PROBABILISTIC SEISMIC ANALYSIS OF LIGHT EQUIPMENT WITHIN BUILDINGS

by

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SYNOPSIS

Major buildings often provide support for relatively light secondary systems (equipment) whose continued performance during earthquakes is essential to safety. Presented in this paper is a nonstationary random vibration analysis which yields the statistical properties of the equipment displacement relative to the motion of the structure at the point at which the equipment is attached. It is assumed that the results of a classical modal analysis of the structure are available, and that the equipment can be modeled as a linear one-degree oscillator. The method presented herein in fact generates the probability distribution of so-called floor response spectra.

ANALYSIS

It is common to separate the seismic analysis of secondary systems (equipment) from that of the primary system (a building) and to view the structural response acceleration at a particular point A (see Fig. 1) as input for the dynamic analysis of equipment supported at that point. Thus we consider a multi-degree-of-freedom structure with an attached single-degree-of-freedom viscously damped oscillator representing equipment. The entire system is excited by a ground motion $\ddot{U}(t)$, which is represented by a suddenly applied weakly stationary Gaussian process whose spectral density and duration are specified. It is assumed that a classical modal analysis of the structure is possible and that the effect of the equipment vibration on the structure (i.e., interaction effect) is negligible. The structural response, $Y(t)$, at the point of attachment can be expressed in terms of the normal modes ϕ_j and the modal coordinates $\eta_j(t)$, j=1 to n, i.e., $Y(t) = \Sigma\ \phi_j\eta_j(t)$. Each component $\eta_j(t)$ represents the output of a single-degree-of-freedom system characterized by its modal frequency Ω_j and modal damping ζ_j, and is exposed to a modal forcing function $-\Gamma_j\ddot{u}(t)$, where Γ_j is the participation factor of the j^{th} mode. The dynamic behavior of the equipment displacement, $z(t)$, relative to its point of support, will depend upon the equipment natural frequency Ω_e and damping ζ_e, and on $\ddot{X}(t)$, the absolute acceleration of the equipment support. The latter can be expressed in terms of the impulse response functions of the individual structural modes (1).

The time-dependent (or evolutionary) spectral density (2) $G_z(\omega,t)$ which characterizes the equipment response $z(t)$ can be derived from an expression of the nonstationary mean square response, $\sigma_z^2(t)$, of the equip-

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ment (1). $G_z(\omega,t)$ indicates how the mean square of z(t) is distributed over different frequencies at different times, i.e., how the frequency content of the equipment response varies with time. In Reference 1, exact analytical expressions are obtained for $G(\omega,t)$. Integration over all frequencies of this function yields an algebraic expression for the time-dependent mean square value $\sigma_z^2(t)$. Other spectral moments are also computed and used in (i) a quantitative study of the variation in time of the narrowness of the frequency content of the equipment response, and (ii) an approximate evaluation of the probability distribution of the floor response spectra, i.e., a plot of the maximum value of the equipment response as a function of Ω_e for a given value of ζ_e. All results depend in a relatively simple way on the building's natural periods, modal damping ratios and participation factors, the ground motion statistics and the equipment period and damping ratio. The median maximum response is expressed as $r\,\sigma_x(s)$, in which s = motion duration, and r = a factor which depends upon the spectral moments $\lambda_i(t) = \int_0^\infty \omega^i G(\omega,t)d\omega$, i = 0, 1, 2. Note that $\lambda_o(t) = \sigma_x^2(t)$. An important related function is $q(t) = [1-\lambda_1^2(t)/\lambda_o(t)\lambda_2(t)]^{1/2}$: it is unitless measure of the narrowness of the frequency content of z(t) at time t, and ranges between 0 and 1 (3,4).

