $H(\omega)$ specifies the shape of spectral density function ($= S|H(\omega)|^2$) of a stationary process

$$x_1(t) = \int_{-\infty}^{\infty} h(t - \tau)\; n(\tau) d\tau \tag{5}$$

and $\Psi(t)$ is a deterministic function of time serving as the envelope to a stationary process $x_1(t)$ in the model given in Eq. 1 and to n(t) in Eq. 2, thus making both processes nonstationary. The difference in the results of structural analysis using Eqs. 1 and 2 appears (2) to be insignificant, if any. Both models will implicitly retain the stationarity of the shape of the spectral density if $\Psi(t)$ is chosen to be a slowly varying function of time.

In Eqs. 3 and 4, the sequence $\{\ldots, t_{-1}, t_0, t_1, \ldots\}$ indicates Poisson arrival times with arrival rate λ and $\{\ldots, A_{-1}, A_0, A_1, \ldots\}$ is a sequence of mutually independent and identically distributed random variables A_n with mean zero and variance σ^2. Defining

$$z_1(t) = \sum_{n=-\infty}^{\infty} A_n \delta(t - t_n) \tag{6}$$

as a series of Poisson impulses with arrival rate λ and random amplitudes A_n,

$$x_2(t) = \sum_{n=-\infty}^{\infty} A_n h(t - t_n) \tag{7}$$

can be considered as the output of a linear filter with the impulse response h(t). Obviously then, if one considers series $z_3(t)$ and $z_4(t)$ of Poisson impulses with arrival rate λ and random amplitudes in the form of either $A_n\Psi(t)$ or $A_n\Psi(t_n)$,

$$z_3(t) = \sum_{n=-\infty}^{\infty} A_n \psi(t) \delta(t - t_n) \qquad (8)$$

$$z_4(t) = \sum_{n=-\infty}^{\infty} A_n \psi(t_n) \delta(t - t_n) \qquad (9)$$

the (nonstationary) filtered Poisson processes $g_3(t)$ and $g_4(t)$ are obtained respectively as outputs of the same linear filter to $z_3(t)$ and $z_4(t)$. As in Eq. 1, $g_3(t)$ can be interpreted as a nonstationary filtered Poisson process obtained by multiplying a stationary filtered Poisson process $x_2(t)$ by a deterministic function of time $\psi(t)$, whereas $\psi(t)$ in Eqs. 4 and 9 introduces a physically acceptable time-dependence into the random amplitude of the underlying Poisson impulses.

It can be shown (2) that

$$E\{g_i(t)\} = 0 \quad (i = 1,2,3,4) \qquad (10)$$

and that if

$$\lambda\sigma^2 = 2\pi S \qquad (11)$$

the autocovariance functions of $g_i(t)$ are such that

$$E\{g_1(t)g_1(s)\} = E\{g_3(t)g_3(s)\}$$

$$= 2\pi S \ \psi(t)\psi(s) \int_{-\infty}^{\infty} h(t - \tau)h(s - \tau)d\tau \qquad (12)$$

$$E\{g_2(t)g_2(s)\} = E\{g_4(t)g_4(s)\}$$

$$= 2\pi S \int_{-\infty}^{\infty} h(t - \tau)h(s - \tau)\psi^2(\tau)d\tau \qquad (13)$$

where $E\{\cdot\}$ indicates expectation. Eqs. 12 and 13 indicate that the filtered Poisson processes and the filtered white noise processes as defined above can be made identical up to the second moment by imposing a condition given in Eq. 11. It can further be shown (2) that (a) the processes $g_1(t)$ and $g_2(t)$ are Gaussian and (b) although $g_3(t)$ and $g_4(t)$ are in general non-Gaussian, they are asymptotically Gaussian as $\lambda \to \infty$ with $\lambda\sigma^2$ kept constant.

Some of the stochastic models of ground accelerations that have been proposed and can be classified as one of the four models in Eqs. 1-4 are presented here.

(i) Tajimi (3): $x_1(t)$ (stationary model) or $g_1(t)$ with $\Psi(t) = 1.0$.

$$h(t) = \left\{\omega_g^2 \sqrt{1 - 2\zeta_g^2}\, e^{-\zeta_g \omega_g t} \sin\omega_g' t/\omega_g' + 2\zeta_g\omega_g\, e^{-\zeta_g\omega_g t} \cos \omega_g' t\right\} H(t) \tag{14}$$

$$|H(\omega)|^2 = \frac{\omega_g^4 + 4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \tag{15}$$

where H(t) is the Heaviside unit step function and $\omega_g' = \sqrt{1 - \zeta_g^2}\,\omega_g$.