RESULTS AND APPLICATIONS

Time Dependent Spectra and Mean Square Values

Some typical time-dependent spectral densities and mean square value functions are shown in Figs. 2 & 3 . The parameters employed in the study are the frequency ratio r_{1e} (ratio of structural frequency to equipment frequency), the time variable $\Omega_e t$ and the two damping ratios ζ_1 and ζ_e. In this particular computation both the structure and the equipment are treated as single-degree-of-freedom systems. The time-dependent power spectral density, $G_z(\omega,t)$ for different values of the parameters appears in Fig. 2. The shape of the spectrum during the initial stages (i.e. for low values of $\Omega_e t$) is similar to that of a wide-band process. It becomes narrower and more peaked at the resonant frequencies when time increases. This time-dependent nature of the power spectrum influences the computation of the distribution of the maximum response, through q(t). When the frequency ratio $r_{1e} = \Omega_1/\Omega_e$ is very close to 1, a strong amplification of the response spectral density due to resonance phenomenon is observed. This indicates that most of the energy is distributed around these two frequencies.

Fig. 3 shows how the mean square response, $\lambda_o(t)$, varies with time. It indicates that the time-dependent nature of the equipment response is important. In particular, the transient growth of the equipment response is quite sensitive to (i) the duration of the excitation, (ii) the frequency of the structure and the equipment, (iii) the damping ratios ζ_j and ζ_e.

Floor Response Spectra

Floor response spectra are currently generated in one of the following two ways: (i) by a time-history analysis of the structure which produces the detailed motion of the point at which the equipment is mounted from which a response spectrum is constructed; and (ii) by an extension of the response spectrum method utilizing only the peak modal responses of the structure (5). Both procedures have the same deficiencies previously attributed to time-history and response spectra-based analyses of multi-degree structures, undoubtedly amplified by extending them to two systems in series. Most of these deficiencies are not shared by the direct random vibration approach.

The validity of the proposed method was tested by making a comparison between the predicted median floor response spectra and the average of four (4) floor response spectra curves, each computed by using a time-history analysis based on a single sample function of the site ground motion process. A common spectral density function provided the input to both the artificial motion generation routine and to the probabilistic dynamic analysis. The comparison was made for two different multi-degree-of-freedom representations of the primary structure. The modal parameters of Structures I and II are listed in Figs. 4 and 5, respectively. These figures also show that the agreement between the spectra obtained by the two methods is quite satisfactory over the entire range of equipment periods considered.

REFERENCES

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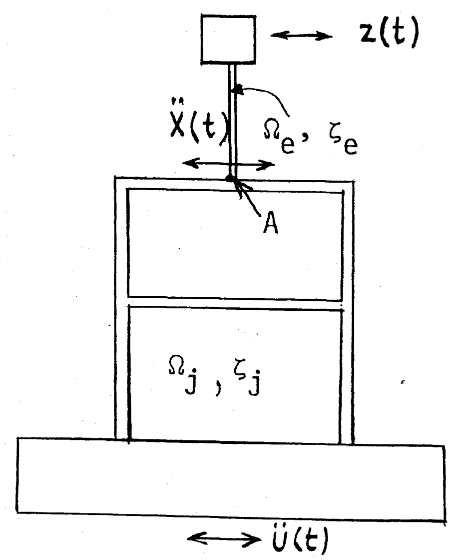


Fig. 1 System Representation

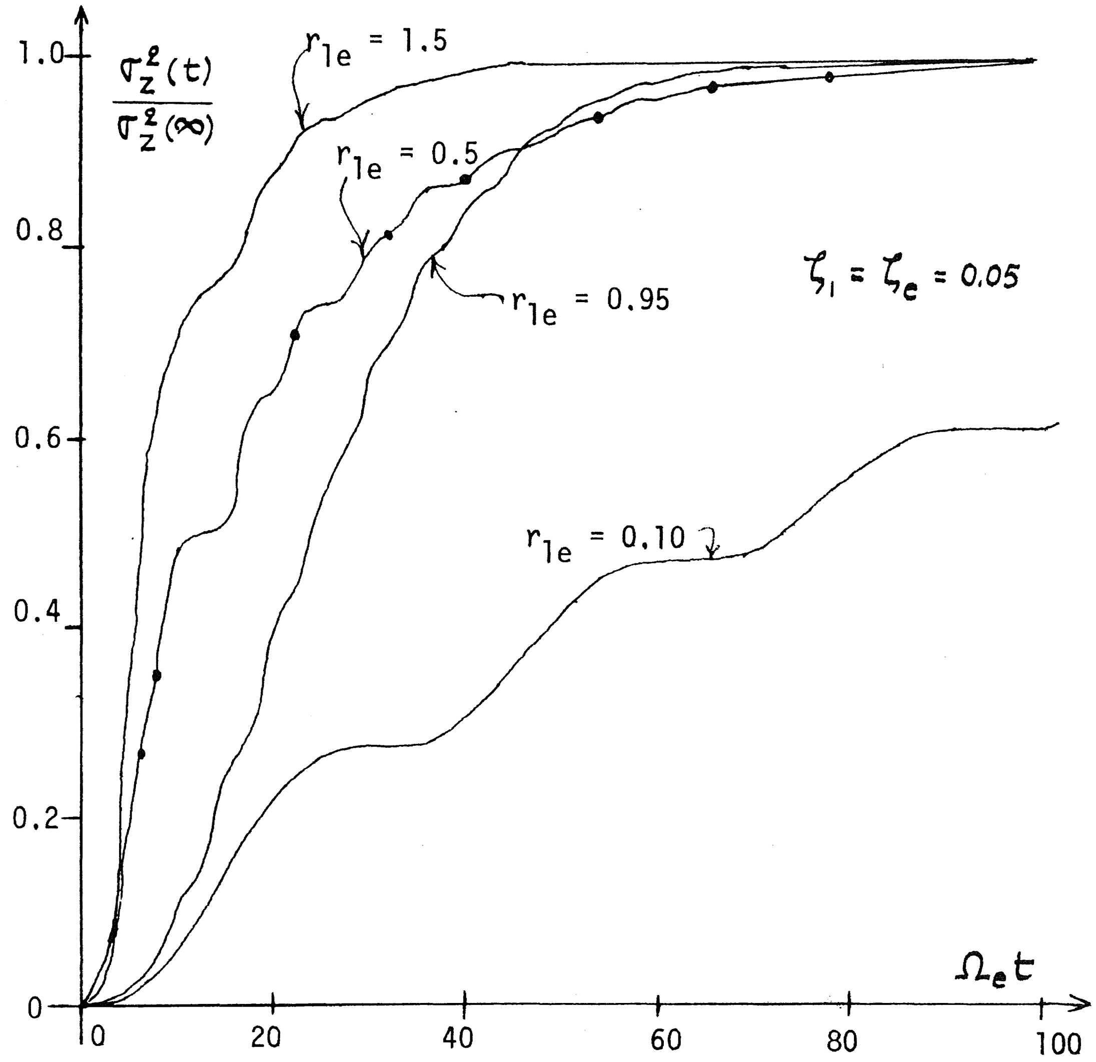