(ii) Housner and Jennings (4): $g_1(t)$ with h(t) as given in Eq. 14 and

$$\Psi(t) = H(t) - H(t - t^*) \tag{16}$$

(iii) Iyengar and Iyengar (5): $g_1(t)$ with

$$\Psi(t) = (a_1 + a_2 t)e^{-pt^n} \quad (n = 1 \text{ or } 2) \tag{17}$$

$$|H(\omega)|^2 = e^{-c^2\omega^2} + A\omega^2 e^{-4c^2\omega^2} \tag{18}$$

(iv) Amin and Ang (6): $g_2(t)$ with

$$2\pi S\Psi^2(t) = \begin{cases} 0 & t < 0 \\ I_0\left(\frac{t}{a}\right)^2 & 0 \le t \le T_1 \\ I_0 & T_1 \le t \le T_2 \end{cases} \tag{19}$$

$$\begin{cases} I_0 e^{-ct} & T_2 \le t \end{cases}$$

$$h(t) = e^{-\zeta_g \omega_g t} \sin\omega_g' t H(t)/\omega_g' \tag{20}$$

(v) Ruiz and Penzien (7): $g_2(t)$ with $h(t)$ as given in Eq. 14 and

$$2\pi S\Psi^2(t) = \begin{cases} 0 & t < 0 \\ \varphi_0 & 0 \le t \le t_0 \\ \varphi_0 e^{-c(t - t_0)} & t_0 < t \end{cases} \tag{21}$$

(vi) Cornell (8): $g_4(t)$ with

$$\Psi(t) = e^{-\alpha t} H(t) \tag{22}$$

$$\begin{aligned} h(t) &= \sin \frac{2\pi}{L}(t - \tau) \quad && 0 \le t - \tau \le L \\ &= 0 && \text{otherwise} \end{aligned} \tag{23}$$

(vii) Shinozuka (2): $g_1(t), g_2(t), g_3(t)$ and $g_4(t)$ as ground velocity with

$$\Psi(t) = (e^{-\alpha t} - e^{-\beta t}) H(t) \tag{24}$$

$$h(t) = e^{-\zeta_g \omega_g t} \sin \omega_g' t\, H(t)/\omega_g' \tag{25}$$

The filtered Poisson process model is more amicable to plausible physical interpretations (2, 8) than the filtered white noise process. In terms of digital simulation, however, the direct use of the filtered Poisson process model as indicated in Eqs. 3 and 4 requires a summation and digital generations of Poisson arrival times and random amplitudes, and it is usually costly. Therefore, even when the filtered Poisson process model appears preferable in terms of physical interpretation, sample functions of the process are usually generated, as in (6, 7), with the aid of corresponding filtered white noise model under the assumption that the condition in Eq. 11 is satisfied or under the premise that it is good enough for the purpose of engineering analysis to use the filtered white noise model in place of the filtered Poisson model since these two produce identical first two moments. Use of Eqs. 1 and 2 requires a digital generation of independent Gaussian numbers and an integration (a summation). From the viewpoint of structural application, it is important to note that, as long as linear responses are considered, these two models of ground accelerations produce structural responses which are identical, asymptotically if Eq. 11 is satisfied and up to the second moment otherwise.

It can be shown (2) that $g_4(t)$ in Eq. 4 is equivalent to

$$g_4^*(t) = \sum_{n=-\infty}^{\infty} A_n h(t - t_n) \tag{26}$$

if the underlying Poisson process has the nonstationary arrival rate $\lambda\Psi^2(t)$. This fact has directly been used by Lin to produce a nonstationarity in his model (9) for ground acceleration. In this respect, the models (iv) and (v) above can be interpreted, under a condition similar to Eq. 11, as filtered Poisson processes either with a constant arrival rate λ and random amplitudes $A_n\Psi(t_n)$ or with a nonhomogeneous arrival rate $\lambda\Psi^2(t)$ and random amplitudes A_n, where $\Psi(t)$ for model (iv) is given in Eq. 19 and for model (v) in Eq. 21.