Fig. 3 Normalized Mean Square Equipment Response

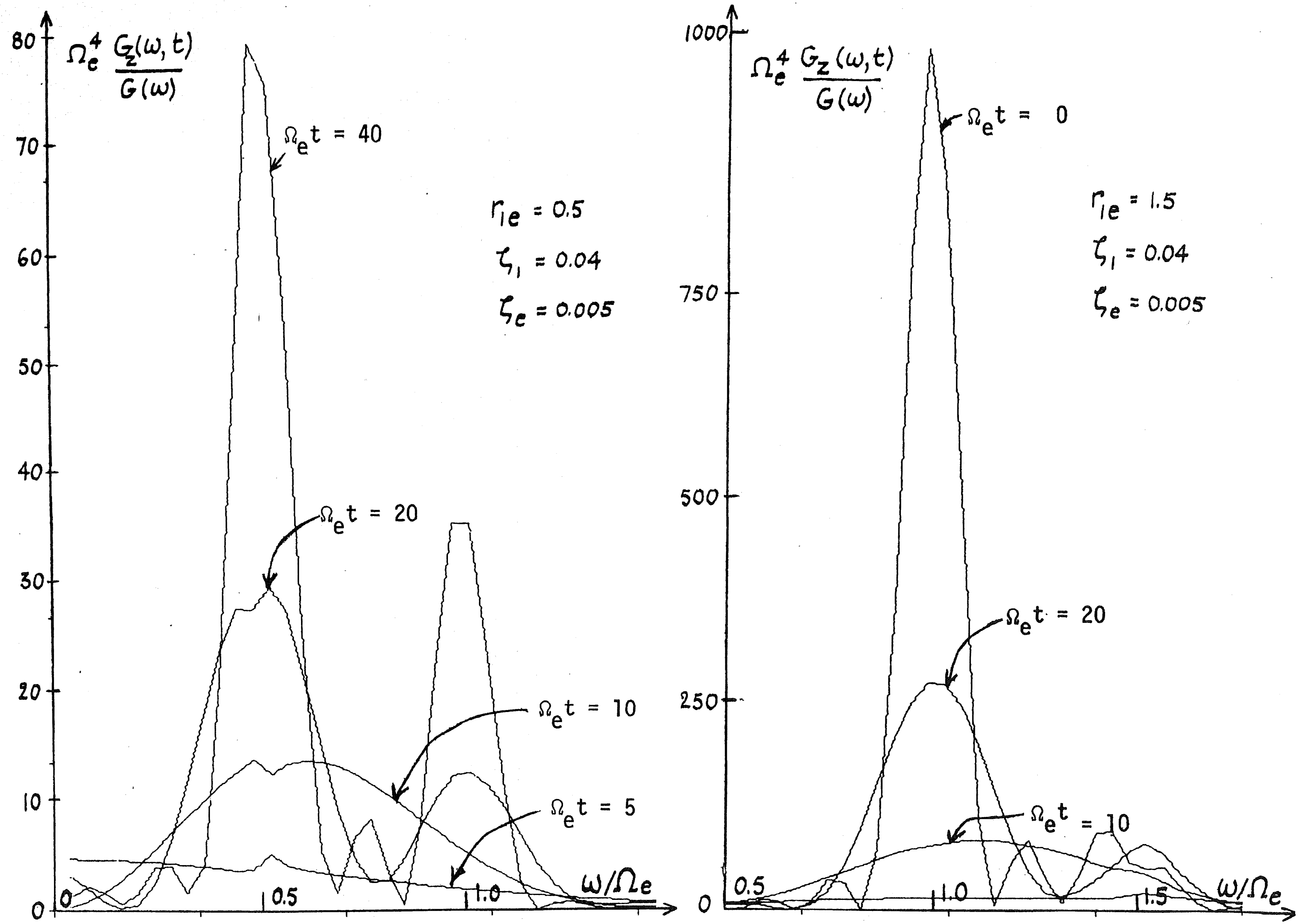
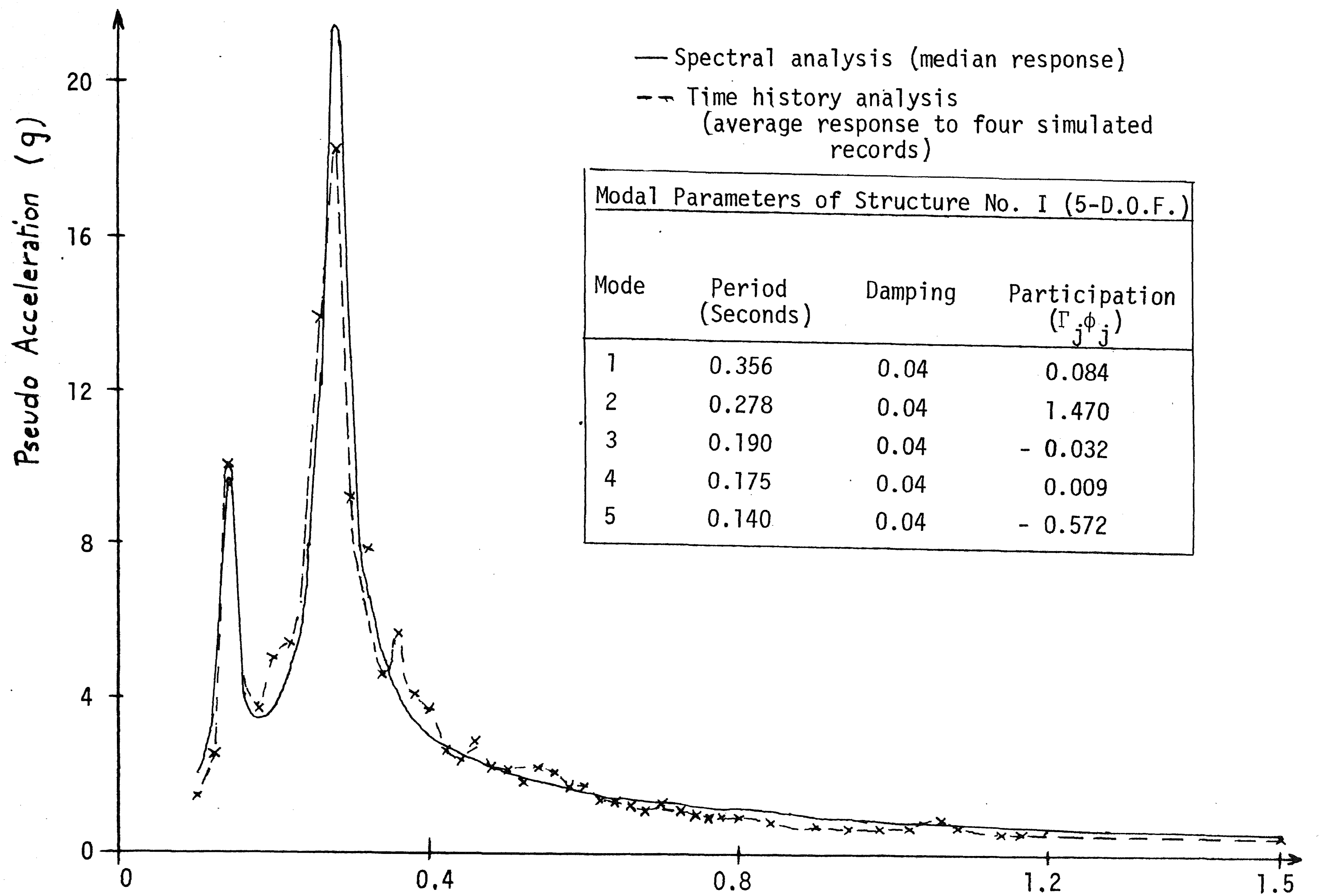


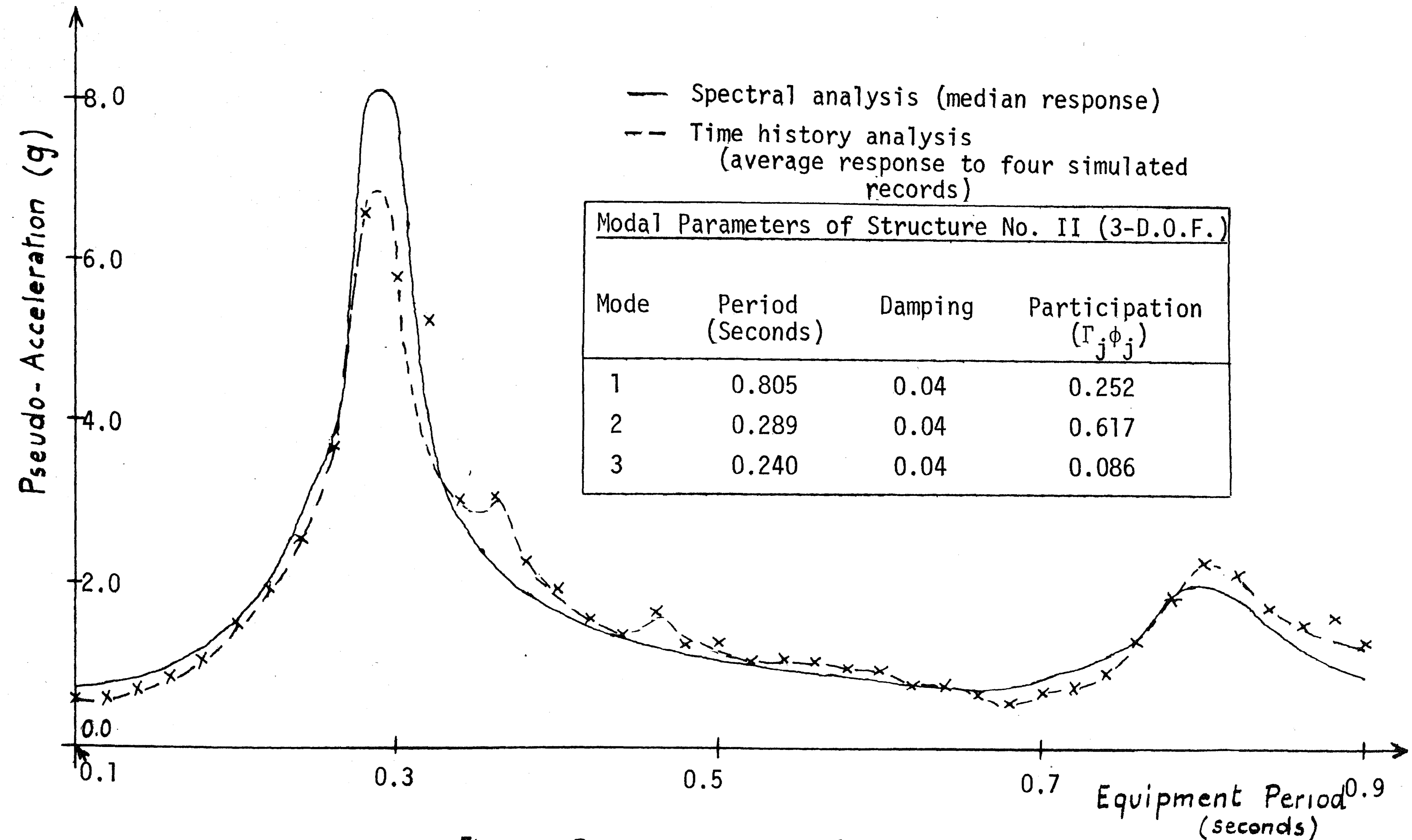
Fig 2 : Time - Dependent Spectral Density of Equipment Response



Modal Parameters of Structure No. I (5-D.O.F.)

Mode	Period (Seconds)	Damping	Participation ($\Gamma_j \phi_j$)
1	0.356	0.04	0.084
2	0.278	0.04	1.470
3	0.190	0.04	- 0.032
4	0.175	0.04	0.009
5	0.140	0.04	- 0.572

Fig. 4 Response Spectra for Equipment in Structure No. I



Modal Parameters of Structure No. II (3-D.O.F.)

Mode	Period (Seconds)	Damping	Participation ($\Gamma_j \phi_j$)
1	0.805	0.04	0.252
2	0.289	0.04	0.617
3	0.240	0.04	0.086

Fig.5 Response Spectra for Equipment in Structure No. II