It appears at this time that the relatively simple model proposed by Housner and Jennings (4) with appropriate choices of earthquake duration t^* (say 10 - 30 sec) and h(t), proves to be practical in many applications, although more sophisticated nonstationary models described above should not be excluded in future use, particularly when such a sophistication becomes justifiable due to gradually increasing amounts of physical and statistical information on strong-motion earthquakes.

Exceptions to the models indicated above include the form

$$g_5(t) = \sqrt{2} \sum_{k=1}^{N} A_k \cos(\omega_k t + \varphi_k) \tag{27}$$

where N = positive integer, φ_k = random variables independently and identically distributed with uniform density between 0 and 2π, $S(\omega)$ = one-sided spectral density of the process for which $g_5(t)$ is a simulation with ω_c being the frequency beyond which $S(\omega)$ is insignificant,

$$A_k = \sqrt{S(\omega_k)\Delta\omega} \quad , \quad \omega_k = k\Delta\omega \quad , \quad \Delta\omega = \omega_c/N \tag{28}$$

If $g_5(t)$ is used as an input to a linear system with the frequency response $K(\omega)$, the output is obtained, without performing the time domain integration, in the same form as Eq. 27 with A_k replaced by $A_k|K(\omega_k)|$ and φ_k by $\varphi_k + \delta_k$ where δ_k is the argument of $K(\omega)$. It can be shown (10, 11) that $g_5(t)$ is ergodic at least up to the second moment with mean zero and that its spectral density converges to $S(\omega)$ as $N \to \infty$. Sample functions of $g_5(t)$ can very easily be obtained by replacing φ_k by their realizations. As indicated in (11), the FFT technique can be applied to Eq. 27 for this purpose, and will drastically reduce the computer time for generation of sample functions. Eq. 27 and its generalization into multidimensional cases have been successfully used in a variety of engineering problems (see references cited in (10, 11)). A slight modification of Eq. 27 produces a simulated oscillatory random process with evolutionary power spectral density (10, 11, 12) whose sample functions can easily be generated digitally.

BIVARIATE MODEL

There is no reason to believe that the two horizontal components and the vertical component of ground accelerations are statistically independent. A possible way in which the dependence among the components can be reproduced in the digital simulation has been described elsewhere (10, 11) in detail. The method requires stationarity of the component processes and the knowledge of cross-spectral density matrix $[S_{ij}]$ (or equivalently cross-correlation function matrix). In this paper, however, only a bivariate model is presented for the sake of brevity.

Suppose that $f_1(t)$ and $f_2(t)$ are stationary models with mean zero of one of horizontal components and vertical component respectively. Then, the two components can be simulated as

$$f_1(t) = \sqrt{2\Delta\omega} \sum_{k=1}^{N} H_{11}(\omega_k)\cos(\omega_k t + \varphi_{1k}) \tag{29}$$

$$f_2(t) = \sqrt{2\Delta\omega} \sum_{k=1}^{N} \Big\{H_{22}(\omega_k)\cos(\omega_k t + \varphi_{2k}) + |H_{21}(\omega_k)|\cos[\omega_k t + \theta(\omega_k) + \varphi_{1k}]\Big\} \tag{30}$$

where

$$\left.\begin{aligned} &H_{11} = \sqrt{S}_{11},\ H_{22} = \sqrt{(S_{11}S_{22} - |S_{12}|^2)/S_{11}} \\ &H_{21} = S_{21}/H_{11},\ \Delta\omega = \omega_c/N,\ \omega_k = k\Delta\omega \\ &\varphi_{1k}, \varphi_{2k} = \text{uniformly distributed random numbers in } (0, 2\pi) \\ &\quad \theta = \tan^{-1}\frac{\mathrm{Im}H_{21}}{\mathrm{Re}H_{21}} \end{aligned}\right\} \quad (31)$$

In Eq. 31, ω_c is the frequency beyond which none of S_{ij} is significant. It is pointed out that even in this case, a nonstationarity can easily be introduced with the aid of an envelope function. Eqs. 29 and 30 are used in investigating the effect of self-weight and vertical acceleration on the behavior of tall structures during earthquake (13).

ACKNOWLEDGMENT

This work was supported by National Science Foundation under Grant GK-24925.

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